Data Analysis and Decision Making - II
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Lecture - 43
MAUT

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you wherever you are placed whether India or abroad. And, as you know this is the DADM-II course which is Data Analysis and Decision Making-II course under the NPTEL MOOC series. And, this DADM-II as was DADM-I is basically for 12 weeks which is 60 lectures; each lecture being from half for half an hour. And, as you already know I do repeat in the beginning of each class each week we have 5 lectures each being for half an hour and after each week we have assignments.

So, we have already completed 8 weeks that is 8 assignments have been solved by you and also we are in a position and we are getting queries in the forum and we are considering it accordingly. And, today is basically the 3rd lecture in the 9th week which as you can see from the slide this is the 43rd lecture in the 9th week. So, we are considering the concept of stochastic dominance, for stochastic dominance was there was first and second. The first was the case when the cdf value of for decision a 1 is greater than equal to cdf value of a 2.

And, in the second order stochastic dominance was when the expected value of a 1 was greater than expected value of a 2 and I have drawn this diagram accordingly if you remember; I will draw it again if required as we proceed So, we will consider few of the different forms of the different way of expressions of the MAUT which is Multi Attribute Utility Theory, I will just mention them. There we would not be solving any problems apart from the very simplistic one.

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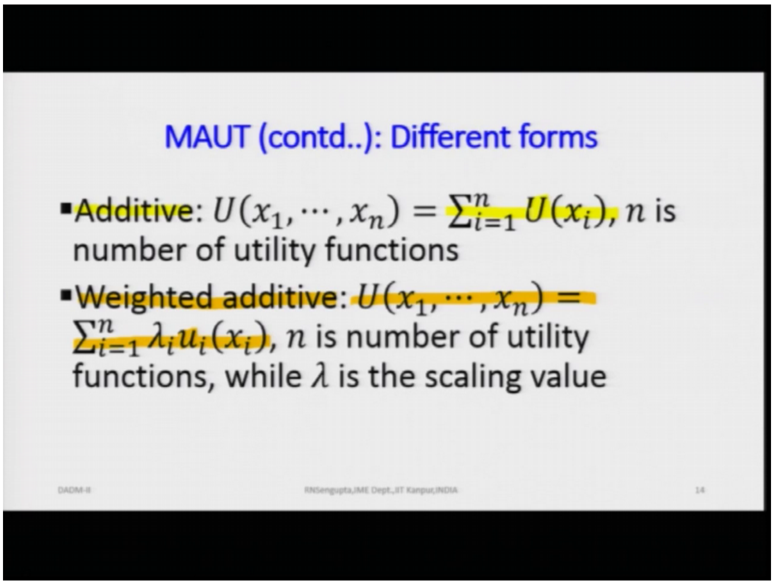

So, for the additive one; the utility function would be where you add up all the; so, technically let me go back a little one step backwards. So, generally you have a bundle of our decision which is bundle of criterias, criteria or attributes maybe for a car I am again given the same example a colour, car, boot space, passenger space, safety, style, mileage, price, resale value, per month maintenance cost all these things. Some are objective, some are subjective, some can be express that equations, some cannot be express the equation, they are just characteristics attributes.

Now, for the utilities of the combination of that vector x x consists of x 1 to x n depending on n dimension or it can be from x 1 to x k depending of k dimension of the properties. If we can express that whole utility as the sum of the utility functions for each individually then we have the additive function, The additive for the case if you can add up. Now, if we basically have the concept where you add up the utility function which is good, but you also give some weightages.

So, you will basically the weighted additive function and these lambdas based on which you find out the weights lambda 1 lambda, 2 lambda, 3 lambda n or lambda 1 to lambda k they would be known as the scaling functions for the utility functions. So, the weighted additive functions and these are the scaling functions. So, and this values of small u and n this capital U are the same, they do not make any difference. I am taking a utility function as a functional form only.

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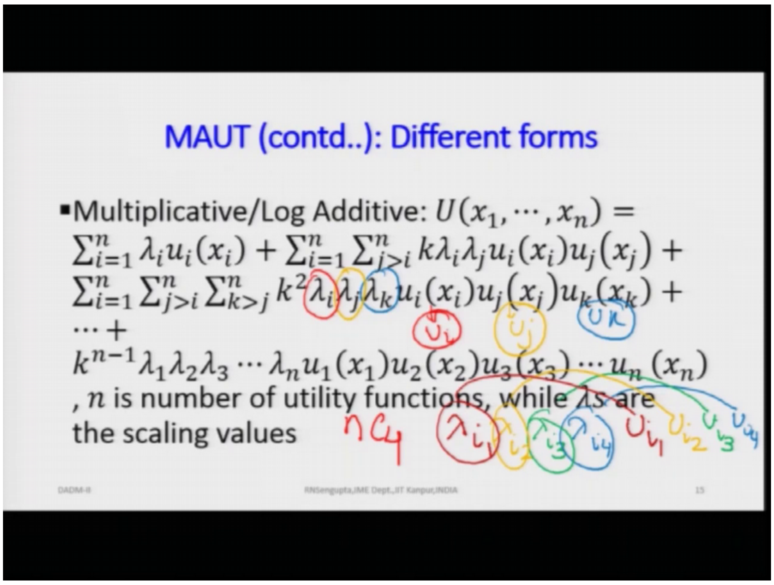

