Data Analysis and Decision Making - II
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Lecture – 05
Utility Analysis

A very warm welcome to my dear friends and students, very good morning, good afternoon good evening to all of you. And as you know this is the DADM 2 lecture which is Data Analysis and Decision Making 2. And this is lecture number 5 and this tour this whole course is for 12 weeks which is 30 hours, each week we have 5 lectures and after each lab and each being lecture being for half an hour and after each week we on assignment. So, we will be wrapping up week 1 and we will have assignments based on assignment 1, then similarly once we finish any week we will have a assignment based on that week and proceed accordingly till we have the final examination and I am Raghu Nandan Sengupta from the IME department IIT Kanpur.

So, if you remember we initially in the last 2 classes I will just wrap it up within 2 minutes and then like recapitulate and then proceed with lecture number 5. So, we had 2 different concept which was basically relative risk aversion and an absolute risk aversion. And these properties were basically which I mentioned was theoretically they have some formulas and I must just give the background based on which you can basically prove it.

But their actual utilization you I said would be coming up later on and for the quadratic utility function, and we did prove I did not go through the details, but you can just simply do it by finding on the first differential, and the second differential that we find out U prime U double prime put the ratio which is U double prime divided by U prime with a minus sign, find out A and then corresponding to find out A prime. Say similarly find out R and R prime and then comment on the this absolute risk aversion property and the relative risk aversion property with respect to whether they are greater than 0 equal to 0 and less than 0 based on that we can say. That it is that quadratic utility function or whatever the utility function is. Which I am mentioning that all the utility functions would have properties based on A prime and R prime and they would be distinct. So, based on that we can comment.

So, let us further continue the discussion with other 3 different utility functions which had briefly mentioned. So, the first we have already done which was quadratic utility function, then there would be power utility function, exponential utility function and logarithmic utility function. So, now, for the quadratic utility function I did not wrap it up the last stages were left. So, if you plot the excel sheet which I just showed was basically the last slide which was there.

In the first column you have the values of W, I took arbitrarily some values of W then you have UW based on the value of b as a parameter which you had. Then in the next 2 columns which I mean, which I did not write or in did not explicitly show it.

But I did mention that you find out the difference of U so; that means, U 2 minus U 1 U 3 minus U 2 U 4 minus U 3; that means, 1 2 3 4 these suffixes are corresponding to the time periods or which W values are there. You find it that that in the third column similarly you find out on the fourth column, again the fourth column was not shown there you find out the values of difference of the values or W that is W 2 minus W 1, W 3 minus W 2 W 4 minus W 3 and so on and so forth.

Then we basically divide the third column values respective values with the fourth column respective values and you find out U prime as the simple definition of differentiation. Then we have to find out the second differentiation which will be basically now the differences of the third column, which is basically U prime values which you have. So, basically you define find out the difference of U prime for the second period minus the U prime for the first period, then you find out the U prime for the third period minus the U prime for the second period and go and so on and so forth.

And then again you divide these values corresponding cell value written in fifth column by the corresponding values of difference of W s that is W 2 minus W 1, W 3 minus W 2 so on and so forth and you find out the U double prime. Once you find out the U double prime, then as per the formulas you can put it and find out a, then similarly you can put these values and find out R and then again follow the same concept to find out the difference of a 2 minus a 1 divided by W 2 minus W 1.

Then find out the value of a 3 minus a 1 divided by a 3 minus a 2 sorry and find out the values of W 3 minus W 2. Similarly you find out the difference of R 2 minus R 1 divided by W 2 minus W 1 find out the difference of R 3 minus R 2 divided by W 3 minus W 2;

that means, you are finding on the differential such that you have A prime and R prime and once you plot it the values which you have in this simple graph.

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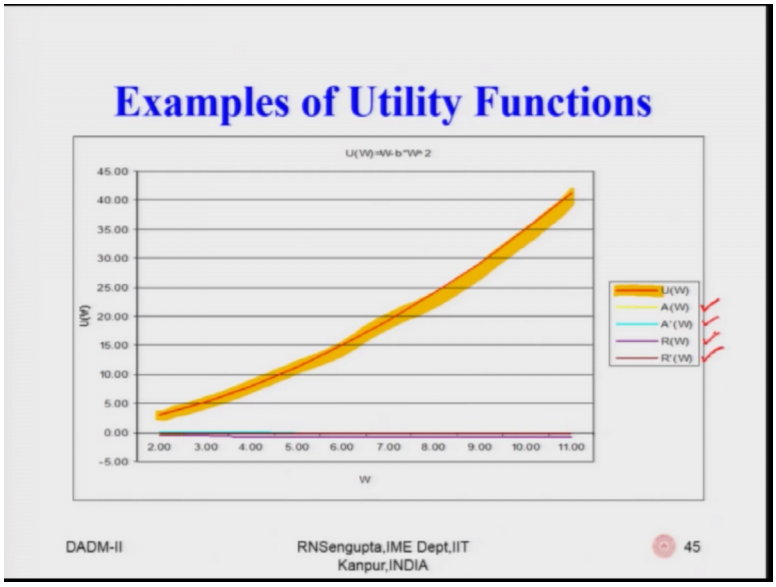

