Data Analysis and Decision Making - II
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Lecture - 44
Multi-Objective Optimization

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are either in India or Abroad. And as you know this is the DADM-II which is Data Analysis and Decision Making-II course under the NPTEL MOOC series and this total course, total number of a contact hours over the lectures is 30 hours which is which can be broken down into 60 lectures because each lecture is for half an hour and this course runs for; at least this DADM-II runs for 12 weeks and each week we have 5 lectures each lecture being for half an hour as already mentioned and after each week we have assignments.

So, we have 2 more lectures to go, before we wrap up the 9th week and similarly we will go into the 10th, 11th and 12th week to wrap up this whole course. So, if you remember in the and my good name is Raghu Nandan Sengupta from the IME department at IIT Kanpur. If you remember in the last class we were discussing about Pareto optimality cons considerations and Pareto optimality considerations means, that two solutions or two outcomes or two alternatives or two decisions whatever they are, they are not worse off to each other with respect to your decision in the sense, the overall bundle of worth which you get by taking any one of these alternatives gives you the same worth.

Now, if you decrease in one you increase another and vice versa. So, hence the total cumulative one remains the same and I will show you examples for that as we proceed with the concepts. So, Pareto optimality conditions are what basically of part of multi criteria decision making which we have been discussing which is or multi attribute decision making and we consider there are many areas of work. It can be say for example, as I did mention it can mean in finance, finance problems we will consider when you are buying a house, when you are buying a car, when you making a decision that you have to choose any one of the people who has person who has applied for a position you want to take him or her considering there are different criterias, different attributes, different characteristics, different objective and subjective criteria based on which you will make a decision to select him or her.

So, it can be say for example, in production line it can be in petroleum engineering and all these things. So, some of the areas where multi objective optimization are used it can be in economics, where you have to give take a decision where the total amount of resources is constrained and you have to get maximum utilization of that and you want they can be different bundle of decisions which gives you the same word so, you want to make a take a decision which is best. In production technology there are different products which can be produced with resource constraints, capacity constraints, supply constraints, utilization constraints based on which you will find out the best optimum solution such that it increases your profit, lowers your losses and also you try to basically make a compromise, compromise in the in the sense that you want to basically make a decision which gives you the best combination of the compromise;which is best worth for you.

In finance it can be where you want to basically invest for a portfolio. Portfolio consists of say for example, n number of assets and assets can be financial goods, it can be bonds, stocks, derivatives or whatever it is. And you want to minimize your loss or minimize your risk maximize your profit which is expected value. So, you want to make a combination which are the n number of assets you will take in order to achieve that criteria. So, it need not be maximization of the expected value minimization loss it can be other criterias also coming into the picture if it becomes more than two objective functions.

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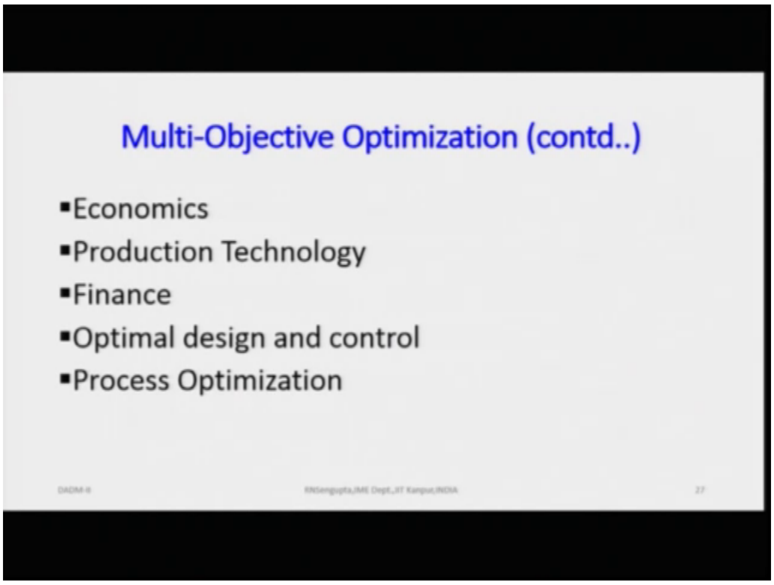

In optimal design and control when you want to basically design a very sophisticated instrument of electrical circuit, a cooling tower for a thermal plant or you want to basically assign or design the flow of heat through different pipes. So, there are different ways, you want to basically maximize the flow, rate flow and minimize the loss also. Or if you want to basically also maintain the ambient temperature between the outside and the inside in such a way that you are the overall process, chemical process works considers a chemical process which you want to basically continue. There you may basically like to minimize the total cost because insulation cost and all these things should be minimized considering this very high cost of insulation. So, all this would be considered.

