

Data Analysis and Decision Making - II
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Lecture – 35
Example of TOPSIS

Welcome back, my dear friends and students. As you know it is a very good morning, good afternoon, good evening wherever you are and you are listening to this course and taking the part in this course. You know this is a DADM II which is Data Analysis and Decision Making 2 under the NPTEL MOOC series and the total number of hours is 30.

So, which is basically 60 lectures because each lecture is half an hour and its spread over total course is spread over 12 weeks. And each week we have 5 lectures after each week we have assignments and we are in the last lecture for the 7th week which is as you can see in the slide this is the 35th lecture and this course is under the NPTEL MOOC series. And my name is Raghu Nandan Sengupta from IME department at IIT Kanpur in India.

And if you remember this we were discussing on the TOPSIS and we have just started by considering that there are 11 parameters or criteria based on which a person is planning to buy a house; house are given as a 1 a 2 a 3 a 4 are the alternatives and the criteria's those 11 onwards from c 1 to c 11 and if you remember we have those points or the criteria matrix for each and every alternatives. So, these are not normalised, we will normalise them accordingly.

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TOPSIS: Example (contd..)

$R =$

0.5116	0.5561	0.4565	0.5180	0.5097	0.0000	0.5647	0.4961	0.3427	0.5908	0.7169
0.5622	0.4629	0.5701	0.4785	0.4248	0.5508	0.0000	0.5177	0.5140	0.4062	0.3700
0.4498	0.5964	0.4278	0.3828	0.3681	0.3390	0.0000	0.2589	0.3855	0.5169	0.5087
0.4689	0.3475	0.5325	0.5968	0.6513	0.7627	0.8253	0.6471	0.6854	0.4677	0.3006

For example
 $r_{3,2} = \frac{0.4498}{\sqrt{40.0^2 + 60.0^2 + 62.5^2 + 6.8^2 + 1.3^2 + 0.8^2 + 0.0^2 + 1.2^2 + 1.8^2 + 4.2^2 + 2.2^2}} = 0.5964$
 $r_{4,8} = \frac{0.6471}{\sqrt{41.7^2 + 25^2 + 77.8^2 + 10.6^2 + 2.3^2 + 1.8^2 + 1.93^2 + 3.23^2 + 0.8^2 + 1.3^2}} = 0.6471$

So, consider that we normalise them and what normalisation we are using if you look this. Let us look at these two blue points. So, this is 0.9564 and 0.6471. So, if you go back to the last class the slides. So, they were basically being normalised you can use the conserve either the column or the row.

So, I am basically doing it along the row. So, the first value was 40; 40 square 42.9 square 62.5 square 6.8 square 1.3 square 0.8 square 0 square 1.2 square 1.8 square 4.2 square 2.2 square. So, from where did this values come? Look at r 3 comma 2. So, this is the space for the 3 comma 2 cell; that means, I am considering the third alternative and the second row. So, the second row values worth this which I said at the squares; now the so, ok.

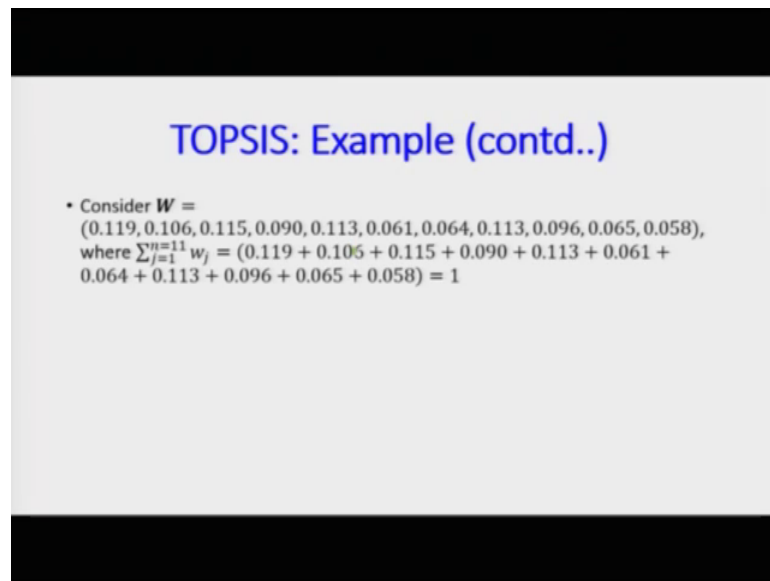
My mistake, it would be basically the third row my mistake sorry because 3 is basically the third alternative and 2 was basically the cell value which is 42.9 which is here I just mark it. I will use different marker here. So, it will be easy sorry for that. So, this is yellow this is yellow this is orange this is orange. So, it will be easy for us to find out then the value of 32 would be given. So, if it is 32, so, 42.9.