It is not possible to see the other values, but at least if you can see where I am just marking this, let me use the color pink if it is available. Let me use the highlighter it could be best the orange one.

So, this is basically the value of U which is this is a quadratic one if you can see. Then similarly in the next the index or the coloring you have I will just mark it. I just put a tick mark you have A W which is in yellow it is somewhere here if you can plot it would be much better for you then the bluish green one is A prime, then the violet one is for R and the brown one is for R prime if you plot it you will get the characteristics curves for this U A A prime R R prime for the quadratic utility function.

Then let us go to the logarithmic utility function which is ln of W and then if you want to find out A and R again the formulas you use b for a would be minus U double prime by U W and for R it will be basic A minus W into U double prime by W put those values accordingly; that means, find out U prime and U double prime for ln of W, U prime is 1 by W and similarly for the value of U double prime would be minus 1 by W square. These formulas are very simple you can check up any class 10 or 11 basic differential equation or basic mathematics book and in basically and can utilize them accordingly.

So, once you find out A prime and R prime the values which are come out to be for A prime would be minus.

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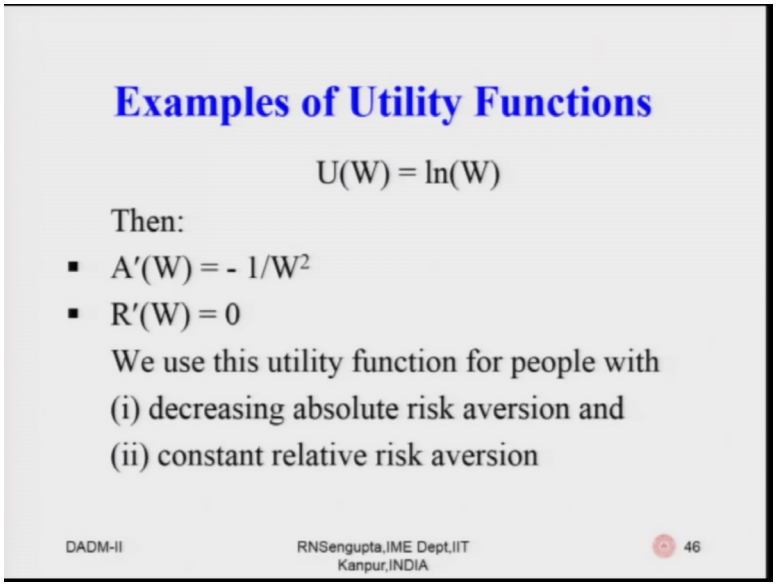

1 by W square and for R prime it comes out to be 0. So, if it is W is positive so; obviously, the square is also; obviously, a positive. So, A prime would be negative and R prime would be 0. So, we can immediately comment we use this type of utility function; that means, if we have this information that is a it has got decreasing absolute risk aversion property and constant relative risk aversion property you can find out and comment that this is a logarithmic utility function.

So, similarly again now if you draw the graph. So, before that we will basically put the values in the excel sheet, follow the same procedure that we as we have done for the quadratic utility function. So, this is also the excel sheet based on which we can solve and it is draw the logarithmic utility function the first columns is the values of W again you can take any values of W I have to make our life simple, I have taken the values as 1 2 3 4 so on and so forth they can be decimal also and they can be any other higher values also. Based on that I find out ln of W and ln of W values are given in the second column which is this values.

