

Data Analysis and Decision Making – II
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Lecture – 39

VIKOR

Welcome back, my dear friends, a very good morning; good afternoon, good evening to all of you wherever you are in this part of this globe whether in India or abroad. And as you know this is the DADM which is Data Analysis and Decision Making II course under the NPTEL MOOC series and this total class for DADM set of classes for DADM II is spread over 12 weeks which is 30 hours of lectures and each lecture is for half an hour; that means, we have 60 lectures.

And each week we have 5 lectures each for half an hour and as you can see we are in the 39th lecture which is in the 8th week and with 39 and 40th lecture which will end at the 8 week we should complete. One of the topics which we are discussing which is a multi attribute decision models which is the VIKOR method and my good name is Raganandan Sengupta from the IME department at IIT Kanpur.

So, yesterday we were just we just discussed yesterday means in the last class 38th class we discussed the concepts of the algorithm of the VIKOR based on the fact that we will take consider the distance norm based on the actual concept of the l_1 norm and also use the l_∞ norm, I will come to that later. And I gave you the definition of l_1 norm being the Manhattan norm where you take the mod of the distance and in the l_∞ norm you take the max of the difference in between the vectors.

And remember the vectors or the set of points which is x bold or y bold, can be of any dimension and I have given an example considering the 3 dimension. So, it was easy for us to understand. And I also gave how the graph looks like for the l_p norm that distance as the p value increases. Here for the discussion we will consider m as the number of alternatives n as the number of criteria, i changes from 1 to m capital small m and j changes from 1 to small n .

So, what we will do is that, we will give the rule based on the algorithm and also give a very simple example to consider.

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VIKOR: Steps # 01 (Construct the normalized decision matrix)

▪ Assume the decision matrix, $X =$

$$\begin{bmatrix} x_{11} & \dots & x_{1n} \\ \vdots & \ddots & \vdots \\ x_{m1} & \dots & x_{mn} \end{bmatrix}_{m \times n}$$

, m is number of Alternatives and n is number of Criterion.

Now we will assume the decision matrix X which basically is the relationship of the alternatives and the criteria's, the criteria's are basically based on some attributes or characteristics it need not be quantitative in nature. So, this matrix is a size m cross n and as it has been discussed in the last 2 classes and also in the first class for VIKOR. Considering the first class I am mentioning that was the beginning of the first this of this 8th like a week lecture.

We had considered that the concept of VIKOR would be such that the attributes may have different units and we will try to consider that, we will assume the units to be such that they can be so called taken care by the normalization factor. So, we will consider the matrix the first row would be x_{11} to x_{1n} , where 1 to n as already has been stated are the criteria and 1 to m along the rows are the alternatives. So, x_{11} to x_{1n} actually means the effect on the alternative 1 by the respective values or the criteria's 1 to n .

Similarly, the second row which will be x_{21} to x_{2n} would be the corresponding effect on alternative 2 by the criteria 1 to n . Similarly when I go to the last row which is x_{m1} to x_{mn} it will basically consider the effect of all the criteria from 1 to n on the m th alternative. Here m is the number as stated here; m is the number of alternatives and n is the number of criteria. So, we will consider m cross n .

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VIKOR: Steps # 01 (Construct the normalized decision matrix)

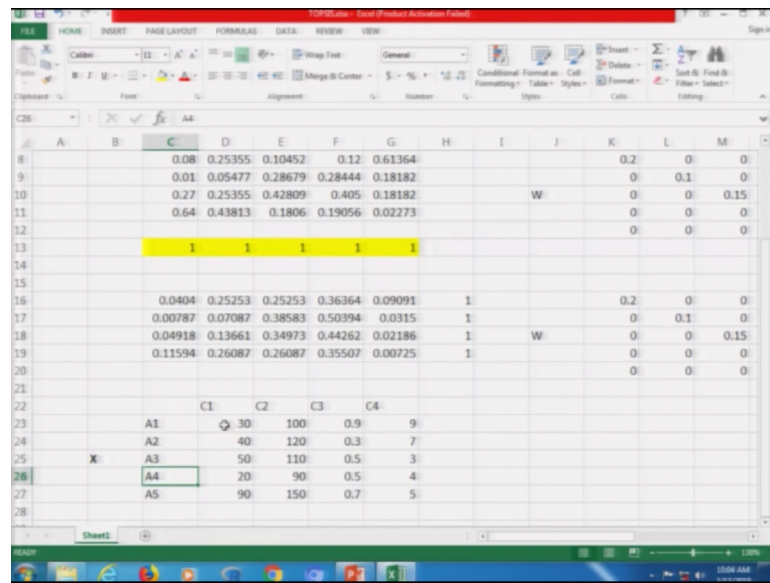
$$\mathbf{X} = \begin{bmatrix} 30 & 100 & 0.9 & 9 \\ 40 & 120 & 0.3 & 7 \\ 50 & 110 & 0.5 & 3 \\ 20 & 190 & 0.5 & 4 \\ 90 & 150 & 0.7 & 5 \end{bmatrix}_{5 \times 4}$$

