Data Analisys and Decision Making – II
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Lecture – 52
GERT

A very warm and a good morning, good afternoon, good evening to all of you wherever you are in this part of this globe. And, now once again welcome to all my dear friends and students in this DADM II course which is Data Analysis and Decision Making II course under the NPTEL, MOOC series.

And, as you know this course duration is for 12 weeks which is 30 hours of contact in the video mode. And this total number of lectures is 60 because each lecture is for half an hour and each week we have 5 classes or lectures, each being for a half an hour and after each 5; that means, after the first week then second week so on and so forth we have assignments. So, if you can notice on the slide we are in the 11th week which is the and we are doing the 52 lecture. And we have already completed these 10 weeks which is 50 lectures and we have solved 10 such assignments.

So, if you remember in the 51st lecture which was the 1st lecture in the 11th week we are discussing on GERT and the launching of the spacecraft with considering the boost rockets. And we are considering two different scenarios; in 1; obviously, in both the cases boost rocket 1 and 2 can be success or unsuccessful that is four combinations, but in the first case we will consider the success-success leads to the actual success of the project.

And, in the second case and; obviously, it will mean there are three unsuccessful ones; that means, one success two failure and this third case being first unsuccessful, second been a success and in the fourth case both being failures. In case 2, we will consider that any one of them being successful is a success for the whole project which means if 1 is successful, 2 is successful, it is a success and in another case 1 success 2 failure; third case 1 failure 2 success and the last case both being failures would all lead all these three things would lead to the failure of the project.

So, we are considering the concept of preparatory reactions which are to be taken for the launch. So, there can be different type of preparatory actions some probabilistic are deterministic.

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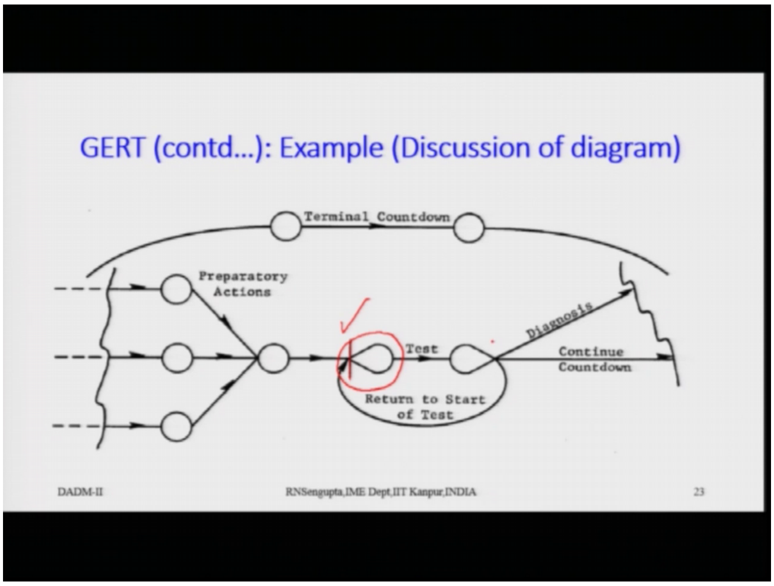

And, based on that we return to the overall known where they return to the start of the test would be performed in such a way. So, that means, we if you if you remember and let me copy it again. Again, as I said it would be easier for us to consider I will take it from, yes. So, if you consider this concept here.

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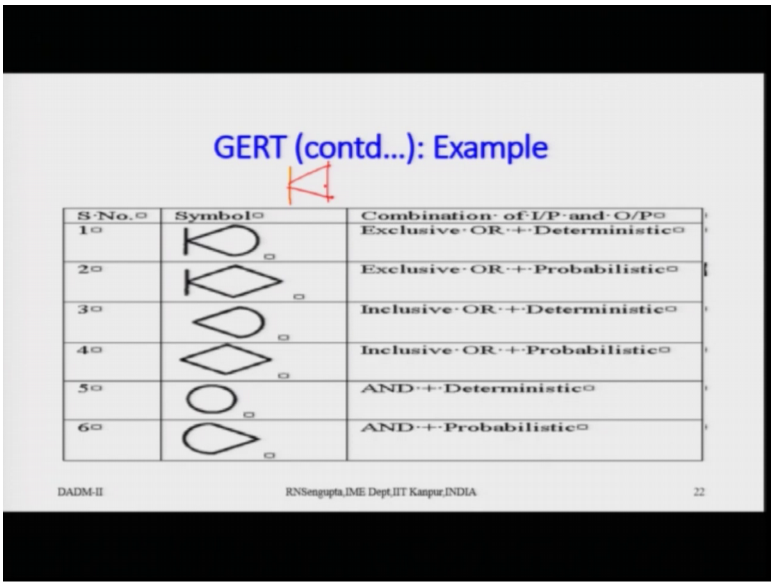

So, this is an INCLUSIVE-OR a deterministic one depending on how the nodes have been formed. And in this case also is INCLUSIVE-OR written, in this case it is an OR 1 it says it is a OR is not there; it means it will be the first case would be EXCLUSIVE-OR and the second case would be inclusive-OR. So, it is EXCLUSIVE-OR and this is an inclusive-OR. Based on that you proceed and finish the concept of how the diagram is made.

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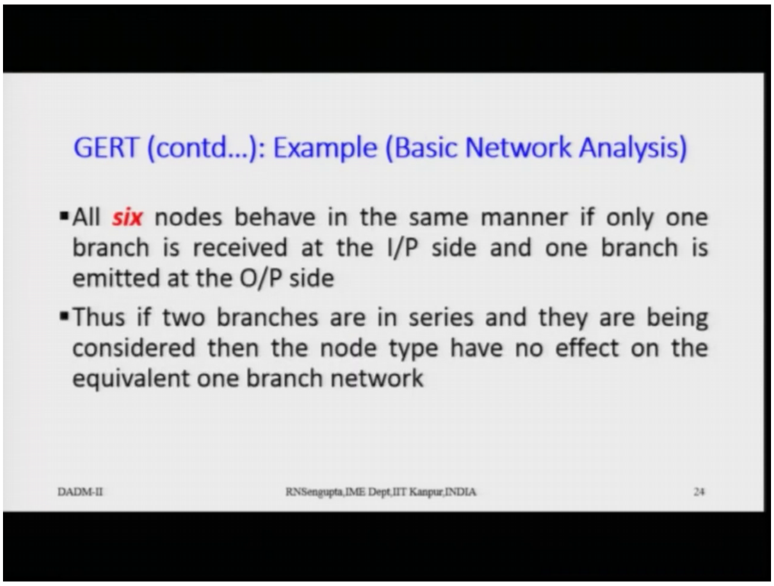

Now, for this all the six nodes; that means, which have been drawn behave in the same manner if and only if the branches received at an input side and one branch is emitted from the output side. So, six are coming based on the output this one.