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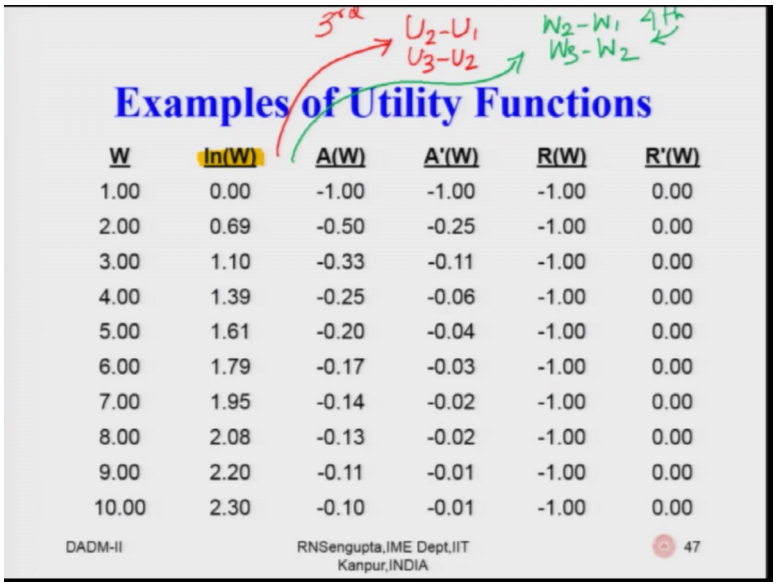

Now third and fourth column are missing which I have not noted down, but I will just because it would have become too cluttered.

But I will just mention it again. I may be going a little bit slow I know that I am repeating few things time and again for each concept, but please bear with me which will it could be much easier for me to follow as sequence point 1 and repeating such things would definitely make things much better for you to understand. If somebody has understood that before end with one of my reputation, I would tell them to have the patience for let me go slowly and cover the concepts.

So, if I basically use the highlighted yellow. So, they have to basically write down. So, what you will have here? One would be basically be. So, these are the use the second column. So, technically I would have one column where I will basically find out U 2 minus U 1 then I will find out U 3 minus U 2 and continue accordingly. So, 1 2 3 are basically the time periods and there the values have been taken purposefully for example, as equidistance. There would be another column and let me use a different color, there would be another column which would be the fourth one which would basically have values of W 2 minus W 1 then the next value in the cell corresponding just below that would be W 3 minus W 2, I will continue accordingly.

So, this would basically be the fourth column values. So, these are the fourth column values and these are the third column values now in the fifth column you will basically

have a. So, how do you find out A? You use the formula minus of U double prime by U prime. So, how do you find out U prime U prime? Would basically be the corresponding value of U 2 minus U 1 divided by W 2 minus W 1.

Next value would be U 3 minus U 2 minus W 3 minus W 2, next value would be U 4 minus U 3 divided by W 4 minus W 3 you find out those values. So, that would basically be the denominator for A prime A sorry A and for finding out the numerator of A again you, different you find out the first differential of the third column values; that means, the difference of this and this similarly the difference of it will be basically U 4 minus U 3 minus of U 3 minus U 2.

So, that would be give me those each corresponding cell values, again divided by W this W values which I have utilizing that you can find out the corresponding values of U double prime. Once you have the U double prime with the minus sign as per the formula you find out A, then find out the first difference of the A values divided by again the difference of Ws find out A prime, put in the formula R find out the R values find out the first differential; that means, R 2 minus R 1 then the next value is R 3 minus R 2 so on and so, forth then divide them by the corresponding values as W s; that means, W 2 minus W 1, W 3 minus W 2 so, and so, forth find out R prime.

So, if you notice done as for the logarithmic one, R prime was mentioned is name is is 0 we have calculated it so, and actually rightly. So, the last column which is shown here comes out to be 0 and if you remember for the values of A prime we mentioned the values was coming out to negative. So, whatever value you take you can just double check in an excel sheet the values really come out to be negative.

So, with this we have been able to at least very in a very simplistic sense check in that the actual calculation based on simple mathematics are differentiation is being corroborated or double check, when we basically take very simple values of wealth and then put them in the utility function which you logarithmic utility function, and check the values of A A prime and R R prime. Now we need to draw the graph, I am going the same sequence of steps as I did for the quadratic utility function.