For the multiplicative or log additive one you will just consider that you will basically have the combination such that for the single units that is u 1, u 1 I am mentioning for u x 1, u 2 I am mentioning for u 1, u 2, u 3 till u n you will multiply with them by the scaling factor add them up. For so, this is you are taking one at a time combination, when you are taking two at a time combination; if there are n of the of them so, it will be n c 2. The combinations are u 1 with u 2, u 1 with u 3 till u 1 with u n, then you will take u 2 with u 1, u 2 2 with u 2, u 2 with u 3; similarly the last one u 2 with un and so on and so forth.

So, you can multiply them with the concepts of the scaling functions or scaling factors which is lambda 1 for the combinations; lambda 1 lambda 2 for the combinations of i and j. So, if you are combining u 3 with u 10 it will be lambda 3 and lambda 10. So, we will combine them and similarly as you increase the combinations you take n c 3. So, you will basically have lambda i, lambda j lambda k for the combinations of u i, u j, u k so, here it is. So, this is I am only marking one so, I use the red colour. So, it would be easy for me to write. So, this is basically U i, this is U j, this is U k that should be utilizing can; let me use the colour difference

So, it would be easy for me; easy, for me to explain also.This is U j, U k . So, the k th one goes with the k th one, the j th one goes with the j th one and the i th one goes with the i th one. So, you have different combinations; if you can put the higher combination it will be given. I am using the colour red only as it is able to differentiate. So, it will be n c 4 combinations. So, I will write; I will do I will write in different colours now. So, these are the combinations n c 4. So, now, if I write it, it would be lambda i 1, I am using the suffix i 1, i 2. So, it will be easy for me to differentiate not i j k l.

I could have used that, but I am not going to use that and obviously the corresponding factors so, I should leave space we can write. So, it will be U i 1, when I use the second colour lambda i 2 U i 2, then green lambda i 3 U i 3; so, lambda i 4 U i 4. So, these combinations go along, this combination go along, this combination go along, means the weightages scaling factor changing and this combination go along. So, we basically multiply them and have that value.

So, here n is the number of utility functions while lambdas are the scaling values which I have for each function. So, what I am doing; I am taking n c 1 combinations, n c 2 combinations, n c 3 combinations till then last one which is n c n combinations. It is basically trying to find out the joint probability distribution, first case is the marginal case then you have the joint distribution, bivariate case, tri variate case so on and so forth.

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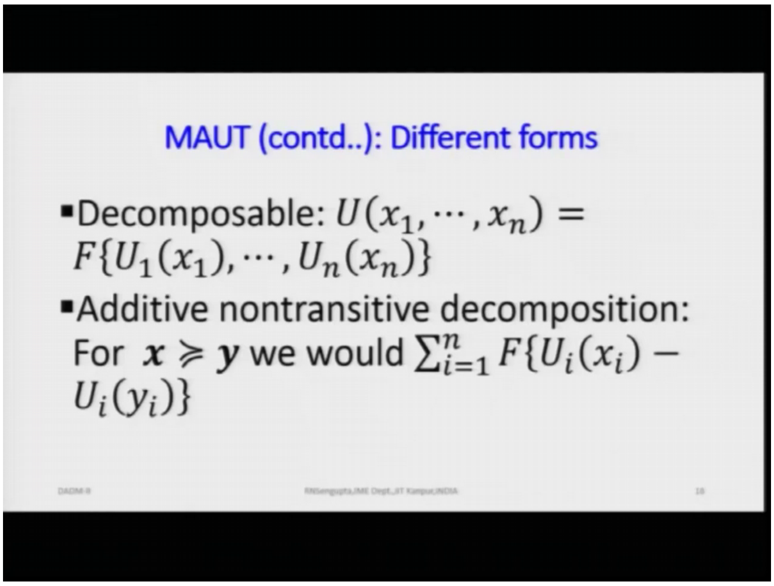

In the decomposable form the utility function U x 1 to U x n is given by a functional form where, it is now a function of the utilities taken individually. So, it is U 1, U 2, U 3 till the last one which is taken as U n . So, I have a functional form which basically gives me in an output is the decomposable form of the MAUT. For the additive nontransitive decomposition, if we have the functional form where x is greater than y; in the sense that it dominates y, x is a bundle, y is a bundle of decision; bundle means those is a vector. So, you will basically have for any combination of U i for x and U j for x you will basically have the functional form always dominating n greater than 0.

