Data Analysis and Decision Making - II
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Lecture - 48
GERT

Welcome back! my dear friends and students a very good morning, good afternoon, good evening to all of you. And as you know this is the DADM II course which is Data Analysis and Decision Making II course under the NPTEL MOOC series. And this total course duration is for 12 weeks which is 30 hours which when converted into lectures it will be 60 in number because each lecture is half an hour and each week we have 5 lectures of half an hour each and after one each week we have assignments.

So, we are in the as you can see from the slide we are in the 48th lecture which is in 10th week and we will start a small new topic about something to do with project management related one, which is in very generally specific in nature. Another thing which I wanted to mention that in the 49th and the 50th lecture on the sessions would be conducted by Ms. Priyanka Sharma a PhD student. She will basically be doing a very specific examples in the area of marketing, trying to basically show that how different type of very simple modelling techniques not from optimization point of view, simple modern techniques can be utilized for different about decision making specific to marketing. And my good name is Raghu Nandan Sengupta from the IME department at IIT Kanpur.

So, I will start with the other topic in this 48th lecture then 49, 50th would be taken in the area for the specific examples; only examples and then starting 11th and 12 week we will cover some topic of GERT and other topics in the area of DADA II as already scheduled.

So, whenever you are doing some activities. So, I will give you very briefly about that ideas of GERT and Q-GERT which is GERT is generalized a review technique and; general evaluation review technique and Q-GERT would be queuing generalize evaluation review technique. Now whenever you are doing project evaluation review technique or you are doing critical path matter. The main concern is that when you start from the source to the sink, starting and there are two ways how you do that, one is activity based on arc and activity based on node. Where activities with their time

duration with the probabilistic or deterministic, whether they are slacks or difference of time between them we will basically denote on the node or the arc depending on whether it is on the activity on arc or activity on node.

Now, the main difference between PERT, CPM and GERT and Q-GERT is that in many of the cases they are looping. Looping allowed means say for example, you do one work and then the work is not yet completed in the full satisfaction then it comes for rework. Like you are doing some welding, the welding work is not proper. So, obviously there is some rework is done or the design from the you, when you are building up small factory building up of wall building up house or trying to basically make some material or make a machine a car whatever they may be some problems, they may be some issues for which or they may be as per the norm some rework is needed.

It need not be only problems that you face that you have to again do the rework. So, say for example, you doing some annulling process, tempering process or where the work has to be done by layers like you are doing the painting. So, the painting has to be coated consider you are doing a very simple painting.

So, red oxide is to be used so that rust this iron, pillars or the iron chairs whatever you are utilizing they are not affected by rust so, it is a protection then you paint it. So, if you do the work and again come to do two or three different times so obviously, rework would be done. So, if you want to consider those simple concept; I am giving very simple examples, if we consider those simple concepts looping would be allowed for which GERT and Q-GERT should be utilized.

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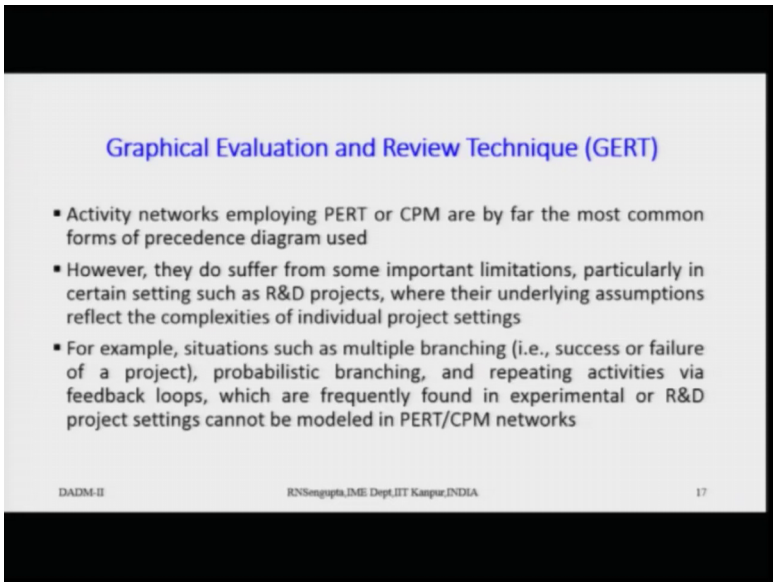