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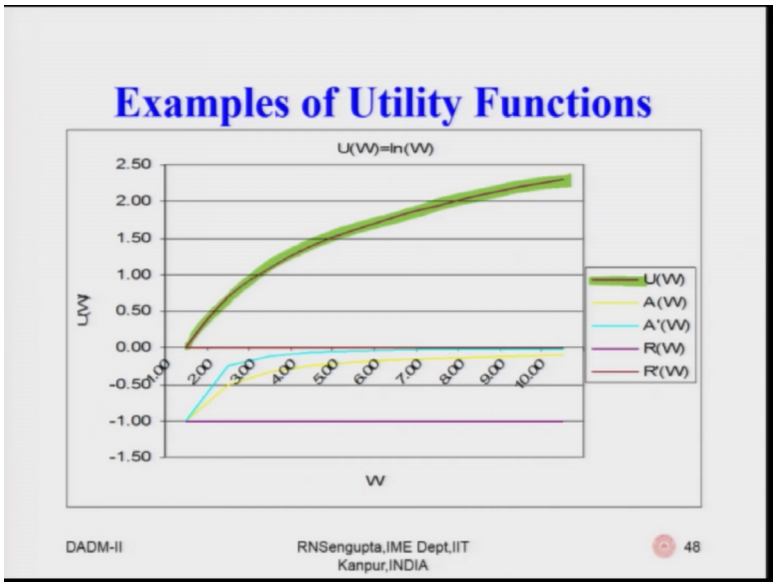

If you draw the graph put those values and then basically find out the and plot the values of U then A, A prime R R prime you can check the values which are given and just use the highlighter of the green one.

So, the pink one is U prime I will just mark that then the yellow one is A which is shown here and I am not going to mark it, but I am just going to move my pointer above that with our marking. Then similarly the bluish green one is A prime which is also there, then R and R prime are given here in these values and they actually corroborate and double verify in the fact which we have already discussed considering the simple differentiation. So, that completes this is a very simplistic way of trying to first do for the quadric one then the log of this one.

So, we are left with 2 more logarithm and utility function and I will push follow the same procedure maybe I am taking time, but that will definitely clear your doubts. The problems just I am dying not digressing I am just changing the topic to some related for the assignments. For the assignments they would definitely not be any diagram part let me make it very clear.

So, it will be more conceptual part of problem solving. The problem solving which you remember I had one of the problems we just solved was basically based on a person investing in a government security or government bond with respect to a nondeterministic one, there were 3 outcomes and each other probability and you had a

utility function. Though those type of simple problems would be coming there would be other problems which I will discuss so constant run on that.

The reason I am I am going slow for drawing the diagrams for the utility function is basically to make you understand that what I actually mean by the concept of RR prime A A prime for, because for each utility function there would have some have some unique characteristics based on which you can basically immediately say that the utility function is either logarithmic or quadratic or exponential or power depending on what are the characteristics of A prime and R prime. So, without much ado let me continue with the third utility function.

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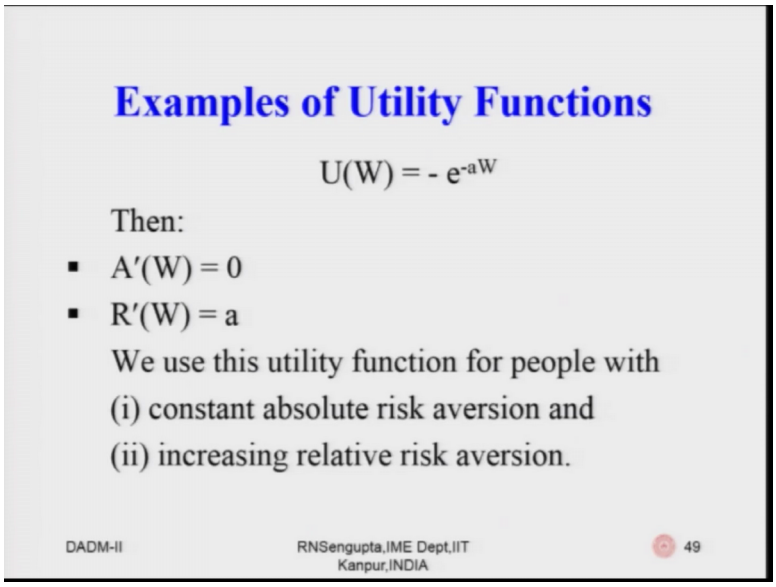

The third utility function is the exponential one. So, we again if you put in the formula and find out the U prime and U double prime using the simple differentiation concepts and if you put them in your equation, the value of A prime comes out to be 0 and the value of R prime comes out to be A A is positive here.

So, we use the utility and if you find out we will immediately say and say that the absolute risk aversion properties is constant as it is 0, by the second first bullet point. And if I consider the value of A as positive then R prime is a sum hence we will say its an increasing relative risk aversion property.

So, if we rather than the utility function if we had these properties mentioned to us correspondingly to the fact that R prime and A prime is e then increasing or decreasing or equal to 0 similarly R prime has this property of either increasing or decreasing or equal to 0. Immediately put them in into a in the right context and basically can mention that what type of utility function it is.