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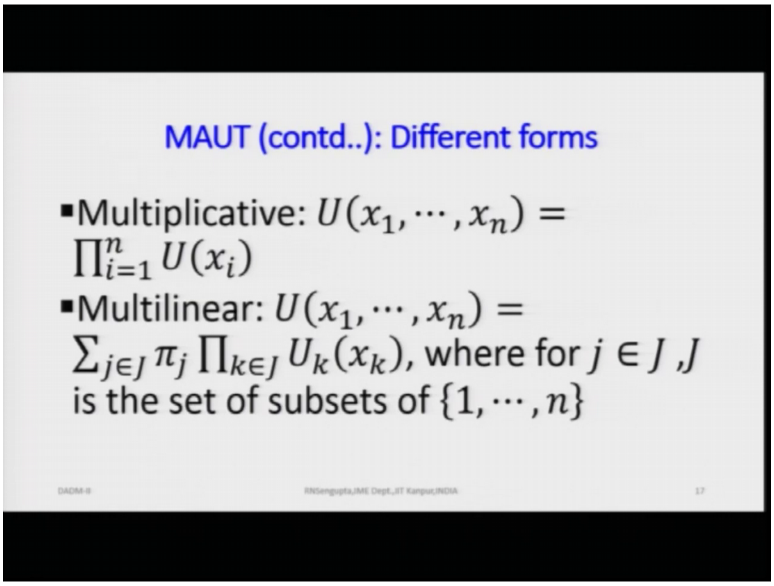

Similarly, you have the multiplicative utility function where you multiply the utilities individually considering that U 1, U 2, U 3, U 4 and till the U nth one and you have been able to break them into individual part. In the multi linear case you will basically have a combination of some of the U k's where you multiply and then multiply those factors by a lambda j corresponding to the fact that you have been able to differentiate the whole utility function into say for example, 3 components of i has been divided into n, n sorry n has been divided into 3 components which is n 1, n 2, n 3.

Such that n union of n 1, n 2, n 3 is the universal set n and the intersection I am considering for the time being as n 1 intersection n 2 or n 2 intersection n 3 or n 1 intersection n 3 are all null set. Then for n 1, n 2, n 3 I have the multiplicative factors as pi j's depending of j which is equal to i 1, j is equal to i 2 and j is equal to i 3. So, if I have that then the utility function which I get is the multi linear utility function.

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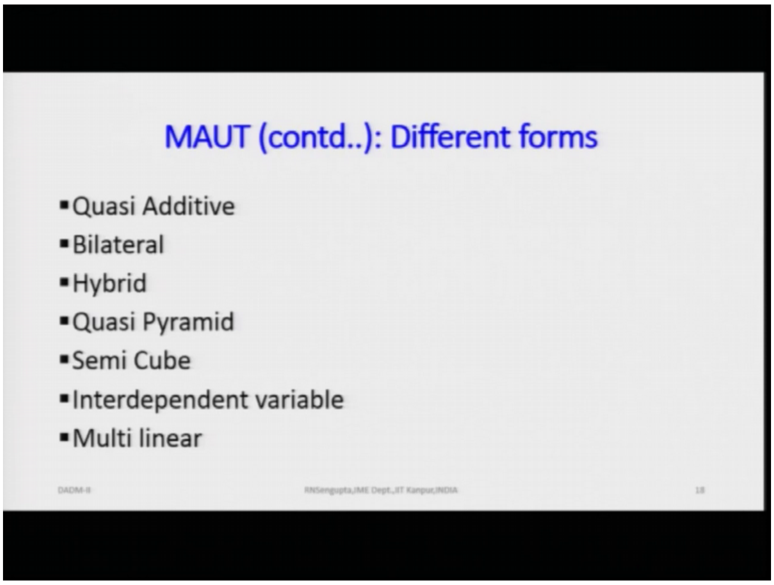

Other utility functions which I am just mentioning is the quasi additive, bilateral, hybrid, quasi pyramid, semi cube, interdependent variable and multi linear which I have already considered.