So, activity network employing PERT or which is Project Evaluation Review Technique or CPM are by far the most common forms of precedence diagram. However, they do suffer from important limitations, particularly in settings of the R and D like if you consider the R and D project of trying to basic come up with a new product in the market or trying to float a new fridge, a new refrigerate, a new AC, a different type of brand of car, if you want to test it; test market it or you want to basically come up with a new drug, a medicine whether for fever or for say for example, cough and cold.

So, you want to basically test it in the market get the response, how fast the drug is able to relieve the pain and then basically float it in the market, and you want to do experience also before on primates, then basically check its efficacy of the drug on human beings and then again market it.

So however, so let me read the second point. However, they do suffer from some important limitations, particularly in certain settings such as R and D projects, where their underlying assumptions reflect the complexity of the individual product project settings. A very complicated project like building, a factory or say for example, building up a nuclear plant or trying to basically check that how the launching of say for example, in this satellite should be done.

I am considering the work being done by say for example by BARC, by ISRO. So, for example, situations such as multiple branching, such there is a success or there is a

failure of the project again you do the reworks. So, they are probabilistic branching methods like you float a product. There is 40 purchase chance the product will be accepted my by the market with good demand, 30 percent may be the case with the product it acts accepted with a low demand and 10 percent with the case where the project is product is not at all accepted.

So, in the probabilistic branching and repeating activities via the feedback loops which are frequently found in experiments or R and D projects cannot be taken into consideration when you are doing the project evaluation review technique or the CPM method.

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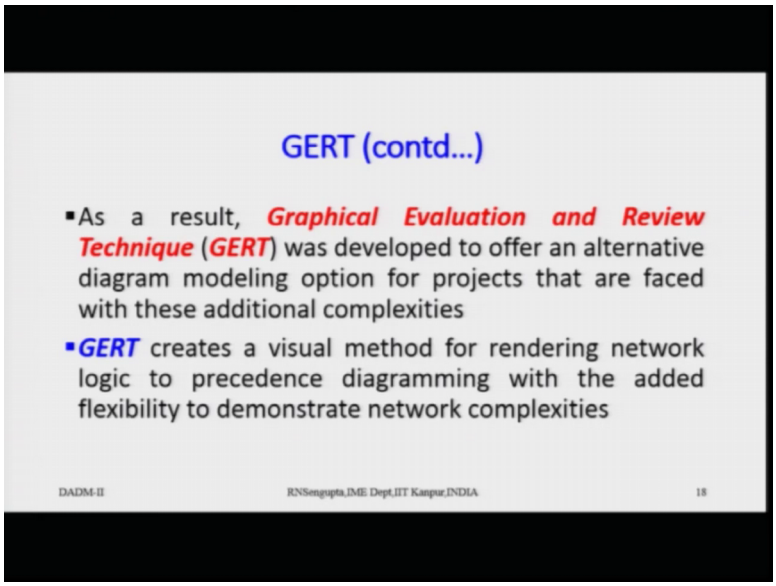

As a result Graphical Evaluation and Review Technique GERT would be employed or can be employed to offer an alternative diagram modelling option for projects that are faced with these additional complexities. So, GERT creates an visual method for rendering the network logical flow.

So, how you go from all the concept would be from the left to the right. So, if you are going from activity 1 to 2, 2 to 3 and then once 3 is done again you come back to 2 to do some rework. So, all these things would be considered and in very minor details in the GERT process. So, GERT creates a vision method for rendering network logic to precedence diagramming with the added flexibility to demonstrate network complexities.