So, I calculate the values when I go to 4 8 again I will consider the 8th column and the 4th row. Row means the fourth alternative values I given 42.7. I am not mentioning the square values 25 78 7.8 10.6 2.3 1.8 1.93 3.23 0.8 1.3. So, these are the accrued values

which are coming from each and every criteria to each and every alternative e1 1criteria 4 alternatives.

So, the third value is basically coming from the sense I am considering the 8th column. So, let me see what is the 8th value 1 2 3 4 5 6 7 8. So, this is threes 8, I will use a different colours let me use the light green 3 with 3 here. So, with this you calculate all the values that is would be use you put it in the excel sheet and calculate.

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The slide contains the following text:

TOPSIS: Example (contd..)

- Consider $W = (0.119, 0.106, 0.115, 0.090, 0.113, 0.061, 0.064, 0.113, 0.096, 0.065, 0.058)$, where $\sum_{j=1}^{11} w_j = (0.119 + 0.106 + 0.115 + 0.090 + 0.113 + 0.061 + 0.064 + 0.113 + 0.096 + 0.065 + 0.058) = 1$

So, once we have this now I consider the weights. So, the weights are I am I am for this problem I am considering for each and every criteria starting from 1 to the 11th one are point.

I am only reading the 2 places of decimal 0.11 0.10 which is about 11 percent 10 percent the third one is again 11 percent fourth one is 9 percent 11 percent 6 percent 6 percent 11 percent 9 percent 6 percent 5 percent; that means, what is the weightages for each and every criteria among themselves. So, the double check the sum should be 1 as it is now what I will do is that I will I will I will.

Now, given the normalised matrix which is R capital R and I have already explained how the normalization was done whether the rows or the columns are being used. You can intension then there is no issues, but remember the R matrix which will be multiplied by

w is of size m cross n which is 4 cross 11 here and the size of the matrix which is w would be 11 cross 11. So, this is the value.

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TOPSIS: Example (contd..)

* V =

0.0609	0.0589	0.0525	0.0466	0.0576	0.0000	0.0361	0.0545	0.0358	0.0385	0.0417
0.0669	0.0493	0.0656	0.0430	0.0480	0.0336	0.0000	0.0569	0.0493	0.0263	0.0215
0.0535	0.0632	0.0493	0.0331	0.0416	0.0207	0.0000	0.0427	0.0370	0.0337	0.0295
0.0558	0.0368	0.0613	0.0586	0.0736	0.0446	0.0528	0.0758	0.0658	0.0304	0.0117

For example

♦ $V_{2,11} = (0.5622 \ 0.4629 \ 0.5701 \ 0.4785 \ 0.4248 \ 0.5508 \ 0.0000 \ 0.5177 \ 0.5140 \ 0.4062 \ 0.3700) \times$

0.000
0.000
0.000
0.000
0.000
0.000
0.000
0.000
0.000
0.000
0.058

-0.0215

$V_{4 \times 11} = R_{4 \times 11} W_{11 \times 11}$

So, once I multiply them I have I am not going to this was not possible in a one slid. So, I have given the actual value of V which is V is equal to V remember I will again write.

I am sure you remember, but still for the sake. So, V was basically 4 cross 11 was being calculated from R which is 4 cross 11 into W which was 11 cross 11. So, the value of V is given here. Now if I want to find out each and every cell value. So, I have only highlighted and then use the blue colour no I will not use the blue colour yellow would be better.

So, I will basically highlight this value which is here. So, how did I calculate I took this is 2 11 which means I am taking the 2nd alternative and the 11th criteria; that means, the second row and the 11 th volume. So, the values are given. So, this 0.562 to 0.4629 and so, on and so, forth will be the last one which is point 0.37 are already calculated normalised values for the vector R or the sorry matrix R. So, that you multiplied with the last column for the weight if you remember the last value was 0.058 which is about 5 percentage weightage being given to the 11 criteria. So, all the other cells would be 0. So, if you multiply the value comes out to be 0.0215.

So, based on that you have the matrix V, now given matrix V now our actual calculation will start. So, let us pause here the V values gives us the weightages which are corresponding to each and every criteria and the normalized over all accrued value which is coming out from each and every criteria on each and every alternatives; that means, they are normalised on a scale of 100 or 1; that means, on a scale 1 what is the value which each criteria is given to each other alternative that was 1, then we multiplied by the weightages corresponding to the fact that the weightage would give you the importance level among the criteria's themselves.