Thus if two branches are in the series and they are being considered, then the node type have no effect on the equivalent one branch because series means what any one of them working would lead to the output being positive. If it is parallel in a series 1 sorry, in the series 1 my mistake in the series 1 if we have two consecutive concepts which are working and if any one of them fail, so obviously, that concept would launching of that this spacecraft is not possible. In the parallel one, if we consider in the simplistic sense any one of the in success would basically lead it lead us to the suspect success of the project.

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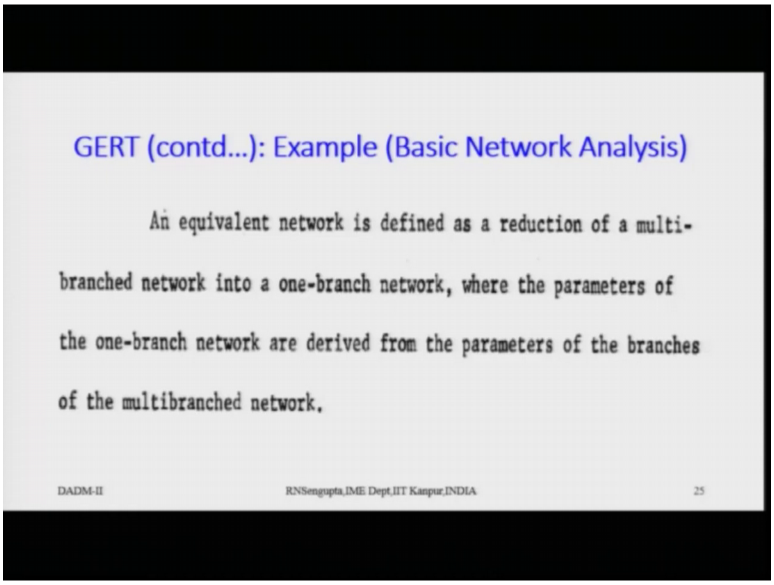

Now, we will basically have to convert them into the equivalent networks. So, an equivalent network would be defined as a reduction of the multi-branched network which we already have into the basic AND, OR, exclusive-OR, INCLUSIVE-OR nodes and network such that the complexity of the diagram is reduced, yet it basically has the same logical sequence of the output and the input combined together in the same way as it was earlier; earlier means before you reduced.

So, let me continue reading it, an equivalent network is defined as a reduction of a multi-branched network into one branch network, where the parameters of the one branch network are derived from the multiple network which you have. So, if you remember we consider the time which was deterministic or probabilistic and for trying to find out the time we consider the moment generating function and the for the probabilistic feedback the probability of that branch being taken was also considered.

So, we will combine both the moment generating function to find out the time as well as the probabilities to find out the probabilities of which path will be taken. So, you will combine them from the overall complicated network in the equivalent simplest form of the network.

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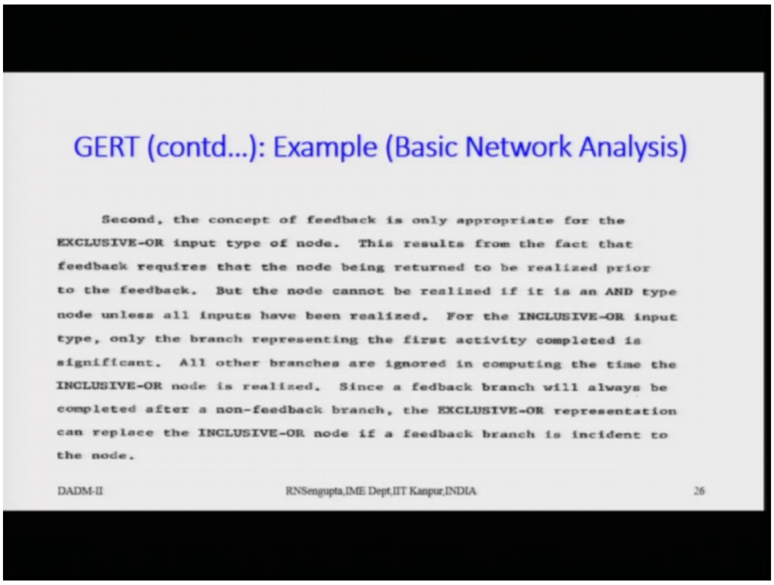

Second case would be the concept of a feed is only appropriate for the EXCLUSIVE-OR input output of the node. So, EXCLUSIVE-OR if you remember, so, you have the EXCLUSIVE-OR is these. So, it was basically let me if you are interested.

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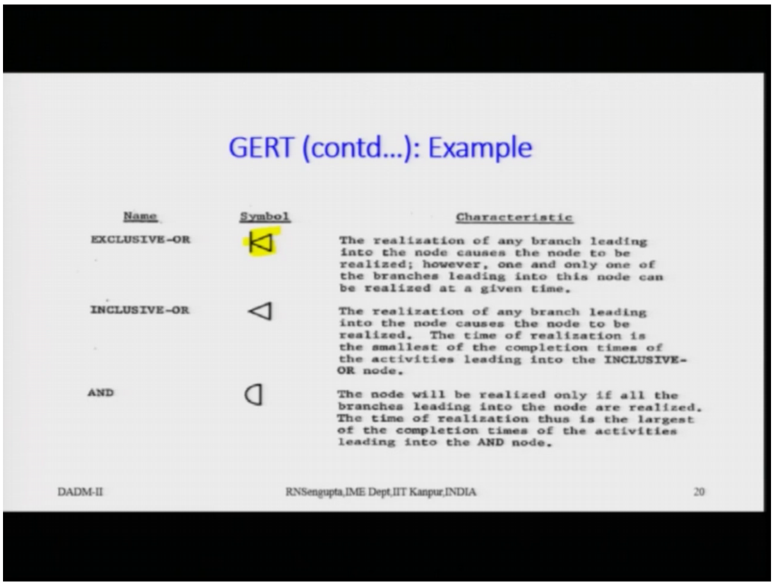

So, this is the EXCLUSIVE-OR. So, it means the realization of an any branch leading with the node causes the node to be realized; that means, yes activated; however, one and only one of the branch leading into this node can be realized at a given point of time. So, that means, only one is being realized, it will be realized, but it only one would be realized.