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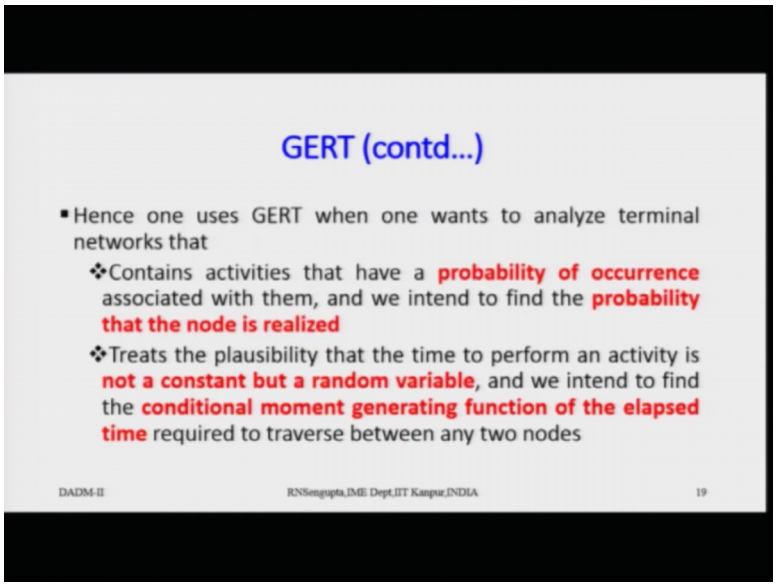

Hence one uses GERT when one wants to analyze terminal networks that contains activities that have a probability of occurrence associated with them and we intend to find the probability that the node is realized. So, the flow which is happens from the left or the right.

So, GERT also treats the possibility that the time to perform an activity is not constant. So, it is probabilistic. So, you will; if somebody has done PERT and CPM, I am not going to go into details of them they would be a most probabilistic time, more or pessimistic time, then most optimistic time and the most probable time. So, you will take these values of t suffix, a t suffix, b and t suffix m and do the calculation considering the type of distribution for the time duration would be there for each and activity which you do in PERT and CPM.

So, GERT creates the plausibility that a time to perform an activity is not a constant, but a random variable with a certain distribution and we intend to find the conditional moment generating function of the elapsed time required to traverse between two nodes, two activities such that we are able to find the particular distribution, the variance, the mean, the median based on which we can find out the average time. And the variances if at all from node 1 to node n considering there are n number of nodes from the start to the finish for that job.

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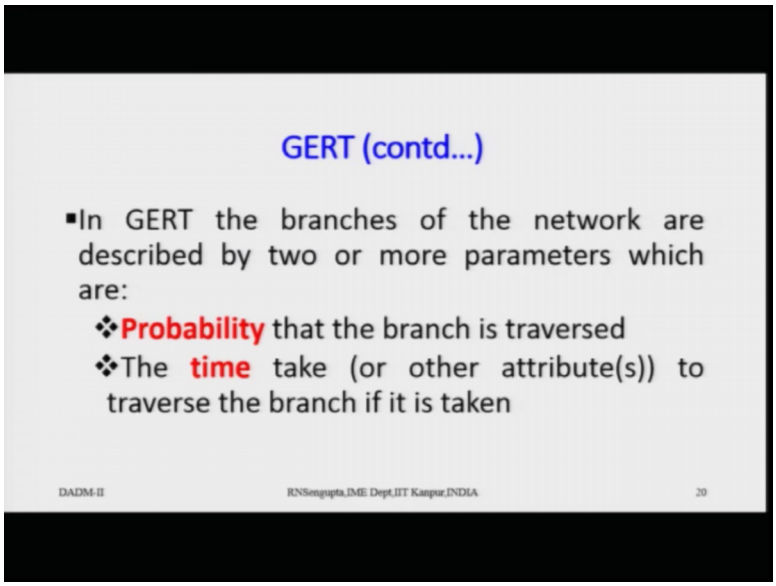

In GERT, the branches of the network are described by two or more parameters which are what is the probability that the branch will be traversed. So, obviously as I said product being successful 40 percent, product not means that successful 30 percent and product be in a failure by can be 20 percent.

Say for example, you are doing some project where you are the main engineer chief engineer of an oil exploration plant or a company and you want to basically test whether their oils in some base in a really can be Godavari base in, it can be Arabian sea, it can be in a North sea whatever it is. And you want to basically do some test marketing of the type of of the seabed or the floor where you want to basically search for the oil or the minerals which are there, considering that you are searching for minerals also.