So, I will try to again go through the same sequence of steps as we did for the quadratic utility function and the logarithmic utility function. You plot the values of W in the first column in excel sheet, put the values of U W; UW is basically that exponential utility function with a certain value of A let us not be too much bothered of A A would be 0.25, but what I am trying to highlight is that how you draw the graphs in order to basically make yourself very acquainted. In a very simplistic and practical sense that how the utility functions can be plotted.

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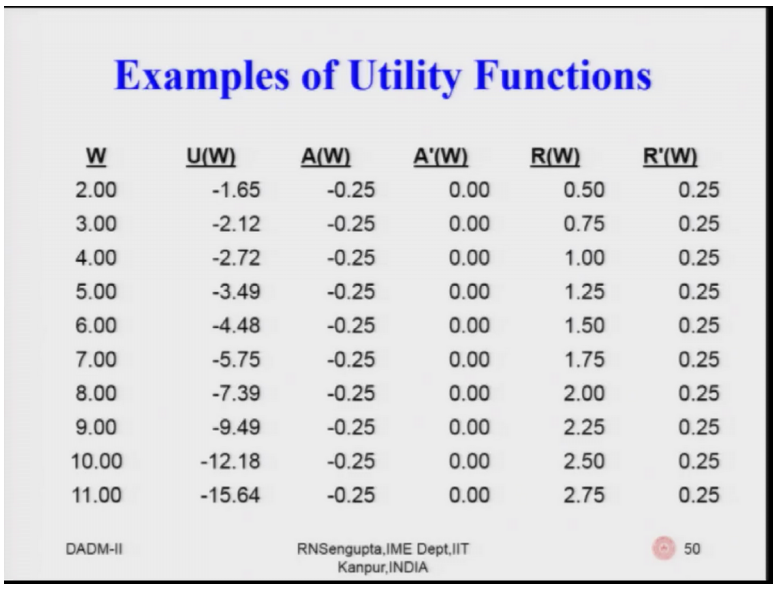

Then again in the third column I am not going to write much in this slide, but again explain to you. So, if you listen to me you will definitely understand. In the third column you will basically have the difference of the utility functions which is U 2 minus U 1 U 2 3 minus U 2 so on and so forth.

In the fourth column you will have the difference of the W values which is given in the first column. So, it will be W 2 minus W 1 W 3 minus W 2. And based on that once you can you are able to find out U prime once U prime is found out, then you will basically

found or the first differential of the U prime values; that means, which will be U 3 minus U 2 minus in the bracket U 2 minus U 1 continuing this way and then divided by the values of del W. Del W is basically W 2 minus W 1 similarly in the next value would be W 3 minus W 2 so on and so forth.

Once you put them in the cell values in the corresponding column. So, the columns are not shown here, those particular columns corresponding to the first difference of use or the second difference of U primes are not shown here. So, once you basically find out the first difference of the of U primes and if I would been them by the delta W, will basically have the U double prime put them in the formula which is minus U double prime by UW you find out A value.

So, if you find out the values are coming out to be constant then finding out A prime would; obviously, be 0 which basically collaborates with R actual in a set of information which is given for this utility function, which we just have covered in the last slide. The value comes out to be 0 correspondingly again we find out R; R is basically minus W into U double prime by U U prime put them in the equation and find out that we do the same steps as we did for a A and A prime only that change would be you are multiplying by the value of W.

So, once you put find out R you find out the values of R prime. R prime comes out to be constant which is A as per the formula. So, it basically has an increasing relative risk aversion property. Now let us draw these curves. So, if this curves if you remember A is positive which is and with a negative sign, it will basically be an exponential function but it will be decreasing.

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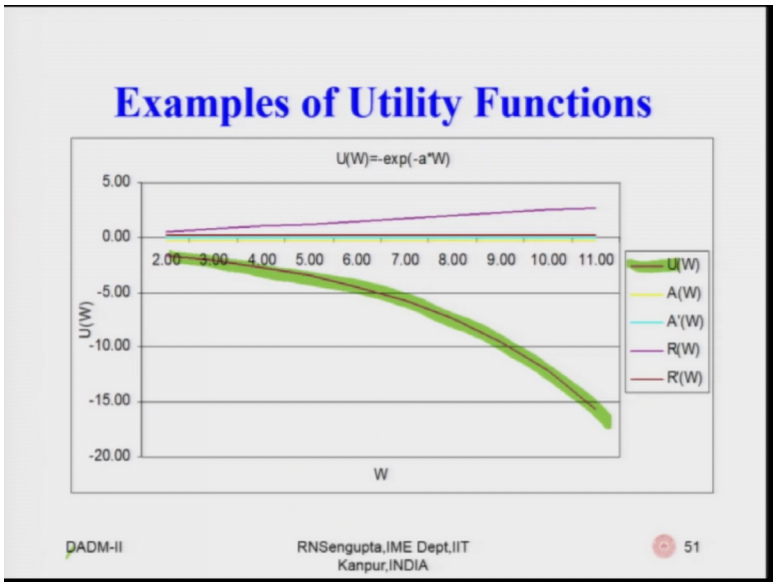