So, it means the concept of feedback is only appropriate for the EXCLUSIVE-OR input type of node. This results from the fact that the feedback requires that the node being returned to be realized prior to the feedback based on which we will proceed. But, then mode cannot be realized if it is an AND type node unless all inputs have been realized. So, the EXCLUSIVE-OR anyone would realize, but in the AND network all of them has to be realized in order that the output is true.

So, if you consider the very simple concept of series and parallel in this case when it is an AND one all the series is all the elements which are there in the series; they should be activated such that current will flow from the point A to point B. And in the concept of the parallel one you will basically have that any one of them working would mean that current will flow from node A to node B. So, you need not be basically all of them have to work at least more than one combination have to work. So, we will basically consider this in the very simplistic sense as I mentioned in the very simple way of a series in and parallel.

For the INCLUSIVE-OR input type only the branch representing the first activity completed is significant. All other branches are ignored in computing the time the INCLUSIVE-OR is in a node is realized. So, in the INCLUSIVE-OR you will consider the first branch being realized would give you all the results as it requires which is unlike the EXCLUSIVE-OR.

Now, since the feedback branch will always be completed after non-feedback branch, so, consider off with a non-feedback branch you will basically a feedback which will give you the output of the second stage going as the as a feedback or an input at the first stage, so, they can be different type of loops as I explained.

So, hence as this is possible the EXCLUSIVE-OR representation which EXCLUSIVE-OR we have already considered is this one EXCLUSIVE-OR which you have. So, the EXCLUSIVE-OR representation can replace the INCLUSIVE-OR node if a feedback branch is incident to that node, hence the overall logic circuit would be maintained.

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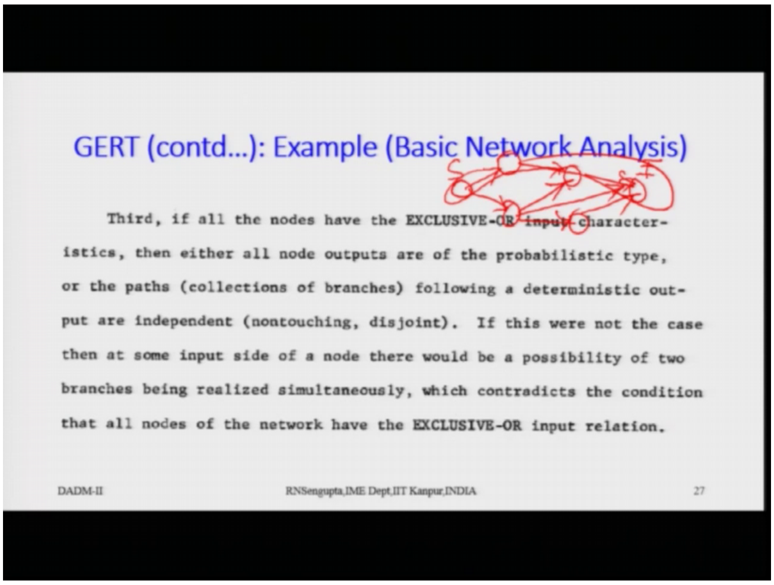

Third case, if all the nodes have the EXCLUSIVE-OR input characteristics then either all node outputs are of the probabilistic type. So, probability means probabilistic output being there or the paths that is the collection of the branches. So, there are different paths you can follow. So, it can be path i 1 to i 2, i 2 to i 3, i 3 to j; so all the combinations can be there. So, I can be i 1 to i 5, i 5 to i 3, i 3 to j. So, there can be different ways how you can do.

So, if I draw the network it can be, this is one of the ways, this is one of the ways. So, you can have one path like this, this is the source and this is the sync I am using the starting and the ending one. So, consider is the starting S, ending E; so there can be different ways how you realize that one.

So, third, if all the nodes are the EXCLUSIVE-OR input characteristics then either all node outputs out of the probabilistic type. So, in this case the probabilistic concept would come or the paths there is the collection of the branches which you have; any one of the paths you take you combine all the branches which are there in that path follow a deterministic output and they are independent with each other, in the sense they are disjoint. So, the independent structure would not be there.

If this was not the case then at some input side of a node they would be possibility of two branches being realized simultaneously. So, if they are not mutually exclusive and exhausting say in statistics so, if that was the case so, it was that case then there would be a possibilities of two branches being realized simultaneously. Then hence how you basically take that path which will be taken up would basically be difficult to find until unless you have a probability of split age; split age means their path 1 and path 2 being taken which are true.

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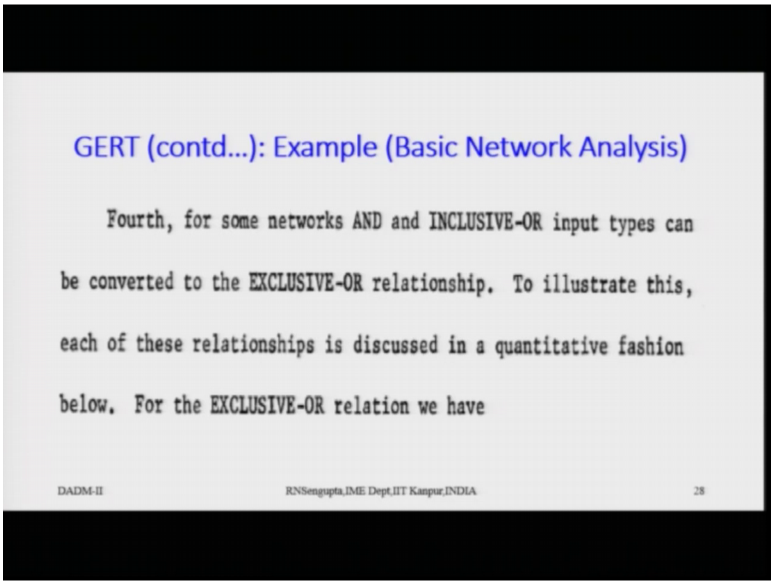

Fourth case, it can be for some network AND on INCLUSIVE-OR input types can be converted into the EXCLUSIVE-OR relationship. So, to illustrate this each of the relationship is discussed in a quantitative fashion for the EXCLUSIVE-OR relationship we can basically have the following diagram.