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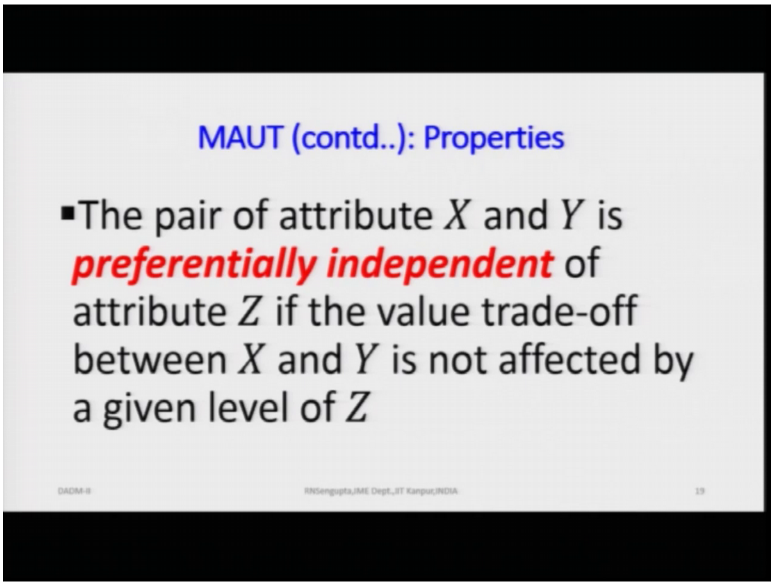

Now, we will consider the properties for the multi attribute utility theory. So, there would be two properties I will come to that, the pair of attributes of X and Y; so, X and Y are basically the bundle of decision which are vectors. So, pair of attributes X or Y which are elements of capital X then we will say that X is preferentially independent of attribute Z if the all the tradeoffs between X and Z is not affected by a given level of Z.

So, if I have decision Z and it is not going to affect the ranking system on X and Y at any stage; wewill basically say they are preferentially independent; based on the fact and X and Y are not affected by the decision or level of Z.

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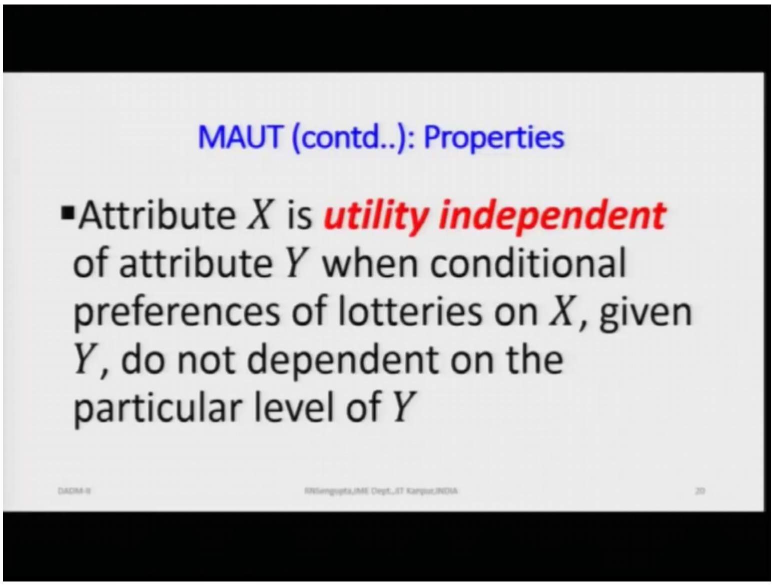

While, we will say that utilities are independent how X will be independent of the attribute Y, when the conditional preferences of the lotteries of X bundle of decision X or bundle of decision Y, where X and Y are element of the subs the overall universal set X or Y whichever you know. So, this X and Y are subset of X and Y is X, small x and small y are subset of either capital X or capital Y whichever you denote.

So, capital X and capital Y are the universal set and the elements are given by the small values and they are obviously vectors. So, when conditional preferences of the lotteries in on X given Y do not do not depend on the particular level of Y. So, they would be independent in the sense, that you can express them as multiplicative factors.

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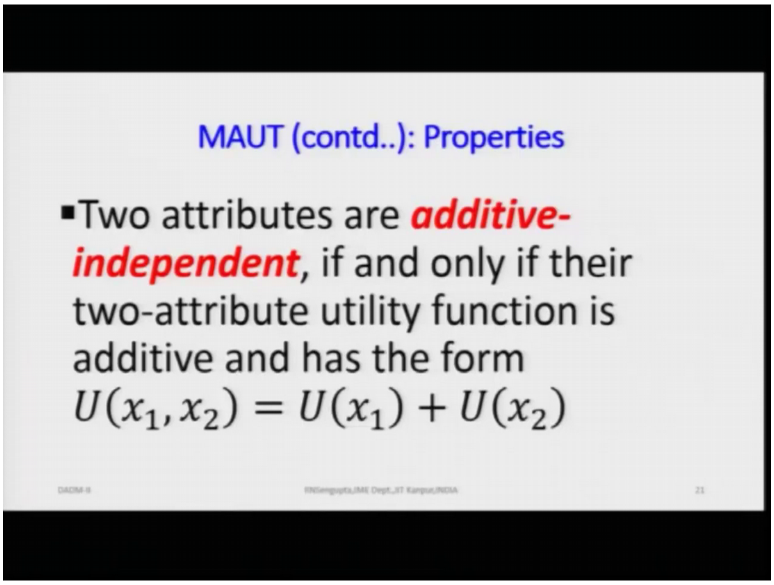

So, two attributes are additive independent when you can basically mention so, the properties of the utility function. So, of U x 1 to x n can be expressed as the sum of U 1 till U n. So, that will be a property of additive independence where utility function of any two bundles x 1 and x 2 can be expressed as the combination of U 1 and U 2 and it can go to n different combinations. So, n or k number of combinations depending on how you denote the vector.