So, once I multiply I merging both these 2 concept one is the weightages among the criteria and one is the value accrued from each criteria to the each alternative. So, now, I have so, called made a ranking corresponding to the fact of the criteria and the alternatives now this view V matrix would be utilised.

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TOPSIS: Example (contd..)

▪ Calculate

$V^+ = (0.0669, 0.0632, 0.0656, 0.0586, 0.0736, 0.0446, 0.0528, 0.0758, 0.0658, 0.0385, 0.0417)$, where $V^+ = \{(\max_{j \in J} v_{ij} \mid j \in J), i = 1, \dots, m\} = \{v_1^+, v_2^+, \dots, v_n^+\}$

$v_1^+ = \max(0.0609, 0.0669, 0.0535, 0.0558) = 0.0669$

$v_2^+ = \max(0.0589, 0.0491, 0.0632, 0.0368) = 0.0632$

Now, if you remember I have to find out the V plus values and the V minus values for each and every these concepts of the columns which are the criteria. So, which if your normal remember one thing which I did not mentioned in the last lecture for which I apologize.

So, when you are trying to find out the V plus or the V minus, remember in which direction you are doing the normalisation. If it is along the rows so, you will find out the V plus for the rows if you are doing along the columns you will find out the V plus for

the column similarly for the V minus. So, here we did normalization corresponding the fact for each row was taken because they were 11 elements in the denominator 11 means squared and thing being added and then find out the square root. So, if I have this I try to find out the maximum.

So, I give you 2 only examples to make you understand. So, let me again highlight it. So, I highlight the first value with the first value, I highlight the second value with the second value. So, what was 0.669 if I take the maximum of all the values which were there corresponding that the fact. So, I have say for example, for the first this alternatives I have 11 values they are starting from 0.0 0.0609 till the value of 0.055.

So, I am taking the maximum that for each and alternatives. So, the maximum is come out to be 0.0669 similarly go to V 2 plus it is a maximum for all this 4 values. So, you have to consider the columns. So, there are basically 4 values in each column. So, first column which is criteria 1 4 alternative; second column second criteria 4 alternatives once you find out you will have basically such 11 values and each being the maximum of the 4 values. Remember that because that can be reversed depending on the concept of the normalization utility you are utilising.

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TOPSIS: Example (contd..)

▪ Calculate
 $V = (0.0535, 0.0368, 0.0493, 0.0331, 0.0416, 0.0000, 0.0000, 0.0427, 0.0358, 0.0265, 0.0117)$, where $V^- = \left\{ \left(\min_i v_{ij} \mid j \in J \right), i = 1, \dots, m \right\} = \{v_1^-, v_2^-, \dots, v_n^-\}$
 $v_1^- = \min(0.0609, 0.0669, 0.0535, 0.0558) = 0.0535$
 $v_2^- = \min(0.0589, 0.0491, 0.0632, 0.0368) = 0.0368$

Similarly, I go to the V minus here in the V minus I take the minimum remember that max minimum is coming.

So, when I do that I try to find out the V min; and the V min value will again what I am doing is that I am again taking one column at a time and in each column there are 4 values. So, the 4 values that are corresponding to alternative 1 2 3 4 and how many columns are there 11; that means, for each criteria I rank the alternatives amongst each other on the weighted scale what is the weighted scale? Again I am repeating that was the multiplication of the array tricks with each and every row for the array tricks with the columns of the w matrix; corresponding to the fact that r was the normalized values accruing from each and every criteria to each and every alternatives point 1 and point number 2 was that the weights give us the level of importance among the criteria themselves.

So, once you have this the V i values which are minimum. So, again I am I will colour it in for ease of understanding is the first value gives us the first value. The second value gives us the value similarly the third fourth and so, on and so, forth we come to find out. So, they would be technically v 1 to v n I am not talking about the minus sign because it would be plus for the positive values. So, v 1 to v n basically we will have such 11 values.

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TOPSIS: Example (contd..)

- Calculate the distance of each project to **most positive ideal solution** using $S_i^+ = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^+)^2}$, thus
- ❖ $S_1^+ = 0.1045$
- ❖ $S_2^+ = 0.1543$
- ❖ $S_3^+ = 0.0870$
- ❖ $S_4^+ = 0.0425$

Now, once you have that you want to basically find out the distance of each project or each or project means the alternatives to the most positive ideal solution similarly you

find try to find out the distance of each (Refer Time: 13:07) alternatives for the most negative ideal solution.