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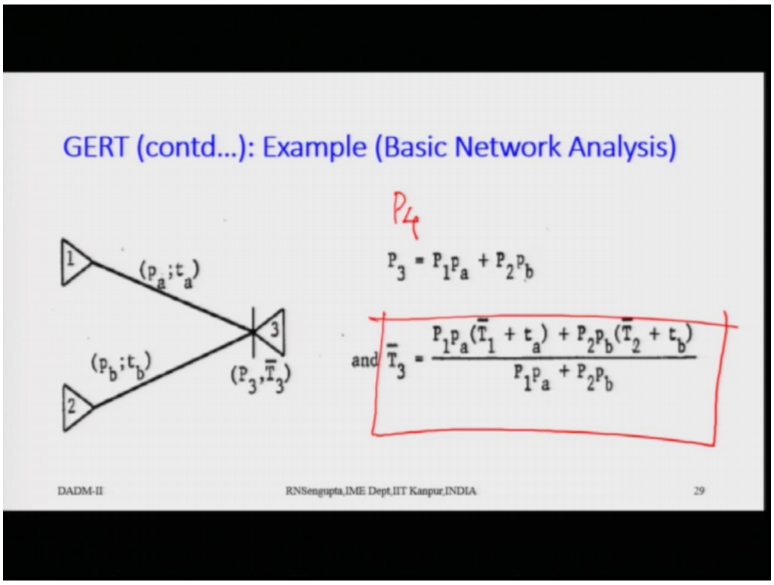

So, consider this; so from the realization which is probabilistic 1 we go to 3, from 2 to go to 3. So, let us again go back to the logic. So, I am going from the logical circuit to the diagrams. So, logical circuit to the diagrams are like this; so this is the INCLUSIVE-OR. So, that would be on the right hand side.

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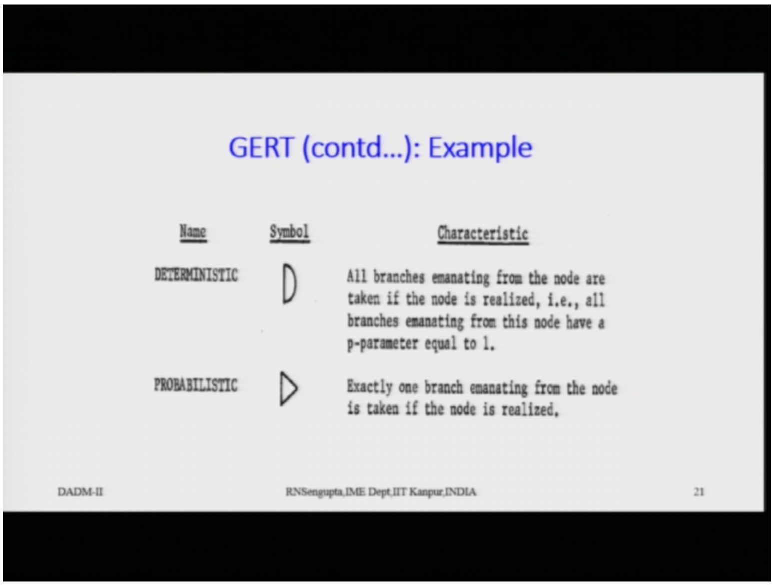

And, if I consider these are the probabilistic one, the diamond shaped with the pointed on to the right the triangle one. So, these are the deterministic probabilistic one the probability is given p a and t a is the time similarly probability p b and t b is given and we basically realize with the time T 3 bar mean the average time. So, in this case P 3 would basically be the corresponding probability P 1 into small p a which is the probability and this paths P 1 being taken P 2 into the probability p b and the average time would be calculated depending on the total average time which is taken in combined by the probabilities.

So, it will be so, if there are three paths, so, technically it would be P 3 would be P 1 into p a plus P 2 into p b plus P 3 into P 4 I consider. So, P 4 would basically be P 1 into p a plus P 2 into p b plus P 3 into p c considering there are three paths which are coming 1, 2, 3. And, the corresponding time period average which is T 4, but would be P 1 into p a. So, I am only repeating the first out terms which are outside the brackets in the numerator. So, it will be P 1 into p a the second term outside the bracket will be P 2 into p b and the third term would basically be P 3 into p c p c because there is a third path also I said.

And, the corresponding time period would be based on the fact that I will try to find on the average T 1, T 2, T 3. So, T 1, T 2, T 3 would be coming from the previous stage. So, we will consider that. So, it will be T 1 bar it would be calculated in the previous stage T 2 bar would be calculated in the previous stage and the corresponding and obviously, they would be T 3 bar also.

Because you are trying to find out the T 4 bar and the multiplicative factor inside the bracket in the numerator would be T 1 bar plus t a plus T 2 bar into t b plus T 3 bar into t c and the denominator will basically have P 1 capital P 1 into p a plus capital P 2 into p b plus capital P 3 into p c. It can be extended depending on number of such connections you have.