So, if you do a testing of the surface of the geophysical characteristics of the rock surface, then obviously it will give you; it can give you different hints in one case it can be it is a high probability that good quality of petroleum ore may be there. In the second case there is a average probability that the petroleum ore would be there and in the third case the; I am only considering three cases so, the third case can be the probability of getting in or is very less.

So, in both the cases you will basically go for a drilling experiment or drilling exploration and if the probability is very high then the drill would give you different high levels of petroleum ore. If the probability of finding ore is medium, then the actual

chance of getting the petroleum ore would be much less than the first case and in the case when the actual the testing scientific testing which you did, geophysical testing which you did, geological testing which you did, if the probability of finding an on the petroleum more is less, obviously in the later case when you actually drill it will be much less.

So, they can be different ways how we can implement that overall thing. So, probability would be there; there would be a probability distribution oh point 1. Point number 2, if you remember, I just I mentioned that you do some geophysical or geological test and then you basically go for the drilling. So, obviously the probability of getting an ore would depend on what type of the rock surface which you have, which is that the rock surface quality the probability of finding a particular type of rock surface will dictate what is the probability in the later stage.

So, stage two when you are drilling would be dependent on stage one such that you will basically have a conditional distribution of finding and the actual ore when you drill. So, say for example, if you have three stages and in consider the stages are like this you would you find do one geophysical test, then you drill and then you market the ores. Consider you have already gone into a contract with different type of companies where you will market the petroleum ores.

So, if the probability of you making a profit depending on whatever the investment which you have done in third stage where you are basically marketing it, it will depend on what is the probability of the second step actually finding the ore and it will also depend on what is the probability of the experiment which you have done in trying to find out whether actual good quality of ore would be available.

So, it will be stage by stage the probabilities would be given and we have very simple concept of Bayesian analysis and if you remember those who have done DADM 1, I have very basically covered the concept of Bayesian analysis and how conditional distributions can be utilized.

The second thing which would be the parameter, which would be required for the GERT process would be the time. So, the time taken to traverse the branches if it is taking and also remember the time taken to traverse a branch from node 1 to node 2 or node i to

node j considering there are no other nodes between these two nodes i and j that can also be a probabilistic one.

So, consider that the work being is doing done by a consider the concept of painting. So, though if the work is being done by automatic machines or semi-automatic machines with respect to the workers so obviously the time taken in the initial case when you are using machines would be much faster, precision would may be much better. But in the other case obviously, when using the workers the precision may not be high that high. But you should consider the cost factor which is also important when you are doing your project.

So obviously, time is probabilistic and the branch which is taking is also probabilistic. And the branches which are taken would basically have a distribution as I mentioned it can be also a conditional distribution and the probability of the time being taken to traverse would also have a particular distribution. If you remember, I did miss in the PERT and CPM case we take the most optimistic, most pessimistic and the mean time or the average time most probable time based on which we try to basically find out.

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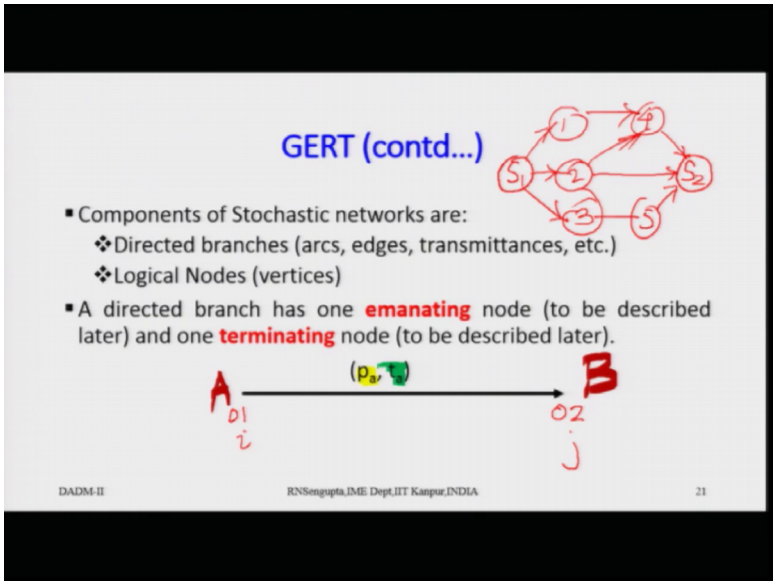