Now, this is the value you are taking. Now, notice this is a hypothetical example and I purposefully kept the so called values for the first criteria second third fourth in such a range that first one is with 2 units before decimal ,the second one is 3 units before decimal, the third one is a 0 unit before decimal and the fourth one is 1 unit before decimal and I have taken it purposefully such and these values are arbitrary like if I consider the first row 30 100 0.9 and 9 or the second last row which is 20 190 0.5 4; they are just arbitrarily.

So, the values of the first second third fourth columns are taken correspondingly. So, there is some semblance between the values corresponding to each and every column. Now if I consider say for example, the value of 90 150 0.7 0.5 in the last row it basically means the value occurring from the first criteria to the last alternative is 90.

If I consider the second cell value in the last row, which is 150 it means the value accruing from the second criteria to the mth alternative which is the fifth alternative is 150. Similarly, the corresponding values accruing to the fifth alternative from the third and the fourth criteria are 0.7 and 5 respectively. So, let me make these values in the excel table. So, it will be easy for me to process it as I solve the problem.

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So, I will put X here which is bold and as I had done I will consider alternatives they are 5 and the criteria's are we just 1 minute; criteria's are 4 in number.

So, values are I will go along the column and note it down 30 40 50 20 90 30 40 50 20 90. So, the second column is 100 120 110 90 and 150; similarly the third column due to corresponding to the third criteria is 0.9 9 3 5 5 7. So, this would be 9 3 5 5 7 and 9 7 3 5 9 7 3 4 5; 9 7 3 4 5. So, this is a 5 cross 4 value.

Now, these are the priority values they are not normalized. So, we will use some normalization scale. So, let us go back to the.

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VIKOR: Steps # 01 (Construct the normalized decision matrix)

$$\mathbf{X} = \begin{bmatrix} 30 & 100 & 0.9 & 9 \\ 40 & 120 & 0.3 & 7 \\ 50 & 110 & 0.5 & 3 \\ 20 & 190 & 0.5 & 4 \\ 90 & 150 & 0.7 & 5 \end{bmatrix}_{5 \times 4}$$

So, these are the normalized we have to consider the normalized decision matrix, this is the priority values.

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VIKOR: Steps # 01 (Construct the normalized decision matrix)

- Convert the entries in \mathbf{X} into scaled **normalized** values, where $r_{ij} = \frac{x_{ij}^2}{\sum_{k=1}^m x_{kj}^2}$, which has no dimension

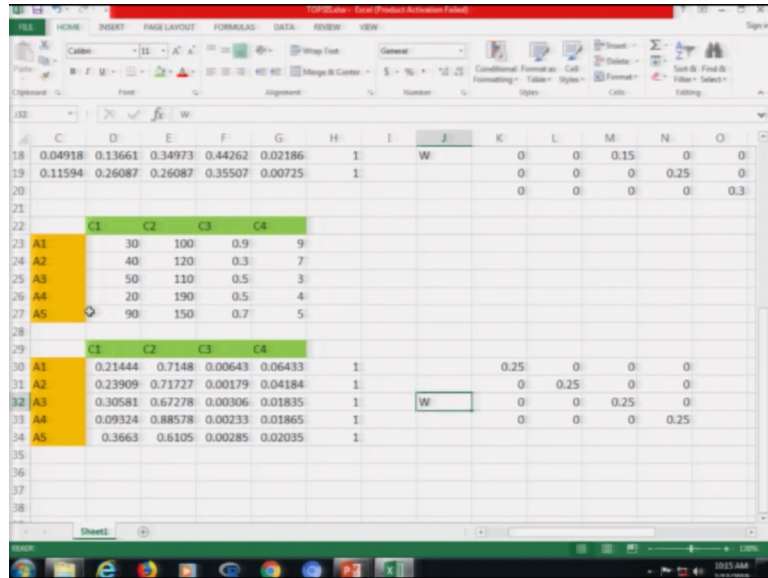
$$\mathbf{R} = \begin{bmatrix} r_{11} & \dots & r_{1n} \\ \vdots & \ddots & \vdots \\ r_{m1} & \dots & r_{mn} \end{bmatrix}_{m \times n} = \begin{bmatrix} \frac{x_{11}}{\sum_{k=1}^m x_{k1}^2} & \dots & \frac{x_{1n}}{\sum_{k=1}^m x_{kn}^2} \\ \vdots & \ddots & \vdots \\ \frac{x_{m1}}{\sum_{k=1}^m x_{k1}^2} & \dots & \frac{x_{mn}}{\sum_{k=1}^m x_{kn}^2} \end{bmatrix}_{m \times n}$$