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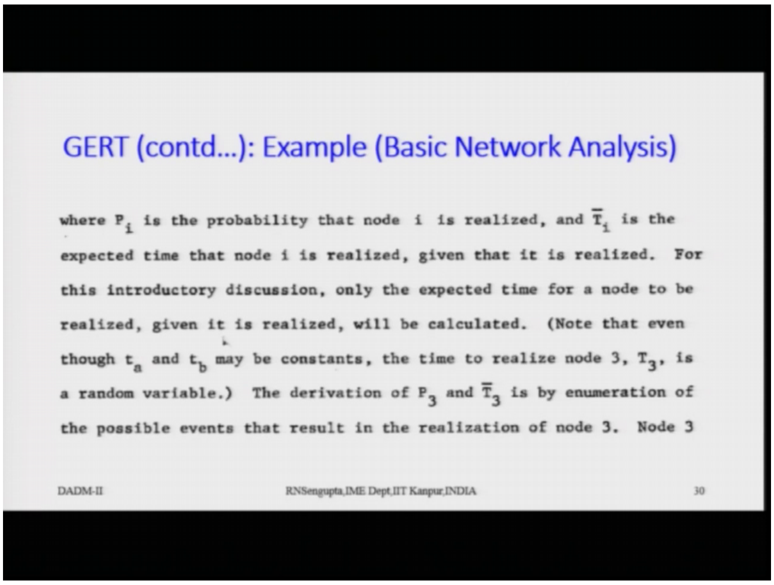

Now, obviously, what our capital P i's? So, where capital P i is the probability that the node I has been realized beforehand. So, if I consider as I said so obviously, 1, 2 and 3 I am considering, so, in this case I consider 1, 2, 3. So, obviously, there would be much more nodes and more connection coming from the from the left.

So, we will consider the corresponding probabilities P 1 capital P 1 capital P 2 capital P 3 so on and so forth for the corresponding probabilities then the average time would be corresponding to those probabilities that is T 1 bar, T 2 bar, T 3 bar. So, once we find out T 4 it will be utilized in the next stage wherein to calculate T 5 will utilize this concept of T 4 bar and the corresponding of P 4 capital P 4 which is the probability.

So, let us let me continue where P i is the probability that node i is realized and T bar i is the expected time that node i is realized based on the probabilities in the time which are already given in the previous stage for and obviously, they should be realized for this introductory discussion only the expected time for a node to be realized given it is realized will be recalculated. So, given it realized, will only find out the average expected time and the average time.

So, note that even though t a, t b may have may be constants depending on the time period. The time to realize note 3 in our case note 4 and corresponding time period T 4 and T 4 bar or T 3, T 3 bar depending on how many such nodes based on which you are proceeding that would be through the utilization and these T 3, T 4 not the bar values are random variables. So, derivations of P 3 and T 3 bar is an enumeration or the possible different type of events that result in the realization on node 3. So, similarly once we want to realize node 4 you will basically find out the different combinations based on which will proceed.

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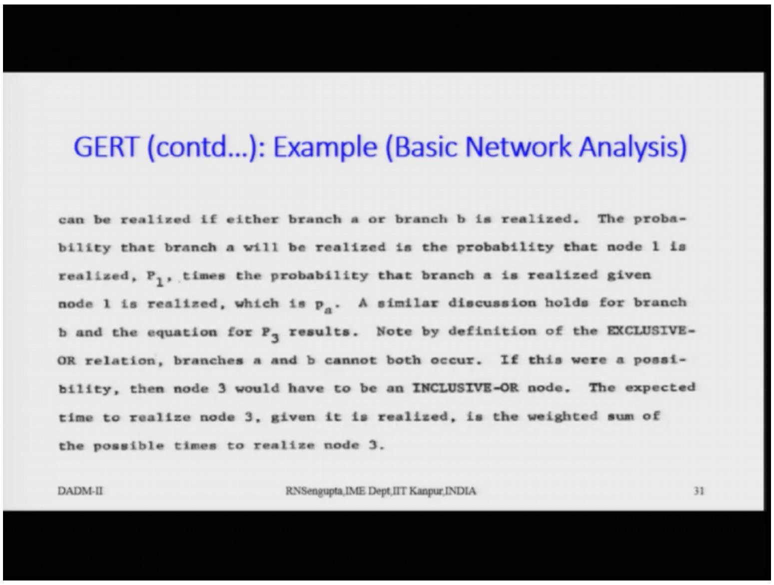

So, they can be realized if either branch A or branch B is realized. So, if you have three paths either a or b or c is realized based on which we will proceed. The probability that branch a will be realized is the probability that the branch one is the one is realized which is basically P 1, times the probability that the branch a is realized given node is realized which is.; so, you will basically have the branches on the nodes. The nodes realization is given by the small p's and the branch realization is given by the capital P's. You will basically multiply them accordingly.

A similar discussion holds for the branch b an equivalent results which you have found out. So, note by the definition of the EXCLUSIVE-OR relationship, branch a and b cannot occur both at the same time. So, that means, if you have three branches a, b, c they cannot occur at the same time. If this was a possibility then node 3 would have to be an INCLUSIVE-OR where we will consider that more than one branch occurring is possible.

The expected time to realize node 3 given it is realized is the weighted sum of the possible times to realized in the node 3. So, node 4 would depend on 1, 2, 3; similarly node 3 would depend on for this diagram which you have would depend on 1 and 2 you will basically proceed it iteratively later on.

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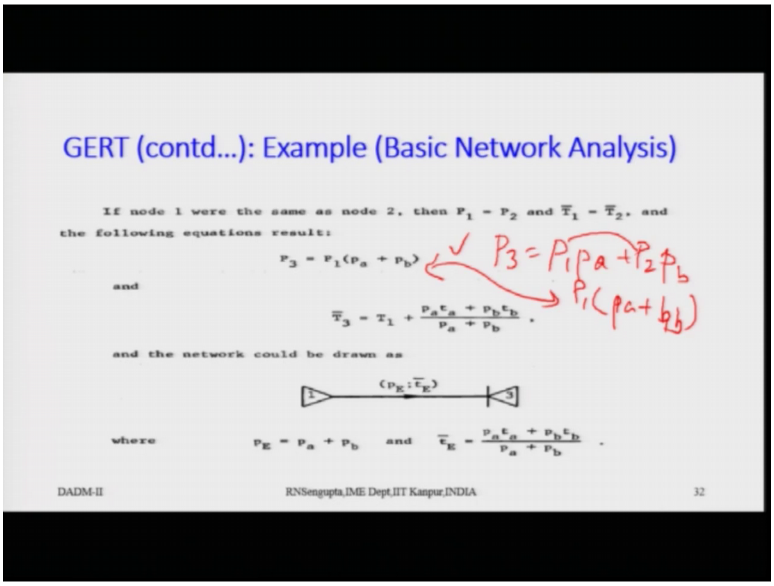

So, consider this if node 1 was over the same as node 2; that means, P 1 is equal to P 2 and T 1 bar is equal to T 2 bar, then the corresponding result equations would be P 3 can be counted. So, let me put this value. So, let me bring it here. So, P 3 is P 1 p a P 2 p b, P 3 P 1 p a P 2 p a and p b. So, consider if they are equal you take it outside this pa plus p b as already mentioned.