In process optimization like you are doing fractional distillation you want to get different type of products from petroleum, from the ore and you want to basically will get the maximum of them utilizing minimum cost. Cost can be energy utilized, time utilized, the losses or the overall residue which comes out from this production process, process optimization process should be minimized. So, there are different ways you can do that, but at the end of the day your main focus would be to consider more than one objective function where the objective functions are such that you want to maximize on some front, minimize on some front and all do not go hand in hand in the sense that if you increase one of them the other would also increase. That means, if you are increasing the maximization concept the minimization value would also be increasing in such a way that it will have a one to one combined detrimental effect.

So, if you increase one the negative sense of the other also increases if you decrease one the negative sense of the other also decreases. So, which one you want to choose; the combination you want to choose to get the best optimum solution. We will consider a very simple example. So, this is the example we are considering. Consider that you have n number of financial assets. This is from the financial engineering and or optimization problem. So, you have a basket of n assets and I am for the simplicity I am considering only stocks, not bonds, not derivatives nothing not forward, not futures. We will consider basket of financial goods, financial assets which is the stocks.

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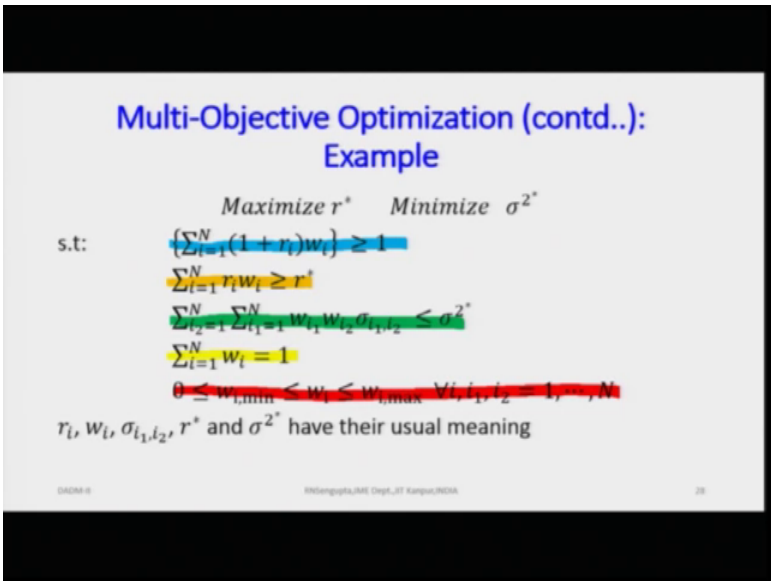

And we will consider that for this basket, the total portfolio which you formulate you want to basically maximize the return and you want to minimize the risk which is the variance. So, we will consider the variance is the best proxy for the loss. And we are considering variance as the best proxy of the loss considering the fact the distribution on the assets you are considering as normally distributed which in the true sense is not right, but we will still continue doing that, because it does it gives us good results. So, you want to maximize r star some value for the returns over which you want the portfolio turn to be and you want to minimize sigma square star which is some value of standard variance below which you want the variance of the portfolio to be.

So, r star is such a value that you want the portfolio returns to be more and sigma square star to be such a value you want the variance of the portfolio to be less. Because loss; obviously, you will try to reduce, profit you also want to increase. Profit and loss I am using in a very generic sense. Profits is something positive, loss is something negative. Now the constraints have a very simple meaning. So, I will go one by one. So, we will consider the weights which you have that the total amount of money which you have say for example, 100 rupees, 100 dirham, 100 dollars whatever it is you are able to proportion in some proportions divide into those n number of assets.

Say for example, if you have 10 assets and you decide that you will spend any 100 rupees and you spend one-tenth of proportion of your total money in each of these assets. So, one-tenth of 100 would be 10 rupees would be into invest in an asset 1. Similarly 10rupees in asset 2. 10 rupees in asset 3 and so on so forth till the last asset which is the 10th one you will invest 10 rupees. So, the here the w 1 to w n where n is 10 would be equal to 1 by 10 such that, that the sum of all the weights actually should be 1 which is true. So, if you consider so I am not going sequentially for each and every constraint I am just discussing from the simple 1 and then make it much more logic. So, it is much more logical to all of you. So, the first one is that weight of all the asset weights which have invested which I just gave an example w 1 to w n they sum up to 1. So, if the first part which we decide that the weights of the; sum of the weights of all the assets is equal to investment being done is 1.