So, I will again just highlight it using the color green. So, this is the pink one graph where I am just now highlighting is the utility function which is here. Similarly it will have the yellow color which is given here I am not highlighting, but I am just pointing out this is the utility function A value for the utility function then the greenish blue one is the A prime then the violet one is R and again the brown colored is R prime.

So, for all these graphs again we have the fourth graph also for all these graphs I am using the same color scheme in order to make you understand. So, its my honest request please at least for this first week of lectures before we solve the problems whatever is given, please draw these graphs in order to make you understand that what are the utility functions and what is the significance of A and R and corresponding to that what is the significance of A prime and R prime.

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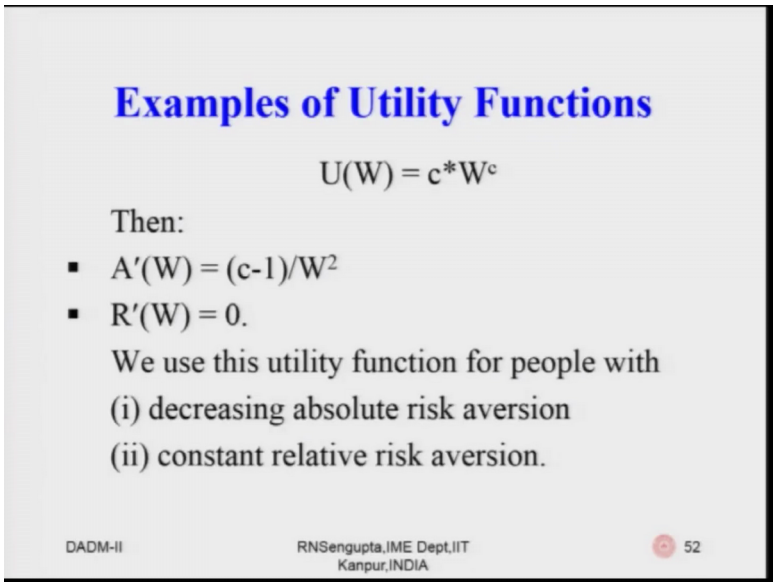

Now, let us come finally, to the power utility function the c values can be taken as positive or negative depending on the problem. I am not going to the nitty gritties of that I am just giving you a feel of how you can find out the characteristic of the utility function. So, you first find out U U prime, then you find out U double prime put them in the equation and then you find out with obviously, with the minus sign that is minus of U double prime by U prime will give you the A value differentiate again with respect to W.

So, whenever I am using the word differentiation for all these problems, it is differentiation with respect to the variable which is W which is the wealth. So, once you differentiate a you find out A prime, it comes out to be minus bracket c minus 1 by dividing W square. So, W square obviously, is positive v c is positive greater than 1 you will have basically A prime as a positive, if it is less than 1 obviously, it will be negative.

So, you will have basically decreasing absolute risk aversion property. So, here we and actually it would be. So, we can immediately comment that there is a decreasing absolute risk aversion property and this and along with the fact that when you find out R and R prime the R prime value comes out to be 0.

So, it will basically have a constant relative risk aversion property. So, with these 2 properties for absolute risk aversion property and relative risk aversion property, which is given as decreasing absolute risk aversion property and constant relative risk aversion property, you can immediately point to the fact that the utility function would be power

utility function. Similarly as I mentioned depending on these combinations of R prime and A prime whatever you have, you can find out the unique utility function based on which you can work.

So, now let us go to the last stage of the utility function, which is the fourth utility function which is true to basically find out the values correspond to some hypothetical values are W and plot the utility function graph. Again with far as follow the same procedure plot the values of W in the first column.