Now, coming to the time; so the time factors are given by this equation. So, this is the time factor. So, you consider that P 1 capital P 1 and P 2 are equal based on that if you want to just recalculate and make that equation look nice without much complications. So, you can find out T 3 bar would be T 1 plus a so called factors. So, it will change and the network would be drawn as we are going from 1 to 3 corresponding to the fact that node 1 and node 2 are the same. If node 2 and node and node 1 are not the same the calculation remains as it is before.

So, you will basically have the equivalent probabilities as p a plus p b and the equivalent time would be given as already calculated out, we know that you have just given the formula. So, only remember the general concept of how you find out the capital P's depending on how many such nodes you have before and the time average time also depending on how many nodes you have before.

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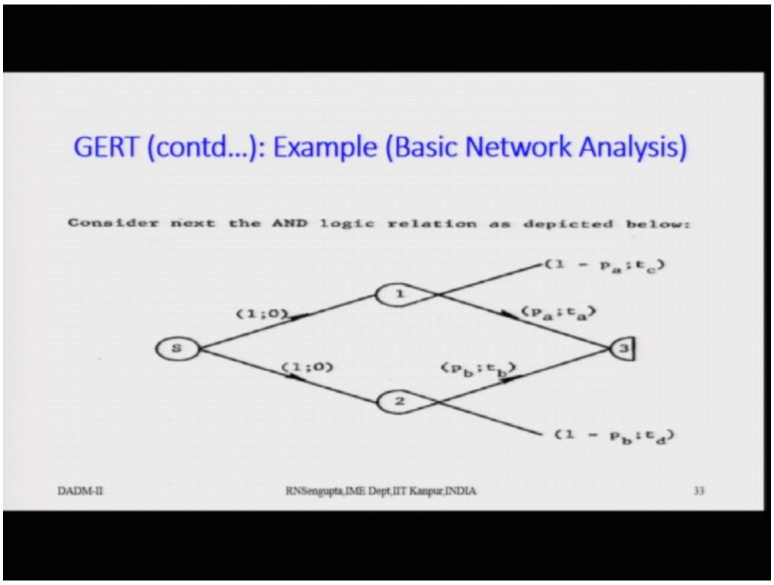

So, now, let us consider the AND logic relationship and it will be depicted as below. So, basically you would be take and these are the probabilities in the time. So, you can basically go from the source or the start to 1 and 2. So, the corresponding probabilities of paths being followed will be taken.

Similarly, there are two outputs happening from 1 and 2 depending on the logic relationship. One would basically with the probability p a; so, the counterpart path because there are two paths would be 1 minus p a and the time taken in these two paths would be different, obviously. In the first case when probabilities p a it will be we are considering as t a and in the second case when the probability 1 minus p a it is considered at small t c.

Similarly, for node 2 you have the output given as the probability and the time given as p b and t b and probabilities values given we can find out the corresponding probability and the time period for the subsequent parts later.

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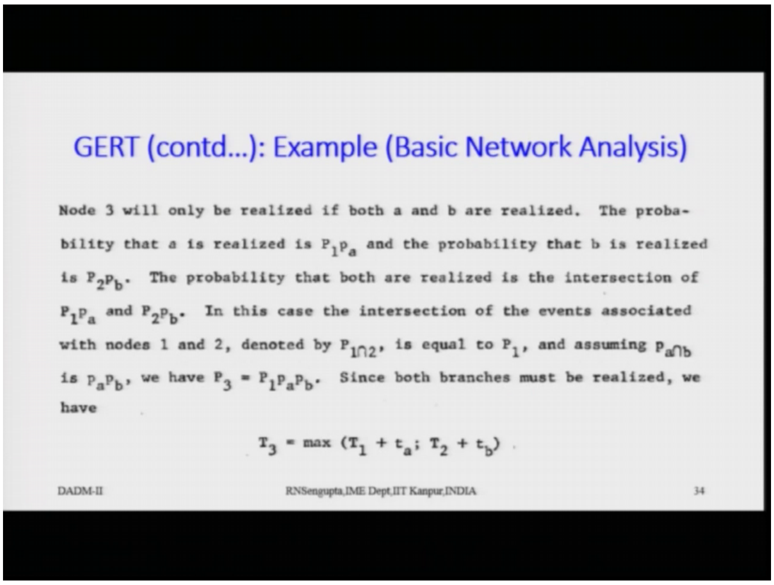

So, node 3 will be will only be realized if both a and b are realized. The probability that that a is realized is capital as I already said; so, considering the simplest case of P 1 and P 2 being equal we are that is the simple one.

So, the probability that a is realized it is given by P 1 into p a and the probability that b is realized is given by P 2 into p b, which you have already seen. The probabilities such that both the both of them are realized is basically the intersection set or the intersection numbers.

In this case, the intersection of the events associated with node 1 and 2 would be depicted by probability 1 intersection b, which is common and that should be equal to P 1 and assuming p a b is p a into b which is means you are trying to basically multiply the probabilities. So, we can find out that the branches must be realized if we have the corresponding time period being the maximum which is being realized at two different points accordingly.

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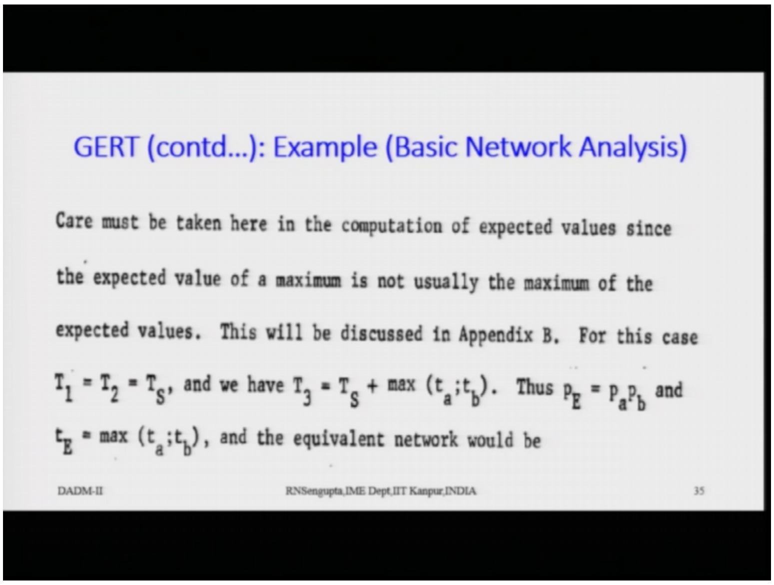