So, if dimensions are n and you are going to combine them n number of times depending on different type of realized values so; obviously, it will be an additive independent to the factor of k. Now, we will consider background of multi objective optimization theory and here I am mentioning again, I am just giving you the picture. No solutions would be done because, that will be considered in the in the area of DADM III which is decision analysis and Data Analysis and Decision Making III, where you will consider operation research as the main focus.

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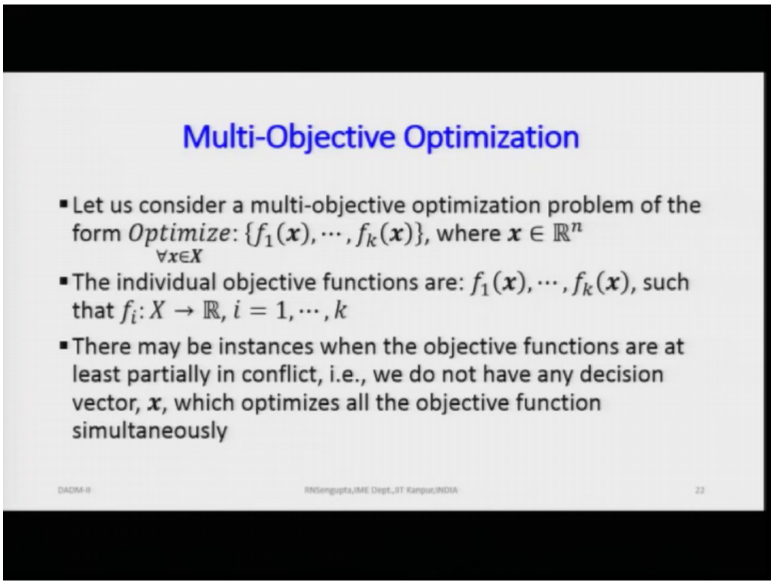

So, let us consider a multi objective optimization problem and where the form of the optimization is that you have k number of functions and each is based on the fact that for some of the x's you basically take the functional form from f 1 to f k. So, consider I have the decision to find out the total cost, total distance moved and consider the total say for example, number of cars; lorries used. So, what is the background of the problem? Background the problem I am transporting goods from any one of the cities, cities considered that is they are say for example, Calcutta, Patna, Bhuvaneshwar, Delhi, Kanpur, Lucknow, Chennai, Bengaluru, Mumbai and all these things, I am basically transporting goods from each location to the other.

So, there are some distributors, they are basically transporting goods to the retailers. So, and I am the main decision maker, I am the owner of the company. So, what is my main task is to find out what is the minimum number of lorries which I utilized to transport goods. What are the goods consider there I am trying to transport some cars say for example, or parts of the cars. Or it can be say for example, I am trying to transport different type of motors, motors being of different ratings. Now, when I do that I need to find out what is the minimum number of lorries used because, I want to I want to have to hire.

So, if the hiring cost is there I want to minimize that point 1. Point number 2 is that if I am going to transfer goods from any one of the cities which I mentioned to the other retailers point from the disturber to the retailers or from the main storehouses of the factory to the other retailer point. Then basically I have to utilize the lorries in a such a way that I am able to visit maximum of the hubs with the least number of lorries. Hubs means the places where I want to deliver and I want to basically travel in such a way that the minimum amount of some of the distance travelled by all the lorries taken together is minimum; that is point number 2 and consider the total cost is also minimum.

Total cost may depend on say for example, the roads which I follow, the toll tax, the charges per hour, kilometres travelled and all these things. So, these are cost which I want to minimize, on the other hand maximization can be what is the total profit, what is the total sales, revenues there can be many things. So, consider for some decisions where consider the decisions are to manufacture some quantum of the motors; the motors the ratings are say for example, there are 50 horsepower, 100 horsepower, 150 horsepower. So, x is a bundle of decision where x 1, x 2, x 3 are the corresponding numbers which I have produced for 50, 100 and 150 horsepower motors.