The second one is which I will consider and mark a different colour; second one is this which is also very simple to understand. What does it mean? That the weight for each and every asset which you are going to invest out of that 100 rupees which we had would always be between 0 and 1. So, 0 is here; that means, the weights cannot be negative which is right and it cannot be more than 1 which is also logic very logical. So, we are considering each of these w 1 to w 10 are bounded between 0 and 1. Now you to make it a lot a little bit more realistic you what you can have is the weights for each would be bounded by their minimum value and the maximum value. So, if say for example, w 1 will be bounded below by w 1 minimum and bounded above by w 1 maximum; that means, there are some values corresponding to w 1 between which w 1 value has to be. Say for example, it can be that for the first asset which you are going to invest; consider that is Tata steel stock or Tata Motors stocks or say for example, Hindustan Unilever stock in India, I am talking from the Indian context and the weights which you have to invest in this stock has to be greater than 20 percent, but they cannot exist 80 percent in that case w 1 for these three cases or examples which I gave would be greater than equal to 0.2 and less than equal to 0.8.

Other case can be say for example, you would consider the State Bank of India stock in the stock market and you are under the obligation or under the condition when you are doing trying to invest is that, the weight of State Bank in India of India stocks cannot exceed more than 50 percent in that case the value of w 2 which is the State Bank in India would be greater than equal to 0, but less than equal to 0.5; that means, the values would always be between 0 to less than equal to 0.5.

So, similarly you can have some constant on w 3, some constant on w 4 some constraint on w 5 and so on and forth. Say for example, for the w tenth one. Whatever the asset is you will say that I have to invest or you put the condition that you have to invest more than 70 percent of your money in that w 10 stock. In that case w 10 would be greater than equal to 0.7 and less than equal to 1. So, based on that you can all these ideas which I gave you can have different bounds for the w is such that the sum of the weights which we have already discussed at the first priority condition which is marked in yellow would always be followed. Now we will come into the subsequent third, fourth and fifth constraint in the sequence or the consequence that how easy it is for us to understand the first one which was yellow, second one was red was very easy for us to understand.

Now, consider the third one and I will mark the color as orange. So, what does it mean? Consider the returns. So, now, here the r i's the returns for each and every stock which we are investing. So, returns are the if you are investing say for example, 100 rupees on day 1 and if I get 110 rupees in day 2 I will consider the returns and we found out in 2 ways, 1 would be 110 minus 100 divided by 100 is the rate of return which will be denoted by small r and if I want to find out the value of capital r then the rate of the rate of the return would be generated by 110 divided by 100. So, we are considering here the small r which is the rate of return which is given by 110 minus 100 divided by r. So, which is actually equal to the investment value which we have done which will come in the denominator and the difference in the net outcome which will get after one day minus the initial investment would give would come into the numerator. So, the numerator value can be both positive and negative.

Now, if I consider the returns as r i for each and every stock consider these are the ten stocks which I which I just mentioned n is 10. Then if I want to find out the overall return on the portfolio which is being formulated by this 10 stock would be the sum of the corresponding returns multiplied by the weights which I am doing for stock 1, similarly for stock 2, stock 3, till the tenth stock which means I will multiply r 1 into w 1 add it up with r 2 into w 2. Then I can add it up with r 3 into w three and so on and so forth till I have all the values of the r i's and w i's multiplied together. Hence the left hand side would mean that overall portfolio return will be the sum of the multiplicative values of the returns and the weights.

Now, why it is greater than r star? Because consider I have told to myself before I invest that I want the return of the portfolio where I am going to invest has to be greater than equal to say for example, 12 percent. So, in that case r star is the value which I have basically fixed for myself for my investment is 12 percent such that the sum of r i into w i would be greater than 12 star. But the question would obviously come up in your mind is that why are we trying to maximize r star also, it means that what I am doing is that I am trying to basically push consider the real line from my side. So, if this is the value of 0; if I go on to the right hand side it increases if I go onto the left hand side it decreases. Consider that 12 percent value which is there. So, I consider the 12 percent value which would be 0.12 and I am trying to basically increase this value of 0.2, 0.1 to more and more on to the right that it becomes 0.13, 1 4, 1 5, 0.15, 0.16 so on and so forth. So, as that the values of the different type of investment which I do and the net return on the portfolio which I get for each and every cases which is the sum of r is w is would be always greater than the r star value which I am trying to fix for this problem. So, as I keep increasing r star the overall combination of the weights for those before that portfolio will be such that the summation of weights into the returns would always be greater than equal to that r star value which I have.