So, the components of the stochastic network because so, if you are coming to the GERT network. So, each path you will take, it will basically have the probability which I will mark. So, it will have a probability. So, this is the probability p a considering that if I

consider the path is 0 1 to 0 2 or i to j. The time taken to traverse from i to j is given by t a and the corresponding probability is given by p a.

So, component of stochastic networks are directed branches which has arcs, edges, transmittances etcetera as it is shown here. and we will mention the starting and the ending say for example, by A and B. They would be logical nodes also or vertices based on which you will proceed from which direction to which direction it will go. It will have a flow and we will consider from the left; from my left to the right. So, if you are considering from your side it will be from your left to the right.

A directed branch has one emanating node from where it is going to start to be described later on we will come to that later on and one terminating node to be described later on which we will discuss that how it starts and how it finishes. So, if I consider very simply the logic would be I start at say for example, the starting node and consider my ending node is here, but it may be possible that all the paths which are taken, are such that I will follow all those paths in order to basically reach my destination.

So, I will consider this as a source S 1, this as a sink S 2 and all the paths would be meantion or the nodes would be mentioned in the logical sequence matter. So, that means, you can basically follow any path and try to reach from the source to the sink in a certain amount of time depending on the time and also what the probabilities that those paths would be followed.

So, I have given a row arrows and where over this each paths your probability of taking that path along with the time to take that part would be given.

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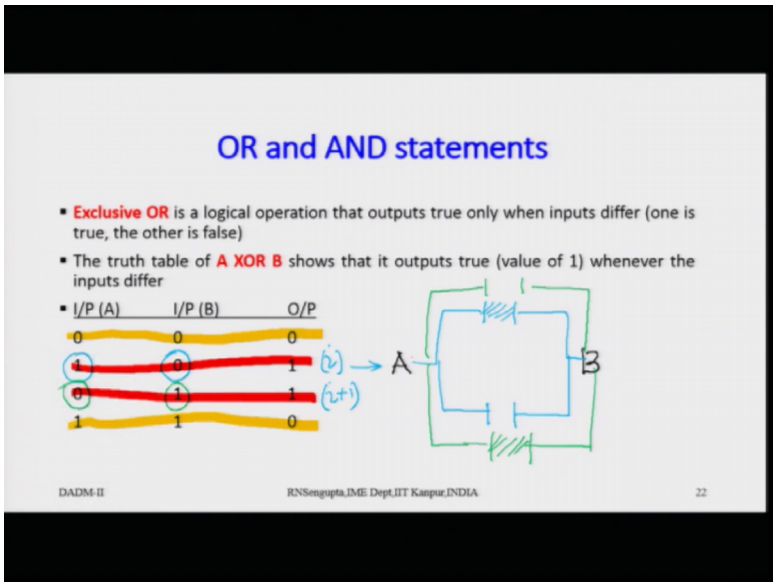

Now, I will consider very simplistically the AND, OR statements and AND, OR logical statements because based on that we will try to basically build up the model. So, the concept of Exclusive OR is a logical operations or process that outputs true only when inputs differ. So, when the inputs are like this.

So, input A and input B, if they are same the output is basically 0. Inputs are same. So, 0 0 gives you a combination of 0, 1 1 gives a combination of 0, but if the inputs two of them on obviously, they can mean more than 2 and you can combine them accordingly. And the inputs are 1 0 and 0 1 opposite to each other, so it should mean that the overall output would be 1 here, no consider this one, how it can be? Consider the 1 is an ON, 0 is in OFF. So, let me draw the simple logic. So, I have A, I have B so, this is electricity is flowing some flow is happening and consider the 1, first one which means it is ON, connection is ON. Consider the second one is 0; that means, there is no connection. So, still current flows from A to B. Hence the output is 1. So, this takes care of this one point on the diagram which I stated.