So, for different values of x these functional forms of f 1 to f k are the total cost of travel, total number of lorries which I use total, total distance travelled; it can be total revenues like I generate and so on. And so, when I want to optimize I want to optimize the combination of them in such a way that for the cost I want to minimize them, for the revenues of the profits I want to maximize them. So, the optimization word I am using is in a very general sense. Here n x which I mentioned x 1, x 2, x 3 which is 50, 100 and 150 hp motors they are basically belong to the real line or in the n dimension n where, n is basically the dimension.

If there are 10 products; obviously, there will be 10 such items. The individual objective functions are given as f 1 to f k, I am not mentioning f 1 x till f k x because, as you put the take out the values of x you get the functional form of f 1 to f k in the numeric value. And, what you do is that for someone bundle of x's which I have I get the functional form of f i, i is equal to 1 to k into the real line and get one value in rupees or dollars or yens whatever it is. So, there may be many instances when the objective function are at least partially in conflict to each other. Because if you increase the cost; obviously, it will mean the increase the, it will also have an effect on the revenues. Or, let me put it the other way if I want to increase the revenues I have to transport more goods, more lorries would be utilized, travel has to be more so; obviously, revenues will also incur on higher cost.

So, that is why we do not have any decision vector x which will optimize in the best possible manner the objective function in all the respect; that means, it is able to maximize the positive values and as well as minimize the negative values all at the same time. But, I will try to find out the best optimum solution.

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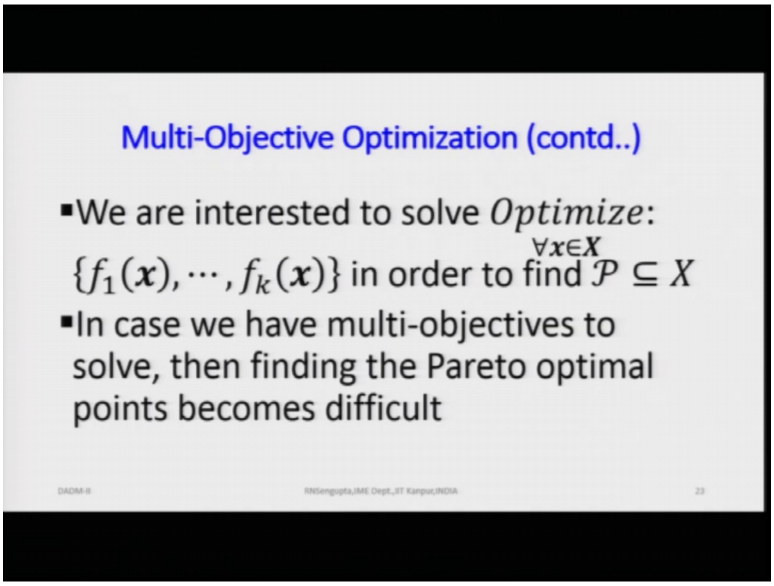

So, we are interested to solve the optimization problem in order to find out some of area or some of the values of x such that we have a multi criteria, multi objective problem. Based on the fact I want to maximize or minimize or optimize some of the objective function based on an outcome which is x.

So, some x's are given which are elements of capital X and I want to choose to some of them x such that I am able to optimize. The fact of the optimization is that they are to be optimized individually, but collectively the overall fact would be that, the best solutions we shall I have may not be same in the quantum front on the vector sense. That means, if I have two bundle of outputs; that means, I have x star 1 and x star 2 and the and the individual values of x star 1 and x star 2 may not be equal. So, what I will try to show is this.

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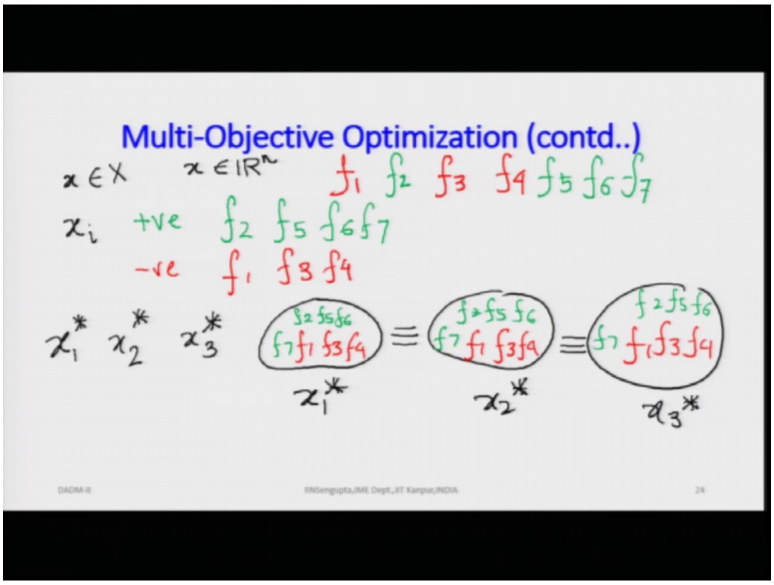

So, I have let me mention so, let me mention it in the. So, I have x is an element of capital X and this x is in the real line. And, now the functional forms are, let me mention the change the colour so, it is easy. So, a negative costs are red, consider f 1, f 3 f 4 and the positive are f 2; that is why it is green it is f 5, f 6, f 7. Now, what you will do is that if I find out say for example, some i.