So, based on that normalization concept are used. I am using where the very simple concept of trying to square them up and write the values.

So, what I will do is that these are different normalization concept which I am using. So, I will write these values as. So, this will be should be and so, the last and the last row values the first one I have stated, this is the last cell in the which is the last value in the

principal diagonal. So, I am just squaring and taking the normalization along the rows. So, these are the.

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So, I come, this would be R as you know, that top values are the let me put a different color. So, it will be easy for us to distinguish. Now we will be seeing normalize along the row as I mentioned square them up. So, this is square. See we have we can take this square one, but here later on we will consider the linear one, but let me first give you the square concept.

So, sum of this you know sum of this plus sum of this, this means I am going along the row. So, I fix these values D would not change. So, it is D would not change, I fix. So, these values are calculated. Let me check whether they are coming as well this is moving. So, I have to fix it. So, these values are given this for the second, this is for the third and this is for the fourth.

So, let us check whether the sum is 1, it should be 1 as it is. Now let us go to the second row which is the alternative to the square it will be sum of the square. So, I can put the dollar at here dollar at here the second value it will be E this would be 24 this would be F 24 square plus dollar G 24 square.

So, the values are already fixed and let us see whether this it is also 1 showing go to the third understand. So, this is done. I have basically found out the values accordingly. Now

these are the normalized values for the priorities for the X matrix which had been converted into R. Now this is the example which I have, but for our actual example which is given what do we do? We basically divide to make it simple we divide by just the linear sum.

So, let us convert into the linear sum. So, if I convert to linear sum these squares will go. So, this becomes 1; just note down this has become 1 which means the process is right, all the values are 1. So, what we do is then?

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VIKOR: Steps # 01 (Construct the normalized decision matrix)

	30	100	0.9	9	
	$\frac{30}{30+100+0.9+9}$	$\frac{100}{30+100+0.9+9}$	$\frac{0.9}{30+100+0.9+9}$	$\frac{9}{30+100+0.9+9}$	
	40	120	0.3	7	
	$\frac{40}{40+120+0.3+7}$	$\frac{120}{40+120+0.3+7}$	$\frac{0.3}{40+120+0.3+7}$	$\frac{7}{40+120+0.3+7}$	
	50	110	0.5	3	
	$\frac{50}{50+110+0.5+3}$	$\frac{110}{50+110+0.5+3}$	$\frac{0.5}{50+110+0.5+3}$	$\frac{3}{50+110+0.5+3}$	
	20	190	0.5	4	
	$\frac{20}{20+190+0.5+4}$	$\frac{190}{20+190+0.5+4}$	$\frac{0.5}{20+190+0.5+4}$	$\frac{4}{20+190+0.5+4}$	
	90	150	0.7	5	
	$\frac{90}{90+150+0.7+5}$	$\frac{150}{90+150+0.7+5}$	$\frac{0.7}{90+150+0.7+5}$	$\frac{5}{90+150+0.7+5}$	

, where $r_{ij} = \frac{x_{ij}}{\sum_{k=1}^m x_{kj}}$

We divide the first cell 30 by the sum of all the corresponding elements in the first row. So, it was in the denominator will be 30 plus 100 plus 0.9 plus 9.

So, the value is coming out to be this one; 0.21 triple 4; if I take the second cell value meaning the first row it is 100 divided by 30 plus 100 plus 0.9 plus 9 the value comes out to be 0.7148. Similarly, if I consider this cell, so, this will be the division of 4 divided by the sum of all these things.