Now, consider I go for the second one. I consider i plus 1. Here I will use the blue colour here now or the green colour so, it is easy. So, this is 0 so, the first one is OFF, second one is ON. So, again the current flows from A to B. So, that takes care of the red marked one where opposite reactions from on opposite output from two different inputs give you the output as positive or 1.

Now considered the other one which has marked in yellow so, let me erase this. So, come to this example later on. So, with examples it will be much easier for me to explain. So, you have YES-YES node and the actual output is 0. So, in this case we will consider the if the outputs are opposite to each other, the output means the outputs happening from A and B which are taken as an input for the final decision. If they are opposite, then your actual output which is happening from A and B is a YES and if the outputs are from A and B which are the inputs; from A and B are same then the combined effect would be a NO. So, the truth table for the A XOR B which is Exclusive ORs would be utilized in such a way that A 1, B 1 output is 0, A 0, B 0 output is 0, A 1, B 0 output is 1 and A 0, B 1 output is 1.

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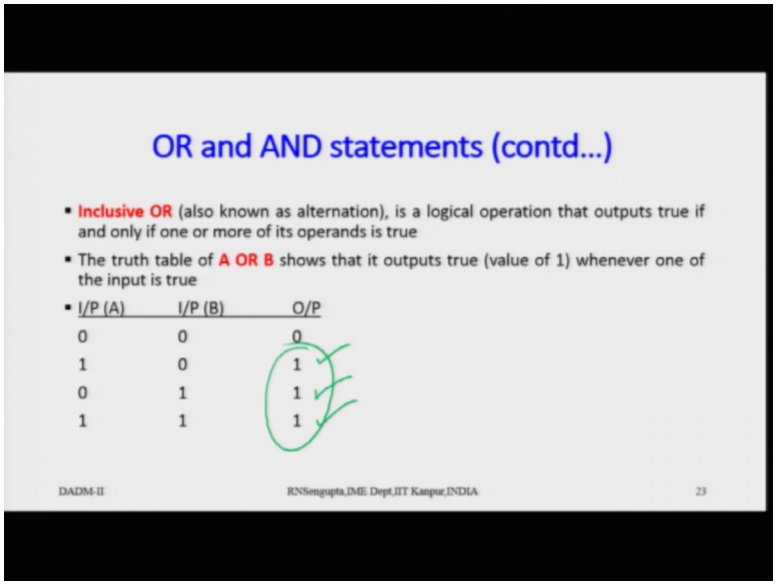

Now, I go to the Inclusive OR on also known as the alternation. It is a logical operation that outputs are true only if one or more of its operands are true. So, in this case we will consider like this. So, this A or B table would be output A is 0 null or null means it is negative, output B is considered as null, in that case you will basically have the total output. So, both are NO, NO output is NO. The moment one of them is YES so, in that case the output for A 1, B 0 is 1, A 1, B 1 is 1 and A 0, B 1 is 1 which means A 1, B 0 1, A 0, B 1 1, A 1, B 1 is 1. So, all these three things are true statement considering the moment you have any one of them or more than one of them as truth statements.

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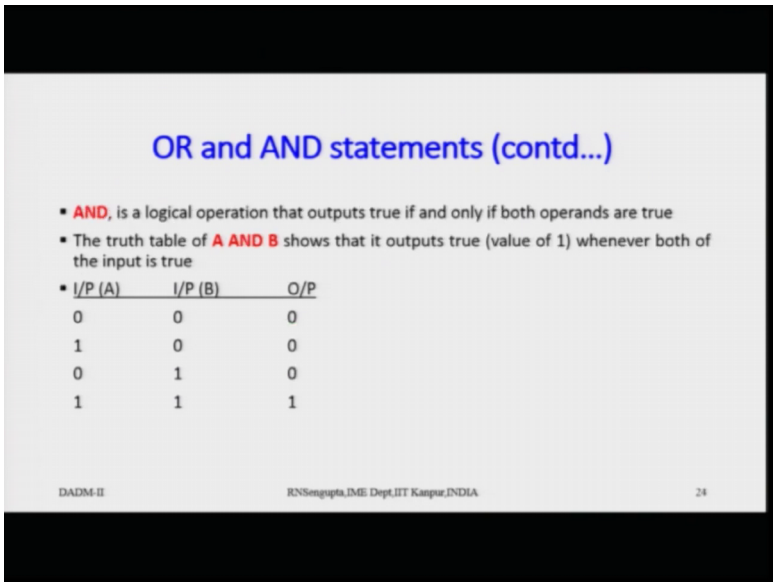