So, once you do that I am I am the same formula. For S plus it will be the difference of that of the distance. Now here you see this is basically the concept which is coming is the distance is basically the L_2 norm which is the Euclidean distance which you talked about. It can be changed depending on what type of problem you are trying to do. So, it will be the difference between each and every cell value with the maximum to find out the most positive ideal solution square it sum it up and find out the square roots such that you are giving one criteria for the S 1 plus which comes out to be 0.1045 if you do the calculations you can find out.

So, if there are if V_j plus V_j s would basically be for each and every rows we have the maximum values. So, if you find it out the values are coming out to be 0.1045 I will highlight them. This once I am I am requesting you once you have that matrix remember that I am again going slowly once you have the matrix for the value of x normalise it be careful what normalization you are using also be careful what is the normalization you have to using in which direction row or column. Once you do that have a have a understanding in that what is the weights remember the size of x should be m cross n size of value of r which you when you.

Which is normalised matrix corresponding to the fact that you are getting it from x also should be m cross n ; m is the number of alternatives n is the number of criteria similarly the weight matrix would be m cross n . So, once you find out s once you want to find out V sorry V which is a matrix would be size m cross n multiplied by n cross n which is m cross n once you have that we basically find out the positive value which is the ideal solution negative value which is the ideal solution, but these being from the positive sense and the negative sense.

So, once you have that you would basically find out you will basically find out the value of v pluses values which are the max for the positive sense and v minus would be the min from the negative sense. And then I also mention that you can find out the min for the max also and min for the not for the max min for the positive distance and max for the negative distance. So, once you have that you find out the most positive ideal solution and most negative ideal solutions which are given as this I will use the colour

scheme as S_i^+ is given S_1^+ S_2^+ is given which is for the first and the second alternative,

similarly, S_3^+ for the 3rd alternative and S_4^+ for the 4th alternatives. So, remember the colour scheme orange yellow green blue I will come to that within few minutes.

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TOPSIS: Example (contd..)

- Calculate the distance of each project to **most negative ideal solution** using $S_i^- = \sqrt{\sum_{j=1}^n (v_{ij} - v_j^-)^2}$, thus

- ❖ $S_1^- = 0.0589$
- ❖ $S_2^- = 0.0477$
- ❖ $S_3^- = 0.0387$
- ❖ $S_4^- = 0.0928$

So, now I do the most negative solution. So, negative solutions I already have the v minuses which is here I am not going to mark it. So, once I find out the v minuses again I find out the dist difference of the distance square them up sum them up find out the square root again I am trying to follow the 1 2 law. So, once I have them I basically have the s_1^- s_2^- s_3^- s_4^- in the negative sense the negative ideal solutions or distances for the first second third fourth alternative.

So, let them mark that orange for the first as I did for S_i^+ , yellow for the second as I did for the S_2^+ , green for the third as I did for the S_3^+ blue for the fourth as I did for the S_4^+ . So, there is a reason for that why I am doing that. So, we have found out S_i^+ values 1 2 3 4 for 4 alternatives I have found out S_i^- for 1 2 3 4 values 4 alternatives this are basically the maximum and the minimum distance corresponding to the fact that I am using the concept of idle distance how far I have close it is.

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TOPSIS: Example (contd..)

▪ Calculate the relative proximity index of each alternative (which is buying the house/apartment) to the ideal solution according to formula $c_i = \frac{s_i^-}{(s_i^- + s_i^+)}$, thus

❖ $c_1 = \frac{0.0589}{(0.0589+0.1045)} = 0.3605$

❖ $c_2 = \frac{0.0477}{(0.0477+0.1543)} = 0.2361$

❖ $c_3 = \frac{0.0387}{(0.0387+0.0870)} = 0.3078$

❖ $c_3 = \frac{0.0928}{(0.0928+0.0425)} = 0.6853$

PIS		NIS	
max	min	min	max
I a	I b	II a	II b

Now, you want to find the relative proximity

So, on the relative proximity index for each and every alternative would be how close or far is on a scaled value. So, the scale value now if here again I repeated one thing in I did mention I did draw that in the last class which I will try to again highlight it here on once again. So, if I know the relative proximity of index for each alternative which is that buying of that house to the idle solutions. So, what is the idle solution? Whatever you have find we will find out how or far it is and basically try to rank them accordingly.

So, I am only taking if you remember I mentioned 4 columns; that means, for the max case for the positive min case for the positive, then I do it for the min case for the negative and max case for the positive. So, now, again listen to me carefully max case for the positive means closer the distance is better; that means, it is absolutely essential that I am I am at that point of the ideal solution. So, higher it is good if I go to the second column what I am said I will basically min of the distances.