So, check let us check. So, it is 4 divided by sum means 20 190 0.5 0.4. So, these are the values 20 90 20 190 wait this should be 190 what yes, 30 40 50 20 90 at 40 50 20 90 100 120 110 190 150 0.9 3 5 5 7 9 3 5 5 7 9 7 3 5 4 5. So, the values calculated are accordingly. So, let us see what are these values are.

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VIKOR: Steps # 01 (Construct the normalized decision matrix)

$$\mathbf{R} = \begin{bmatrix} 0.214439 & 0.714796 & 0.006433 & 0.064332 \\ 0.239091 & 0.717274 & 0.001793 & 0.041841 \\ 0.305810 & 0.672783 & 0.003058 & 0.018349 \\ 0.093240 & 0.885781 & 0.002331 & 0.018648 \\ 0.366300 & 0.610501 & 0.002849 & 0.020350 \end{bmatrix}_{5 \times 4}$$

So, these are the R matrix normalized considering the along the row. So, these are the values point. I only read the first 3 decimal, 0.214 0.718 0.006 0.064 and 0.239 0.717 0.001 0.041. So, I am just flipping from the to the PPT in order to make you understand the calculations I am doing a right.

The third rows are 0.305 0.672 0.305 0.672 0.003 0.018 0.003 0.018. The last, but one values are 0932 8857 0932 8897 0023 0186 0023 0186 and the last values are 0.366 0.610 0.008 0.366 0.610 0.002 and last value being 0.0203.

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VIKOR: Step # 02 (Construct the weighted normalized decision matrix)

▪ If the decision maker decides on the set of weights, depending on his/her preference, then the weight, $\mathbf{W} = \begin{bmatrix} w_1 & \dots & 0 \\ \vdots & \ddots & \vdots \\ 0 & \dots & w_n \end{bmatrix}_{n \times n}$, such that $\sum_{j=1}^n w_j = 1$

Now, I will basically have the weights. Now the weights will be assumed based on the fact that what is the weight you as a decision maker are planning to give to each and every criteria's amongst themselves when you compare. So, the weight would basically be of size n cross n depending on the number of criteria's. So, there are 4 criteria's, the overall matrix for the weights would be 4 cross 4 and the principle diagonal will basically have all the values.

So, let us check what we have done.

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VIKOR: Step # 02 (Construct the weighted normalized decision matrix)

$$\mathbf{W} = \begin{bmatrix} 0.25 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.25 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.25 \end{bmatrix}_{4 \times 4}, \text{ such that}$$

$$\sum_{j=1}^n w_j = 1$$

So we take a value of 0.25 which is of equal weight for all the 4 criteria's. So, let us put the weights accordingly. So, I will copy it 0 0 0, should be 0. So, it should be 0 0 0 0 the third value in the principal diagonal and the last. So, you have the weighted matrix. So, I will remove it a little bit on to the right to note down, now this is W.

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VIKOR: Step # 02 (Construct the weighted normalized decision matrix)

▪ Calculate $F = RW =$

$$\begin{bmatrix} f_{1,1} & \cdots & f_{1,n} \\ \vdots & \ddots & \vdots \\ f_{m,1} & \cdots & f_{m,n} \end{bmatrix}_{m \times n}$$

Now, once you have that you want to find out the weighted normalized decision matrix. So, you are normalized plus multiplying the weights. So, this is F. I will show you the cell which obviously would be a matrix of size 5 cross 4 into 4 cross 4 which is 5 cross 4; let us check.

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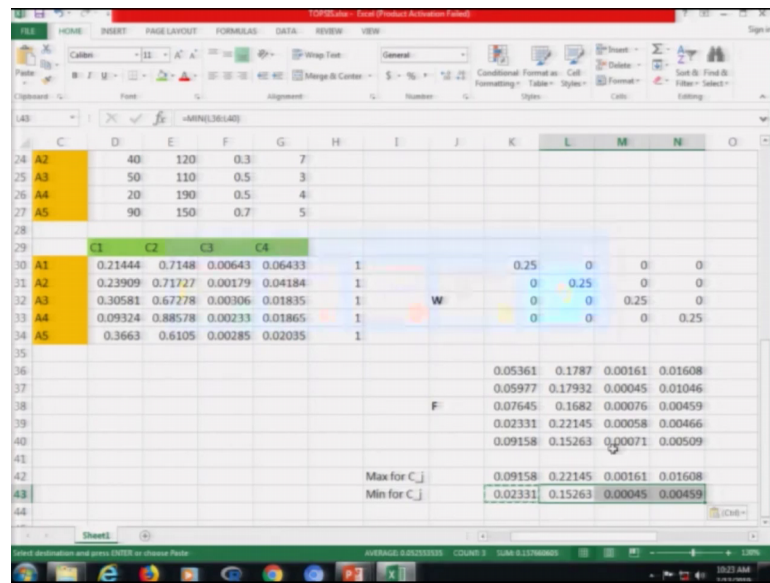
VIKOR: Step # 02 (Construct the weighted normalized decision matrix)