In the AND in the logical statement you will consider that if both of them not true, then only you will have the truth statement otherwise not. That means, if A is 0, B is 0; obviously, it will be 0 if A is 1 or B is 1; that means, if A is 1, B is 0 or A is 0 and B is 1 in both the cases the truth statement will be a null or 0. Null means it is false and only if A and B both are true you will basically have the truth step and as 1. So, this is an AND logical operation that outputs are true if and only if both operands are true and the truth table for A and B are given.

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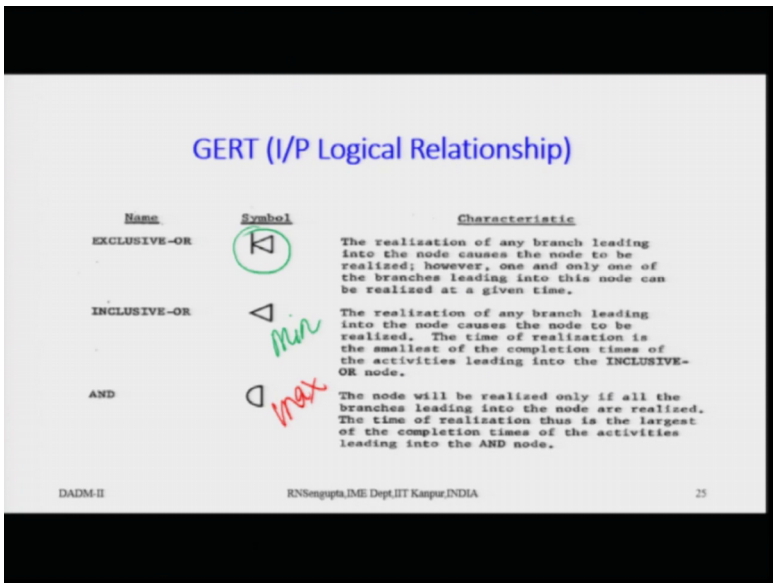

Now, in this case we will formulate with the symbols for the GERT. So, the Exclusive-OR would be a triangle operator with a vertical line. So, the realization of any branch leading into the node causes the node to be realized. However, one and only one of the branches leading to this node can be realized at any given point of time. So, it is an Exclusive-OR.

Inclusive-OR would be the triangle only. So, the realization of any branch leading into the node causes the node to be realized. The time of realization is the smallest of the; so, consider there are different time periods you will take the minimum of them. So, it says that the time of realization is the smallest of the completion time. So, of the all the activities leading into the Inclusive-OR node and the AND node would be the node will be realized only if all the branches leading to the node are realized, the time of realization would be the maximum of them. So, in this case you will take the minimum and in this case you will take the maximum and based on that you will try to basically formulate the GERT network.

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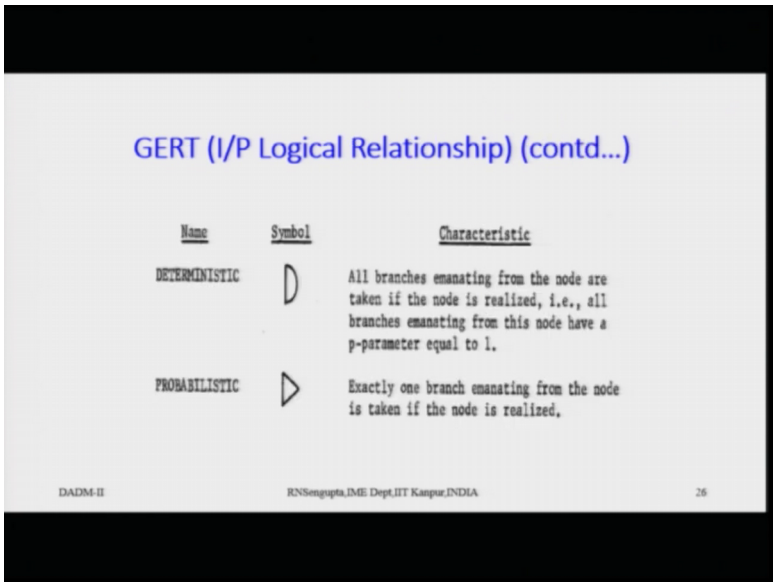