Now, in the similar way as I mention the third constraint. In the similar way, let us consider the forth constraint. This will now make things are much more simpler for you to understand. Consider that if your returns are to be increased then in the same sense; the risk or the loss has to be decreased. So, can again consider the real line. Now consider what is risk? Risk I mentioned is the variance. So, consider sigma 2 star, star is the basically fixed value which I have consider it is say for example, 0.20 in percentage sense. So, what I would want for my portfolio it overall variance would be the value of that variance for the portfolio would be always be less than equal to 0.2. So, it will be on to the left. So, what I will try to do is that I will formulate my portfolio in such a way that the variance value would be pushed more on to the left that will keep decreasing and it as it decreases that the total combination the weights which I have for the portfolio when multiplied by the corresponding variance covariance matrix or by the standard deviation of each and every portfolio is taken together is multiplied along with the correlation coefficient would be such that the value of that overall portfolio's risk would always be less than that sigma square star value which I have.

Now, the value which I have which is double summation and I am not talking about the summation values i 1 from 1 to N and i 2 from 1 to N. It is basically w 1 i and w 2, w i 1, w i 2 into sigma i 1 and i 2 is basically the multiplicative value of the weights multiplied by the covariance variance matrix which I have for the whole set of combination of the assets which are being taken into consideration to formulate the portfolio where I am going to invest. So obviously, that value would always be on to the left hand side less than equal to sigma square star. The final values of the constraint is this one that we use the blue color, is this one which is also very logical in the sense consider that initially when I start investing my total amount of returns which I have would be such that the final return after one time period, two time period whatever I have would always be greater than equal to the total investment which have already done in the initial time.

So, say for example, if I have invested say for example, 100 rupees I would ensure that the combination of the total portfolio would be such that the total returns would be 100 plus some positive value such that it be makes sense to invest in some combination in those in that portfolio such that I am able to meet all the three criterias. Point 1, the overall return on the portfolio is always greater than equal to r star value. I am trying to maximize r star value. Point number 2, the overall variance covariance or overall risk of the portfolio is always less than equal to sigma star square star value which I am trying to minimize the sigma square star value. And number 3, is that the overall return which I am getting from the investment would be always be greater than the total amount which I based on which I am already investing from 0.1 or 0.0 which means that if I as I mentioned; if I have 100 rupees the total return which I will get in the value sense from the total investment which is being done in the portfolio would be definitely greater than equal to 100 rupees.

Now, here in the last line it mentions r i, w i, sigma i 1, sigma i 2, sigma i 1 i 2, r star and sigma square star have these usual meaning. Where r I am again I am mentioning is the return of each and every stock w i is the weight which you are investing in each and every stock, where i is equal to 1 to N sigma i 1, i 2 is basically the covariance values for the combination of the i 1 and i 2 stock taken to get to together. So, if there are capital N number of such number of stocks the total number of combinations would be n c 2. So, for any values where i 1 and i 2 are unequal then you will basically have the covariance values, if they are equal you will have the variance values of each taken with itself. So, you will basically go through the principal diagonal in the covariance variance matrix when i 1 is equal to i 2 and if they are not; obviously, you will go to the off the diagonal element which is the covariance values.

Now based on that what we do is that we analyze the data and so you have to optimize it this problem. It will utilizing the data for the Dow Jones Industrial Average we take the data from the wikipedia, It is easily available. And we take the time frame from 1st January 2012 to 31st January 2015 and we basically formulate the portfolio based on the fact that you want to maximum r star minima sigma square star subject to the constraints as mentioned.