▪ $F = RW =$

$$\begin{bmatrix} 0.214439 & 0.714796 & 0.006433 & 0.064332 \\ 0.239091 & 0.717274 & 0.001793 & 0.041841 \\ 0.305810 & 0.672783 & 0.003058 & 0.018349 \\ 0.093240 & 0.885781 & 0.002331 & 0.018648 \\ 0.366300 & 0.610501 & 0.002849 & 0.020350 \end{bmatrix}_{5 \times 4} \begin{bmatrix} 0.25 & 0.00 & 0.00 & 0.00 \\ 0.00 & 0.25 & 0.00 & 0.00 \\ 0.00 & 0.00 & 0.25 & 0.00 \\ 0.00 & 0.00 & 0.00 & 0.25 \end{bmatrix}_{4 \times 4} =$$
$$\begin{bmatrix} 0.0536 & 0.1787 & 0.0016 & 0.0161 \\ 0.0598 & 0.1793 & 0.0004 & 0.0105 \\ 0.0765 & 0.1682 & 0.0008 & 0.0046 \\ 0.0233 & 0.2214 & 0.0006 & 0.0047 \\ 0.0916 & 0.1526 & 0.0007 & 0.0051 \end{bmatrix}_{5 \times 4}$$

So, this is a 5 cross 4. So, let us basically find out the F value and that is.

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So, this will be first row by the first column, it will be 30 to 34. So, these are the values. So, 0.05361 let me check yes. So, the first value matches 05361. So, now, in this the first row remains fixed.

So, first row means? So, let us check, yes it is me. So, let us check the values to make a double sure; 0.178 0.001. 0.178 which is here I am just hovering the electronic pin 0.001 and 0.0161. Similarly I go to the second row multiplied by the corresponding. So, here the row 1 change corresponding to the R matrix.

So, this is dollar dollar dollar dollar. So, let me make the, calculate the values and then show you a table. So, this would be this is the third. So, I will make the changes then and there. So, is easy for me to copy, done and finally, may be the last of R row multiplied by the corresponding column for W. So, now, let this.

So, let us check the values whether they are exactly the same one and they would be. So, this is there would be another ones let me check, first we have missed this one. So, this would be 33, 33, it will be 34, 34 and this would be 30 and 33. So, the values I will read out along the column is 059 076 023 091.

Then 17 I am not reading the decimal 178 179 168 178 169 179 168 221 152 then it is I am omitting the 2 decimals after 2 0's after decimal. So, it is 161 045 161 045 008 because I take 76 are 80, 0006 as it and last one is 07; similarly the values are there.

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VIKOR: Step # 03 (Determine the maximum/best from the criterion values)

- Determine the **maximum/best**: $f_j^* = \max_{i=1, \dots, m} f_{i,j}, j = 1, \dots, n$
 - ❖ $f_1^* = \max(f_{1,1}, f_{2,1}, \dots, f_{m-1,1}, f_{m,1})$
 - ❖ $f_2^* = \max(f_{1,2}, f_{2,2}, \dots, f_{m-1,2}, f_{m,2})$
 - ❖ .
 - ❖ .
 - ❖ $f_{n-1}^* = \max(f_{1,n-1}, f_{2,n-1}, \dots, f_{m-1,n-1}, f_{m,n-1})$
 - ❖ $f_n^* = \max(f_{1,n}, f_{2,n}, \dots, f_{m-1,n}, f_{m,n})$

Now, once you have this you will try basically try to find out the maxima and the minima based on the distance function. So, what I will do is that for each and every criteria, I will take the maximum corresponding to each and every set of alternatives. So, here if you note I will take for any corresponding criteria, I will take all the alternatives and find out the maximum for that. So, I have the f_{ij} and I will basically keep j as fixed. So, it will be f_{1j}, f_{2j}, f_{3j} so and so forth.