In the deterministic sense and the probabilistic sense also you will have different symbols. So, if all the branches emanating from the nodes are taken, if the node is realized that is all the branches emanating from this have a p-parameter equal. So, the if you basically find out the probability in the sense and if the probability of traversing those paths is; obviously 1, that is they have to be traversed and obviously, the

corresponding probability would be 1. And if exactly one of the branches, so in the initial case they would be the branches would be such that the path should be taken with probability 1 and in the second case when the probabilistic exactly one branch emanating from the node is taken if the node is realized and in that case all the branches emanating from the nodes would be taken.

So, it does not mean the flow is going to happen in such a way that only one path is taken in the first case. So, there are branches OR are taken and OR with probability 1 and in the second case probabilistic one exactly one of the branches emanating from that node would taken with the corner corresponding probabilities already mentioned.

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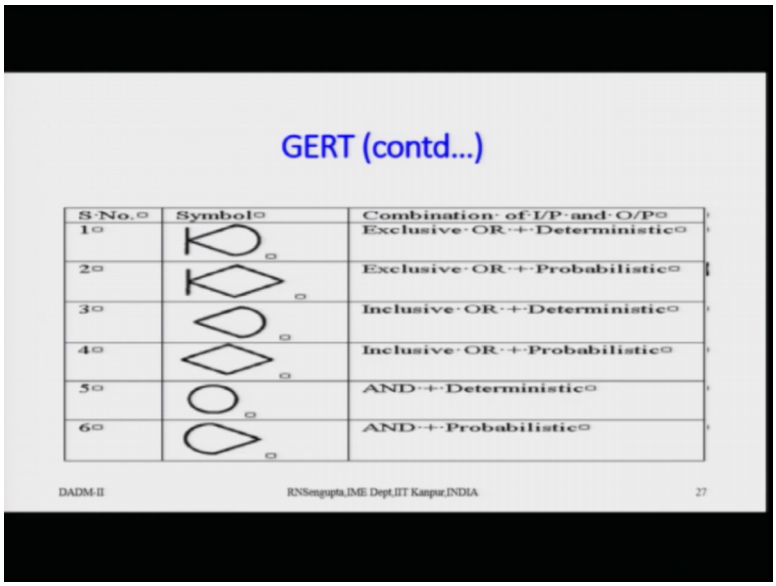

So now, if we combine the whole combinations which will come out from these Probabilistic, Deterministic, Exclusive this AND on OR concepts are like this. See in the first case it is an Exclusive-OR and a Deterministic one, you have the symbol as given like a parachute which is put on a horizontal plane. Now in the Exclusive-OR probabilistic one, you have the diamond shaped and in the first case the parachute with the base. In the third case Inclusive-OR a deterministic one who are parachute without the base and in the Inclusive-OR or probabilistic one you have the diamond shape without the base which was there in the second case and the AND and deterministic part and the AND the probabilistic part with the circular one and the parachute which is 180 degrees opposite.

So, with this I will end this 48th lecture and as I told you already the 49th and the 50th lecture for this week which is the 10th week would be simple examples from the area of marketing in related areas which will be taken by a PhD student and she will basically discuss the overall application of these methods which are quite heavily used in the concept of marketing, in the concept of statistical learning, in the area of say for example, flow process in mechanical engineering in operation research in a big way.

So, thank you for your attention and have a nice day.

Thank you.