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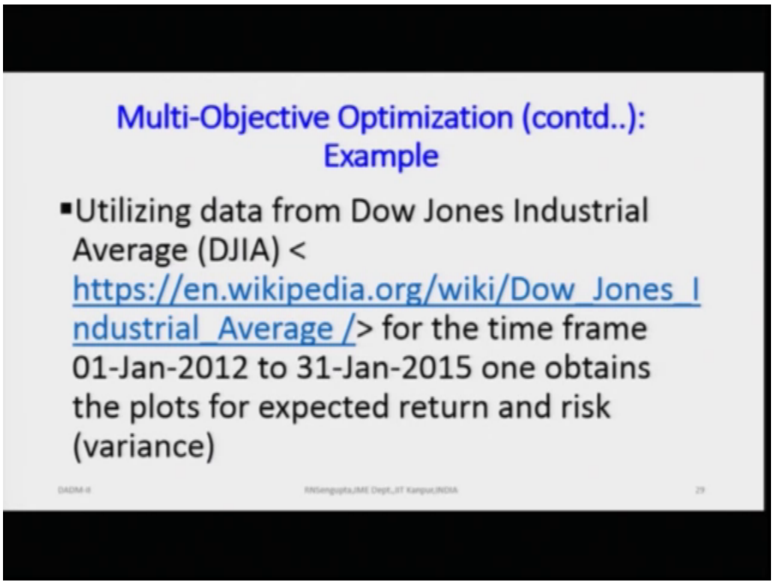

I am again mentioning; 1 will be the initial investment which I am doing should always be less than equal to the final investment value which I get. Number 2 is that the overall portfolio returns is always greater than r star such that we are trying to push r star more on to the right that means, increasing it the overall variance or the portfolio will always be less than equal to sigma square star such that we are pushing sigma square star more on to the left and finally the weights or the portfolio should add up to 1, along with the fact that the weights are always bounded between w i max for the i th stock and w and it will be bounded lower by the case of w i min which is the i th stocks lower values of the weights.

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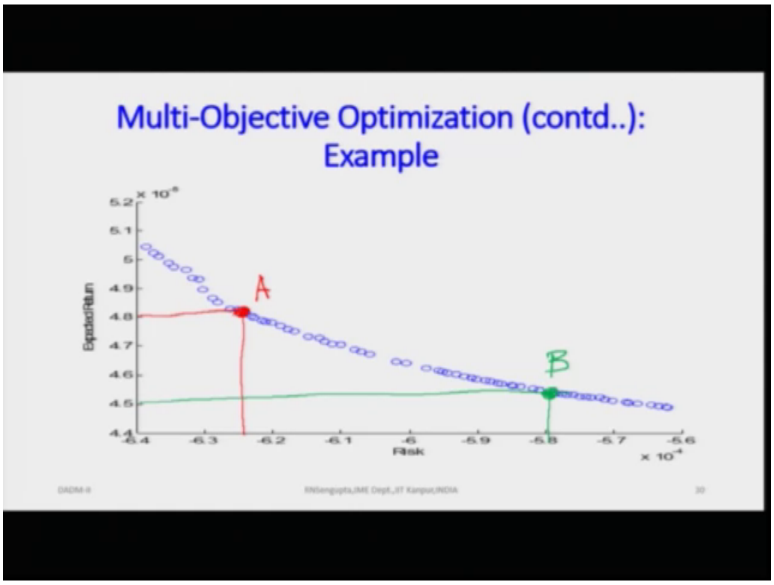

So, once we have this when once we plot it. So, what we do is that we in the simulation problem we generate about say for example, 100 values for the simulation case and as we keep changing the values of r star and sigma square star values which are pre determined from my n are from the investors n, i have a very simple Pareto optimal solution which means that the expected return if I consider, let me use the red color for 4.8. The overall return value comes out is value comes out to be about minus 6.25. If I consider a return expected return a 4.5; the overall value of risk. So, this; remember this is 10 to the power minus 4. This is the expected value there is no and it is 10 to the power minus 5. So, the values which I have here, based on the fact I want to maximize r star and minimize sigma square star would be such that under those consideration I am equally disposed both for A as well as for B So, there can be different combinations along the Pareto optimal solutions where I am plotting expected returns and risk to get the best combination of the stocks which will give me this optimization value which is true to my consideration.