So, care tan must be taken here in the competition of the expected value since the expected value of a maximum is not usually in the maximum of the expected value that should be remembered. So, this will be discussed in the later parts, for this course not with what you will just mentioning that. For this case were, obviously, the time period T 1 and T 2 and the time period from the source from where we start is given and we will consider T 3 is basically the T S plus the max values and we proceed accordingly. And, equivalent probability and that the equivalent time would be equivalent would be found out and utilized that for that network.

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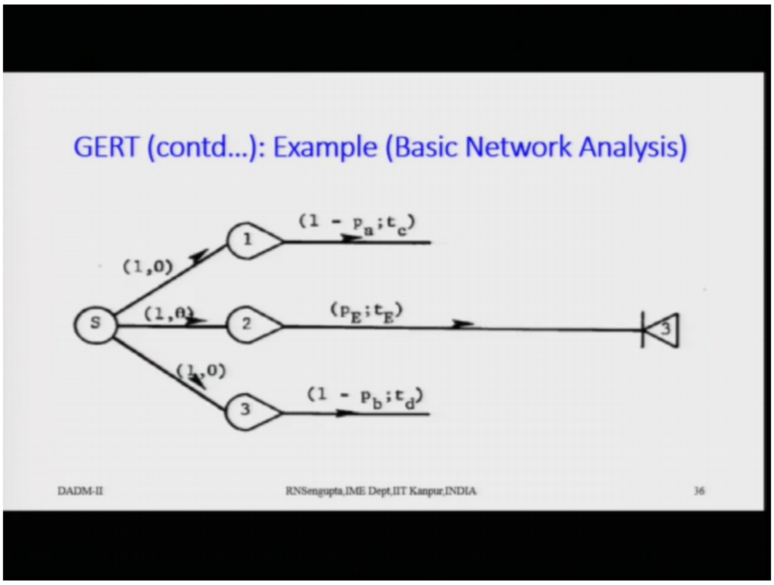

So, we have from the source 1, 2, 3 outputs the probabilities are one for the second one the cumulative the equivalent probabilities p E, equivalent time is t E and the corresponding probabilities and time periods for the so called paths emanating from 1 and 3 are corresponding to 1 minus p a and 1 minus p b because you have that one part is realized another part is not realized would be given by 1 minus p a and similarly for the third part it would be 1 minus p b.

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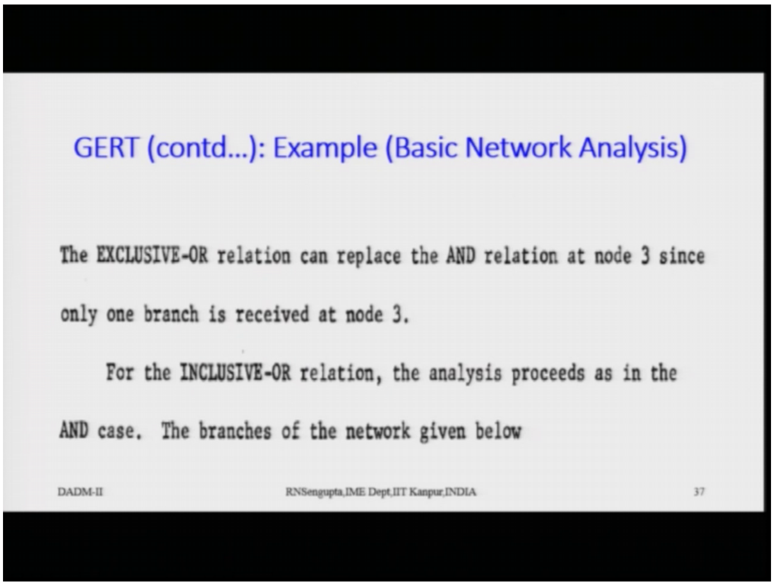

So, once this is logical structure is done so, the EXCLUSIVE-OR relationship can replace the AND relationship at node 3 since only one branch is received is received at node 3. So, if there are more than two branches, obviously, caution has to be taken into consideration where you can basically replace any one of the exclusive network concept in the other one; so, depending on the node structure which is there. For the INCLUSIVE-OR relationship the analysis procedure as proceeds as in this case the branches of the network given below would basically give you the idea.

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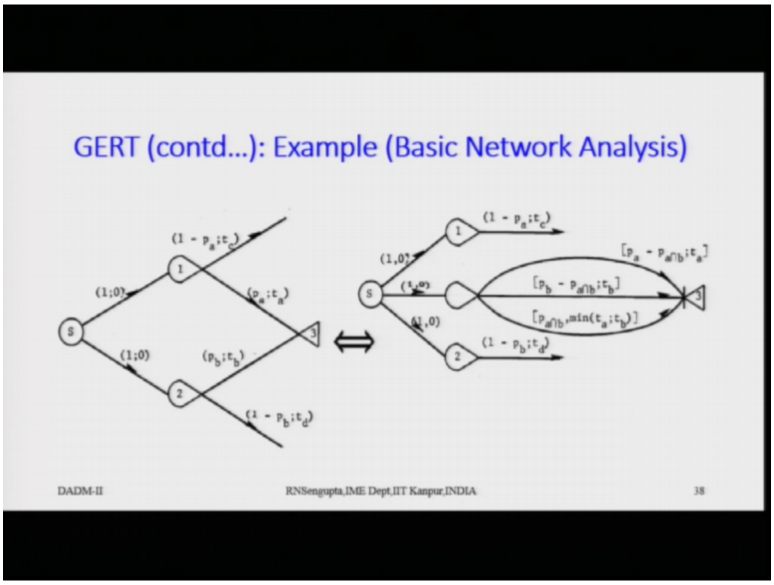

So, the left part of the concept is basically equal to the right part. And, we will basically have the equivalent part being considered in such a way we will consider the probabilities and the equivalent probabilities of p a, p b and p E and the time periods as given.