So, what you will have is that they would be a positive sense, positive values would accrue from f 2, f 5, f 6 and f 7. Negative values with accrue from so, I am trying to draw it f 1, f 3 and f 4. So, increasing one would also have decreasing f 1 the other, if it is true then; obviously, we will try to increase the green one and automatically decrease the red one. Green and red being the positive and negative one, but it is not that. Increasing one also would have an increasing out on another and vice versa because, as I said increasing the revenue would also mean increasing the cost. Because, you have to sell more of the profit products, you have to basically utilize the transportation mode and so on and so forth.

Now, what I want to do is that consider there are some x i's and consider this x i's are there are x; let me mention them as x 1 star. This say; let me this is x say for example, 2 star this x 1 and 2 have nothing to do with the values of (Refer Time: 22:06) and consider x 3 star . So, they give me the best solution. So, what is best? It may be possible that the bundle of output which I have so, they may be equal. So, what is that I am mentioning consider this is x 1. star x 2 star x 3 star when I have

These values which I have f 2, f 5, f 6, f 7, f 2, f 5, f 6, f 7, f 2, f 5, f 6, f 7 and the negative values you have I am drawing the using f 1, f 3, f 4, f 1, f 3, f 4, f 1, f 3, f 4. The combinations of the output which I get the bundle net worth if they are equal, but the individual factor of f 1, f 2, f 3, f 4, f 5, and f 6 and f 7 individual they may not be equal. That means, the total benefit which I get for f 7 either for x 1 star or x 2 star or x 3 star are definitely different. But, collectively if I if all of them are same I will say that I am going using the very simple concept, the net worth if I get the same, but then increase or decrease on individual front.

Then I will say that they are Pareto optimal such that I am equally disposed by taking the decision x 1 star or x 2 star or x 3 star, because individually I get the net worth. I am combine I get the net worth, individually may be different that is a different question.

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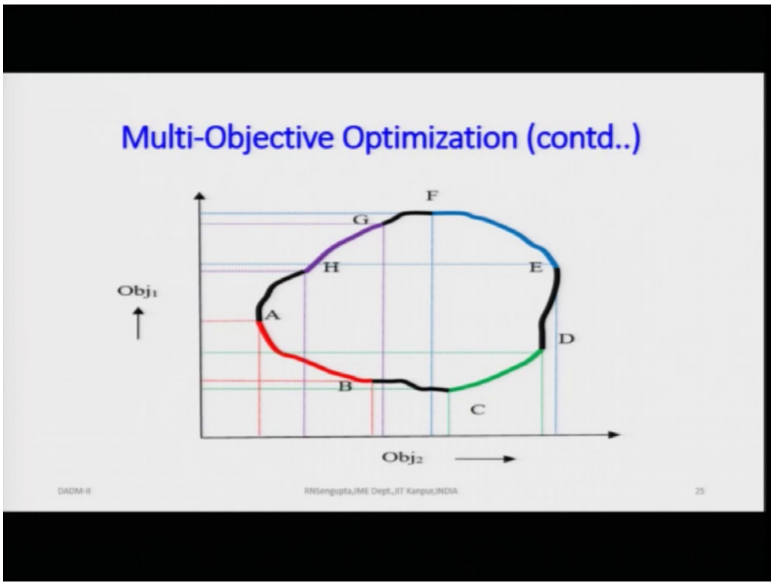

So, now here is the diagonal and it is only possible for me to draw in the 2 dimensional case; I will try to draw it in the 3 dimension case; let me see how I am able to do that. So, consider the objectives we mentioned one is along the x axis objective 2 along objective 1 is along the y axis. And, if you see there are in this overall space of optimization there are 4 colours I have utilized.

So, colours if you can see A B is red in colour, C D is green in colour please note it down, it will be for your own good you will understand. So, if you draw the diagram and if you are seeing this diagram again I mentioning mark A B as red, then area or the line C D as green because, the colour scheme will have important fact while, the line E F is blue and the line G H is violet. So, again I am mentioning A B is red, C D is green, E F is blue and G H is violet while the other portions of the line for this whole area B C is black. So, it would not have any meaning because, it would not be possible for to us to differentiate.

So, black colour I am using for the fact that I am not going to draw any conclusions from here. B C is black again I am mentioning, D E is black, F G is black and H A is black. So, if you have drawn this now, we will proceed based on which I will mention the concept. So, let us pay more attention on the coloured line which is A B red, C D green, E F blue and G H as violet.