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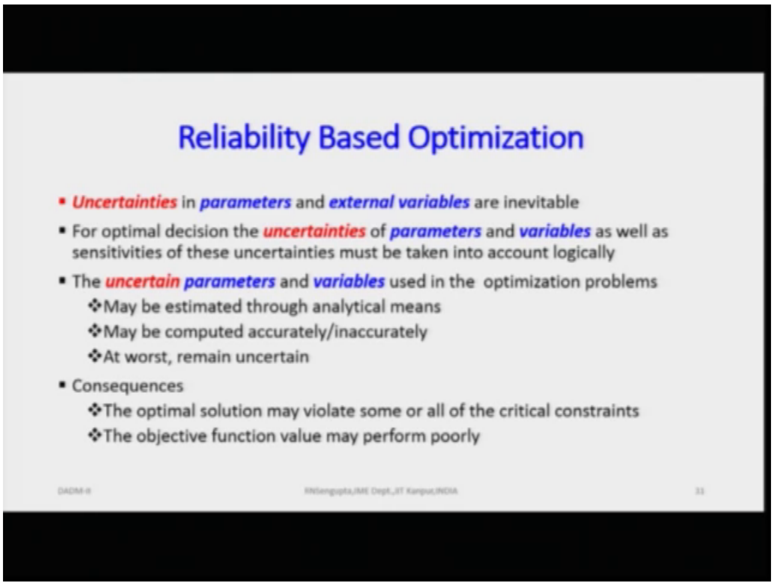

Now, we will consider different idea which is basically known as the Reliability Based Optimization. I will just give the background of that. So, whenever you are solving a problem there would be uncertainties both in the parameter values as well as in the external variables based on which you are trying to formulate the problem. So, what can this can be? Say for example, I am trying to optimize the design of a car and the parameter values can be say for example, which are internal to me can be the strength of the steel which I am going to use can be say for example, the type of quality of door or door handle which I am going to use I am giving an example or it can be say for example. How thick or how strong the glass is the windscreen and or means or it can also be what is the overall speed at which the crowd would be traveling on an average and average as it continues either covers 100 kilometers or 200 kilometers at 1 stretch. And the external variables of the variables can be say for example, what is the road condition? What is the wind speed? Or what is the say for example, tiltration of the road or what is the actual say for example, the effect of humidity? What is the effect of rain? What is the effect of temperature? So, these are variables which are not under a control when you are designing it.

But you consider these variables are even though they are external they are not fixed they will change and some of the parameter values you which you want to design for yourself are in general under your control in the sense that you know the distribution based on which you are going to model them. Hence uncertainties in parameter values which are internal to the system and external variables are inevitable and you have to take them into consideration. For optimal decision the uncertainties of parameters both the internal one and the variables as well as the sensitivityness of these uncertainties, uncertainties for the parameters and the variables must be taken into account logically to arrive at some logical conclusion.

The uncertain parameters and variables used in the observation problem or in the optimization means multi criteria decision problem which I am going to solve or which we will discuss. They may be estimated through analytical means; that means, you have say for example, some statistical tools, you have the mathematical formulation, you either differentiated, use the concept of maximum likelihood estimation, use the concept of general methods of movements whatever it is you are able to find out analytical solutions for those parameters, from the samples which you have based on which you are trying to work.

If not say for example, you have good computing facilities you have some very nice ideas of bootstrapping, you have and you use the sample in such a way that based on the sample size, based on the way you want to bootstrap you are able to estimate these parameters from the simulation runs. And the worst case scenario would be where you are you do not have any mathematical formulation to solve them, you do not have any simulation result to solve them you cannot assume any values they are just kept uncertain and based on that you have to basically proceed with the problem in order to consider those uncertain values in your optimization problem.

What are the consequences? The consequences of these three things; if the parameters and the variables both are unknown, they can be found out as using some parametric estimation method or using some computing method or at the worst case they are unknown. So, in that case the optimum solutions may violate some of the constraints based on which you are trying to solve the problem. So, what are the consequence? If you remember that when you are trying to solve the problem it can be that the total number of trucks which you can use cannot exceed 4. I am just given an example say for example, you want to transport goods. Or say for example, total number of inventory which you can have in factory one cannot exceed say for example, 100 degree units whatever the type of goods are. So, if those constraints are there it may be possible that if you do not consider the concept of uncertainty in the parameters in the variables and they are kept unknown.

Then you may violate some of the constraints based on which you are trying to solve the problem and find out the optimum value. Even if you do not violate the objective function could be such that the actual value based on which you will try to deduce the answer, would not be true because it would give you suboptimal results. So obviously, the answer would be how would you solve it? So, here the overall emphasis would be to find out a way how you can basically formulate the uncertainty in the parameters, uncertainty into the variables such that you do not violate the constraints as well as on the other hand you find out the best possible solution from the objective function based on this conditions. So, with this I will end the 44th lecture and continue in the discussion the 45th one and give us some and give you some idea about reliability based optimization which is a tool under the ambit of multi criteria decision making have a nice day and.

Thank you very much for your attention.