See if I take j is equal to 1 it will be $f_{11}, f_{21}, f_{31}, f_{41}$ till the last one. So, basically I am going to consider each and every column for our consideration. So, let me first write it down. So, I would find max for this. So, let us check whether they are right. So, I will try I mention them as f^*_1 for the first criteria, f^*_2 for the second one, f^*_3 for the third one, f^*_4 the fourth one because there are 4 number of criteria, let us see.

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VIKOR: Step # 03 (Determine the maximum/best from the criterion values)

- $f_1^* = \max(0.0536, 0.0598, 0.0765, 0.0233, 0.0916) = 0.0916$
- $f_2^* = \max(0.1787, 0.1793, 0.1682, 0.2214, 0.1526) = 0.2214$
- $f_3^* = \max(0.0016, 0.0004, 0.0008, 0.0006, 0.0007) = 0.0016$
- $f_4^* = \max(0.0161, 0.0105, 0.0046, 0.0047, 0.0051) = 0.0161$

So, the first one it will be the max along the column. So, you see if I stick the value is 0.09158 which is 09158, its 16; 0.2214 0.0016 0.0161; so, 0.2214 0.0016 0.016.

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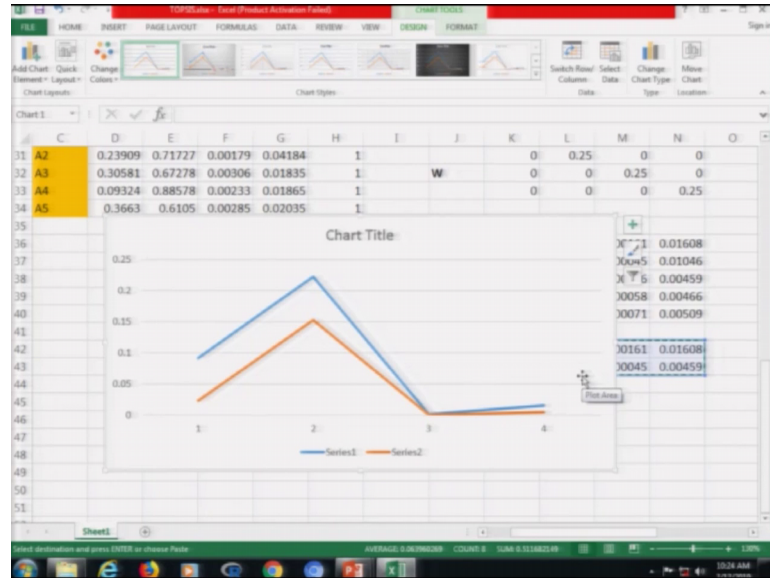
VIKOR: Step # 03 (Determine the minimum/worst from the criterion values)

- Determine the **minimum/worst** $f_j^- = \min_{\forall i} f_{i,j}, j = 1, \dots, n$
- ❖ $f_1^- = \min(f_{1,1}, f_{2,1}, \dots, f_{m-1,1}, f_{m,1})$
- ❖ $f_2^- = \min(f_{1,2}, f_{2,2}, \dots, f_{m-1,2}, f_{m,2})$
- ❖ .
- ❖ .
- ❖ $f_{n-1}^- = \min(f_{1,n-1}, f_{2,n-1}, \dots, f_{m-1,n-1}, f_{m,n-1})$
- ❖ $f_n^- = \min(f_{1,n}, f_{2,n}, \dots, f_{m-1,n}, f_{m,n})$

So, once I find out the maximum I also find the worst. So, I am trying to find out in each and every criteria, the scaled values of the distance based on the maximum the minima. So, I will find out the maxima again for each and every criteria along the alternatives. So, I put this value of max for C 1. I will write it down, so, it is easy. So, this is max for J then I will find the min. So, once I find the min I have the minimum values.

So, let us check the minimum values; 0233 1526 0233 1526 00048 45 and 0046. So, I have the maxima and the minima. Now based on that we will.

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So, what we are doing is that we are trying to find out. So, if I draw the number line, see for example, if I have these values, let me do a small.

So, what we have is for the maxima and the minima. So, the blue one is the maxima. So, I am trying to basically normalize along for the first criteria this distance second criteria this distant third this is 0 4 so and so on. I am basically normalize and find it accordingly where they stand. So, I will continue discussing this in the last class for the eighth week and hopefully we will wrap up the VIKOR all its details. Have a nice day and.

Thank you very much.