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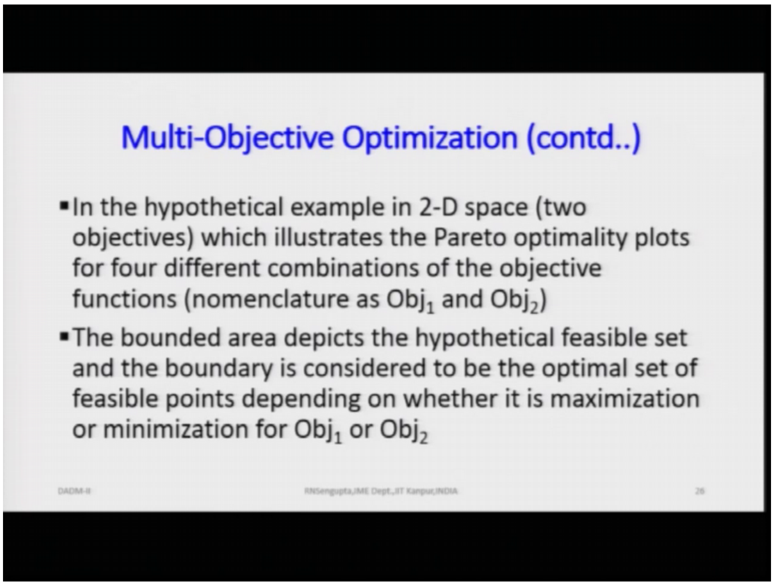

So, in this hypothetical example which I have drawn in a 2 dimension, it can be extend to the 3 dimension also as I mentioned. There are two objectives: it will illustrate the Pareto optimality plots for four different combinations of the objective functions where, the objective functions are Ob O 1 and O 2 which is objective function 1 and objective function 2. The bounded areas depicts the hypothetical feasible set and the boundary is considered to be the optimal set of feasible points depending on whether it is a maximization or a minimization problem based on the fact that you are considering the object function 1 or 2. So, the colour schemes will now become important.

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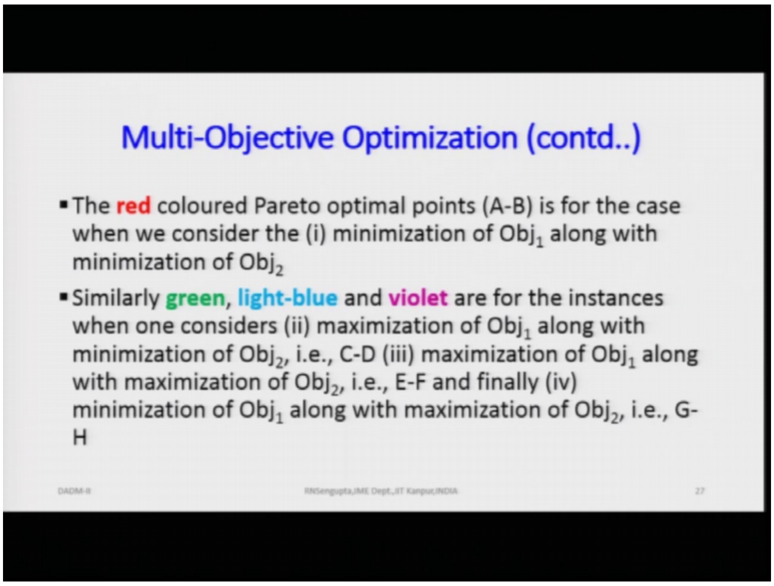

Point 1 the red line which you considered which was I will for the first time I will switch between this graph. So, let me make it just next to each other so, this is the grind. So, here it is the red one. So, A B if I consider is for the case when we consider minimization objective 1 along with the minimization objective 2 let us see. So, I am trying to minimize both of them. So, considering this I am; I would have this area.

If I similarly consider the green, light blue or the blue and the violet one, they are the instances which are like this. Maximization of objective 1 with minimization of the 2 which is for the green one, for the light blue one and it is it is maximization of objective 1 and minimize maximization objective 2. And finally, violet one will be minimization of objective 1 and maximization of objective 2.

So, violet one or minimization of objective 1 and maximization objective 2. So, for the light blue it is a maximization of both. For green one it is basically a minimization of objective 2 and maximization objective 1, while the red one is minimization of both. So, in the third 3 dimension how it looks, I will consider that in the subsequent class. So, this will be more of understanding the concept of multi utility objective problem.

And in the last two class for the 9th week, I will try to wrap it up with some interesting examples as required to make the things much easier for you to understand from the very simplistic point of view. Obviously, we will solve such problems in DADM III which I had already mentioned. With this I will end this class and have a nice day.

Thank you very much.