So, when I am taking the min of the distances is basically I am going as far as a away from the positive solution; that means, it is positive, but I am slowly trying to go away such that it will not accrued to mean that higher value. When I go to the third column concept is basically min of the negative values; that means how far I am with respect to the negative solution.

So, if you take the first and the third column if both the values are high; high in the sense max value high min value will be as low as possible that is the best combination; that means, is a win situation in both the cases I am winning; that means, in first case; that means, I am closer to the positive solution and its min for the third column which is I am as far as away from the negative solution.

The worst case scenario is the second column and the fourth column; that means, I am minimum from the as per as away from the positive solution and I am as close as possible to the negative solutions which is the worst case. and the other 2 cases which are the combination for the first and the third first and say for example, the fourth and the second and the third would give me a combination which you have to basically rank.

So, these things I am not included, but I am again repeating it not to make it too much complicated. So, you have the solution. So, I have basically C 1 C 2 C 3 C 4. So, these S values were already given. So, if you remember this I will I will highlight it now let me do let me do the writing again. So, whatever I am saying if I write it down it will easy for us to understand.

So, there were again 4 columns as I mentioned. So, let me mention them as positive solution negative ideal solution. So, this is the column where I have divided them into 2 parts then under each I divided into 2 parts what are these. So, let me talk. When I use the positive solution I am using max when I am using the negative I am using a min I am using the same colour scheme remember. When I use the min for the PIS I use the light green and when I use the max for the NIS I use the dark red colour.

Now all these things would give me the combinations are there are 1 2 here and there are 2 each here. So, that will be 4, I am only using 1 of them I can do the relative ranking and give a much better picture sum would will be very difficult to differentiate because if you take the first and the see for example, the first and the first here; that means, first and the third would give you the best solution second and the fourth will give you the worst solution any combination of first with respect to say for example, fourth and considering with respect to third will give me an intimate solution.

So, what I am trying to do is that, here o what is coming out here is this one. So, I will mention it as first a first b second a second b. So, the combinations can be done accordingly the values of C 1 C 2 C 3 this should C 4 sorry for that I will change it here.

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TOPSIS: Example (contd..)

▪ Calculate the relative proximity index of each alternative (which is buying the house/apartment) to the ideal solution according to formula $c_i = \frac{S_i^-}{(S_i^- + S_i^+)}$, thus

$$c_1 = \frac{0.0589}{(0.0589 + 0.1045)} = 0.3605$$
$$c_2 = \frac{0.0477}{(0.0477 + 0.1543)} = 0.2361$$
$$c_3 = \frac{0.0387}{(0.0387 + 0.0870)} = 0.3078$$
$$c_4 = \frac{0.0928}{(0.0928 + 0.0425)} = 0.6853$$

So, the values are coming out to be as it is given just be at no pay attention here.

So, C 1 is 0.36 I am only reading the 2 decimals C 2 is 0.23 C 3 is 0.30 C 4.68 once we have that we can find out the values corresponding to that.

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TOPSIS: Example (contd..)

▪ Thus the ranking is $c_4 > c_1 > c_3 > c_2$

▪ Hence Alternative # 04 is the best (position # 01) choice followed by Alternative # 01 (position # 02), then by Alternative # 03 (position # 03) and finally followed by Alternative # 02 (position # 04)

Now, once you are done the ranking values would give that when you rank this C 1 C 2 C 3 C 4 corresponding the max for the positive and min for the negative then you have C 4 is better than C 1 is better than C 3 which is better than C 2.

Hence it will mean the alternative 4 is the best which is in position 1 that is the house where you are trying to buy depending on the 11 criteria's. Alternative 1 is the second one which is in second position on alternative 3 is in the third position third one and finally, alternative 2 would be in the position 4. Now that ranking which you have to use now this is the simple step which I followed with the problem ranking which you have used was for max min, again I am repeating; that means, first and third.

Similarly, I can get 3 different other ranking system based on the second and the fourth which is also quite easy to understand. The things would basically be difficult to make a judgement would with the case when I am considering the second and the third and when I am considering basically the first and the fourth based on that I am doing.

So, with this I will I will end the TOPSIS method discussion and this is the end of the seventh week which is the 34th 5th lecture and you will only have assignments based on the concept of top TOPSIS. And from next week which is the 8th we will start the other different multi criteria decision making and hope any queries which you have you can basically put in the forum and we will definitely try to answer or the TAs would definitely answer as soon as possible. Have a nice day and.

Thank you very much for your attention.