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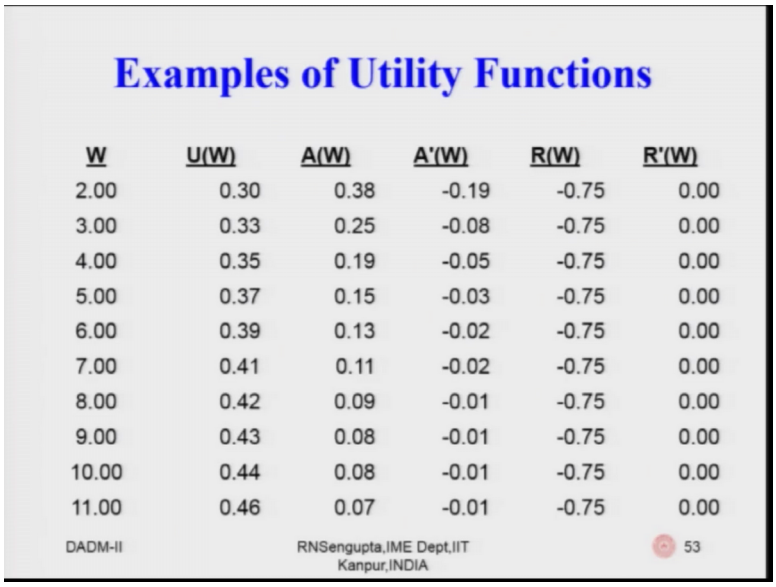

Then in the second column you have the values of U W depending on the c values which you take in the third and fourth column which is not given you basically find out U U difference of U; that means, the differences corresponding differences of the values in the cells for the second column. Then you find out in the fourth column the corresponding difference in the cell values, corresponding to the fact that it is for the first column which is basically W 2 minus W 1 W 3 minus W 2 so on and so forth.

Divide the corresponding cell values of the third and the fourth column you get basically U prime. Similarly you find out the difference of the U prime values and divide those corresponding values with W difference or or del W you get U double prime. Once you prime and U double prime I found out, you immediately know put in the formula find out A find out A prime; that means, A prime would again be calculated in the same way that

you find out the first difference of A divided by the first difference of W that is del A by del W. Once you plot it the A values are 8prime values are given here in this the slide.

So, number 53 is given in the fourth column. Similarly you go into the next step find out given A you find out R which is given in the second last column and then again find out the difference of R divided by the difference of W, which is del R by del W will give you the values of R prime. So, R prime you find out the 0 is exactly matches with what are the calculations we have done and based on that you can basically immediately comment that what are the absolute risk aversion property what are the relative risk aversion property based on that you can comment is a power utility function. So, let us draw it.

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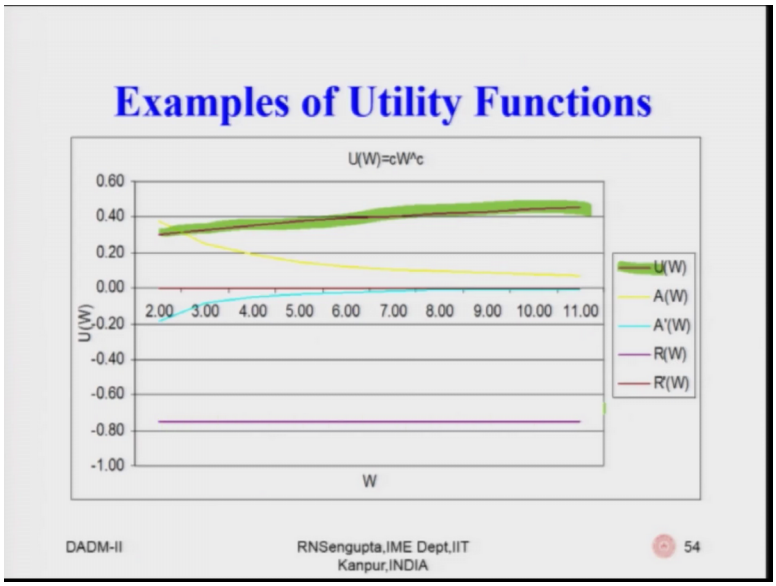

So, again I follow the same scheme in the same set of information which I have you plot this I am doing each time, but I did not mention it.

So, what people do is that plot the values of W on the x axis, the values of U A A prime R R prime you can I have not drawn in 5 different excel sheets, but I just merge them together in order to make you understand. Obviously, it becomes its not cluttered if you even zoom in and when you draw it zoom in and zoom out you can have a much better feel that how the graphs look like for different values of W.

So, the values of U W is given by the pink curve which is this similarly the values of A, A prime R and R prime R respectively given by the colors of yellow which is this one I

am not highlighting just following the pointer then the bluish green green one is the A prime and the R and R prime which are colored as violet and brown are given here.