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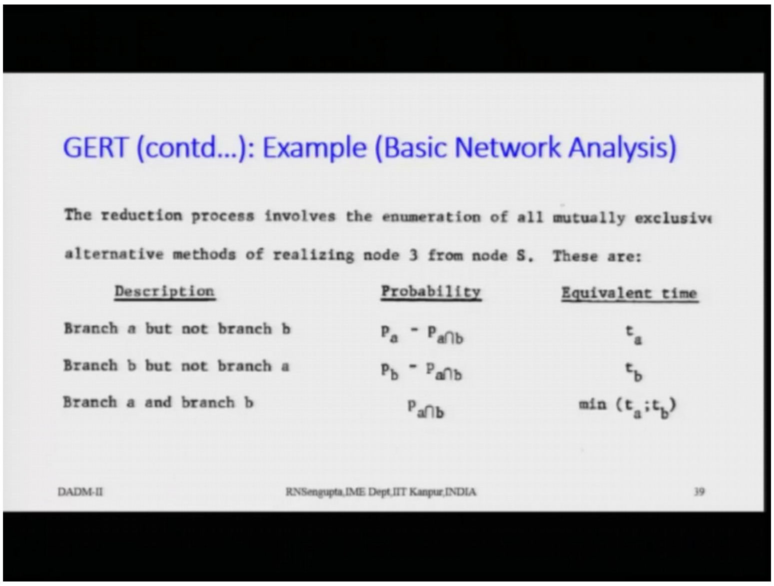

So, thus the reduction process would now involve the enumeration of all the mutually exclusive and alternative nodes in and will basically combine clearly show them each step such that you can find out what is the equivalent network. So, description being branch a, but not branch b is being is being realized. So, it will be probability of p a minus intersection of the probabilities in a b and the equivalent time is given by t a.

Similarly, b is the branch and not a is the branch in that case is just swap the numbers. It will be p b minus p suffix a intersection b and the equivalent time is p b. And, in case branch a and branch b both are realized it will take the probability of a intersection b which is the overall common so called area if you consider the concept of Venn diagram and the equivalent time would basically the minimum time based on which you will proceed.

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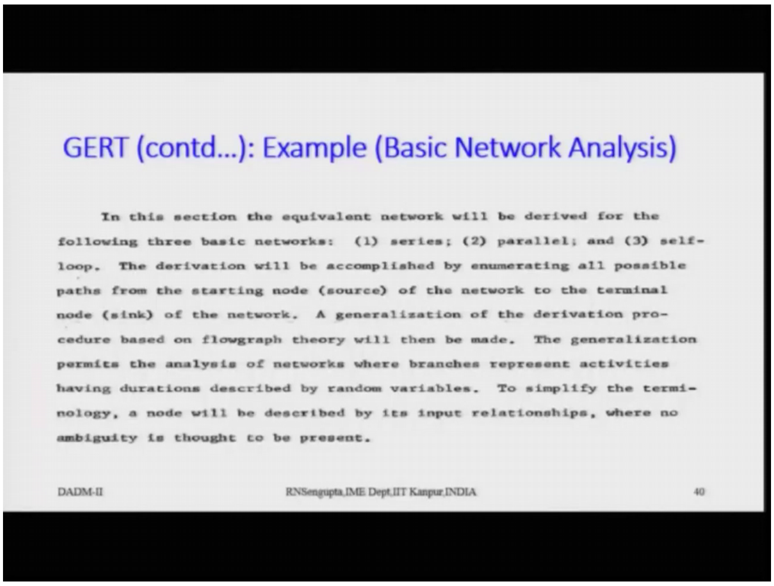

So, in this section the equivalent network will be derived from the following three basic networks which is the series one parallel one and the self loop one depending on whether the loops are allowed for the parallel and the and the series concept. The derivation will be accomplished by enumerating all the possible outcomes in order to show which is the which are the equivalent network possibility that the or the network to the terminal node. So, it means, the derivation will be accomplished by enumerating all the possible paths from the starting node of the network to the terminal node of the network. So, will basically start till the end.

A generalization on the derivation procedure based on the flow graph diagram will then be utilized in order to reduce it to the maximum possible extent. To simplify the terminology, a node will be described by the input output relationship, where no ambiguity would be there for this diagram.

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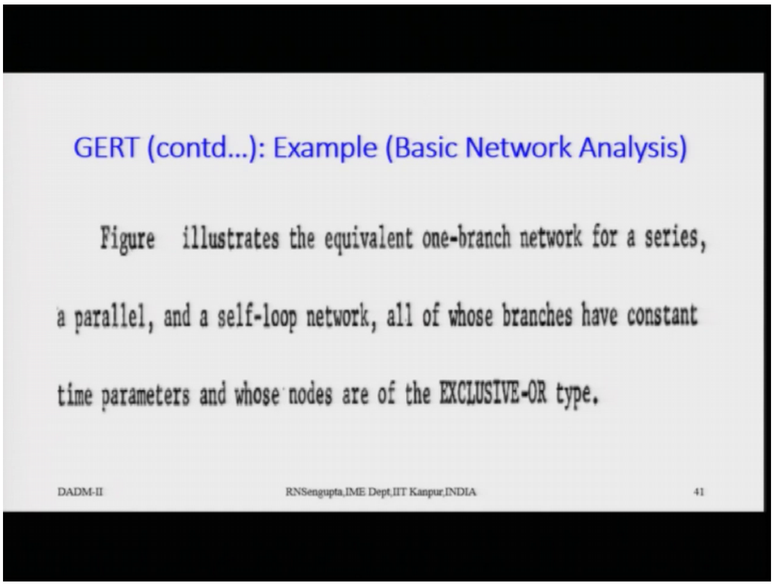

So, in this figure it will illustrate the equivalent one-branch network for a series, a parallel, a self-loop and a this series concept. So, there are three – series, parallel and self loop depending on the loops you have all of whose branches must have constant time parameters, but whose nodes are of the EXCLUSIVE-OR type.

So, with this I will end the 52nd lecture and continue more discussion about the GERT in a very simplistic sense and then start off a new topic as required. Have a nice day and thank you very much.