So, it can basically find out using the graphs the properties of all the 4 utility functions. Whatever you 2 functions you can have follow the simple concept of trying to basically plot the value of given any hypothetical values are W plot U then plot from the fine of the value the plot means mark the values of U prime mark the values of U double prime then go in to find out the using these values of U prime and U double prime. You find out A then find out A prime then proceed to find out R and plot and find out the value R prime plot them and basically pass your comments accordingly.

So, you are basically double checking whatever you have done mathematically using simple derivatives which are already done. Now let us go into very simple examples. So, this is the examples which I am talking about so, but I will again repeat it please please draw this excel sheet in order to make your a custom, with the way how the utility functions look like depending on the different values of the parameters, which is a c or whatever there is or b which are therefore, different of the utility functions. Technically for the quadratic utility function you will have the parameters if it is a quadratic equation it will you will have basically abc, similarly for the exponential one utility function you will have the parameter a, for the power utility functions you had the parameter of c.

So, depending on the change and also on their range and to between which value to which value they can change, you can plot the values and look like how the utility functions themselves look like depending on the different parameter values. Now suppose the utility function is given us as W to the power 0.25 which is W to the power one fourth and we are required to find the properties of the utility function and also draw the utility function graph. I have not drawn the graph but I am just going through the simple calculation.

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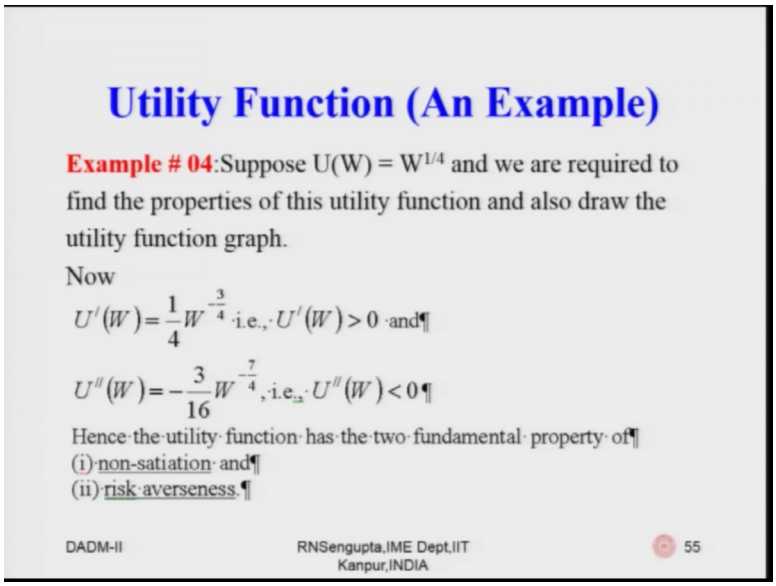

Find out U prime U prime comes out to one for W to the power minus 3 by 4. Hence we have proved as per the first property of non satiation that the U prime value is greater than 0. Because one-fourth is positive W is also positive W to the power any higher negative value would be 1 by that W value. So, it would be 1 by W to the power minus not minus 3 by 4.

So, W being mean positive; obviously, this world would always be positive. Now main question is whether that increase in the U prime value is happening it is increasing rate or the constant rate as or at a decreasing rate based on that we can comment whether the person loves risk whether person is indifferent to risks or with the person hate risks. Now we find out U U double prime; that U double prime value comes out with the minus value minus 3 by 16 W to the power minus 7 by 4.

So, 3 by 16 is positive W to the power minus 7 by 4 is also positive. So, it is negative hence the utility functions has 2 fundamental properties or non-satiation and risk aversion properties is basically agreed upon. So, with this I will end this fifth lecture with; that means, we have we complete our first as week of classes and the examination or the assignments which would be there sorry not nothing the examination. The assignment we they would be based on the simple problems of trying to find out the utilities very simple problems.

But again I will stress the fact that please read the concept of utilities, the books which are already being discussed in the website which is there, and I have discussed that the syllabus which is already given. These are a little bit on the higher level, but there are definitely very simple economics books from which you can get the concert of utilities and based on that brush up your concepts if you know.

Do the problems, solve the assignments and once the utility function is concept is over which I think should be over by within 1 or 2 lectures in the second week, we will start the concepts of different type of multi criteria decision making, multi attribute utility theory and then proceed with the actual content for DADM 2 have a nice day and.

Thank you very much for your attention.