Data Analysis and Decision Making – II

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Lecture – 32
Algorithms for TOPSIS

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you, wherever you are in this part of the world, whether in India or Abroad. And this is as you know this the DADM which is Data Analysis and Decision Making 2 course under the NPTEL, MOOC series. And this score total course duration is for 12 weeks which will be 12 set of lectures and total number of hours is 30, which converted considering the fact that you have each lecture for half an hour, so it will be 60 lectures.

And as you know that in each week, we have 5 lectures. And after each week, we have assignment 1, 2, 3, 4 so on and so forth. And as you can see from the slide we are in the 7th week and we are in the 32nd lecture that means, we have started the 7th week. And my name is Raghu Nandan Sengupta from IME department, IIT Kanpur.

So, if we remember, in the last class just lecturers concluded the last day. So, we were considering the concept of the algorithm for the TOPSIS method. And if we remember, I also mention that we will basically have 2 mattresses; one is the decision matrices, which will technically considered as either x or a or d, whatever it is which is not normalised and we will have a the weight matrix.

Now, the d or the a or the x which is non-normalised would be of the size m cross n, when m is the number of alternatives and n is the number of criteria. So, the cell value i comma j would basically I explain each and every row accordingly to the fact that we had the n comma m, but they remain the same you just take the transpose.

So, the cell for the m cross n matrix would be the effect of the ith criteria on the jth value of the alternative. Here i and j are interchangeable used, here i would basically change from 1 to n and j would basically change from 1 to m. Similarly, you can have the no in this case sorry in when it is m cross n, I am using i, j and it remain the same. So, i would change from 1 to m and j would change from 1 to n.

And we can basically find it accordingly, so and based on that we will proceed. So, first I will go through the algorithm and discuss the problem. Both in the slide, it will be easy for us to make the values change in the excel sheets such that you will be in a much better position to appreciate that.

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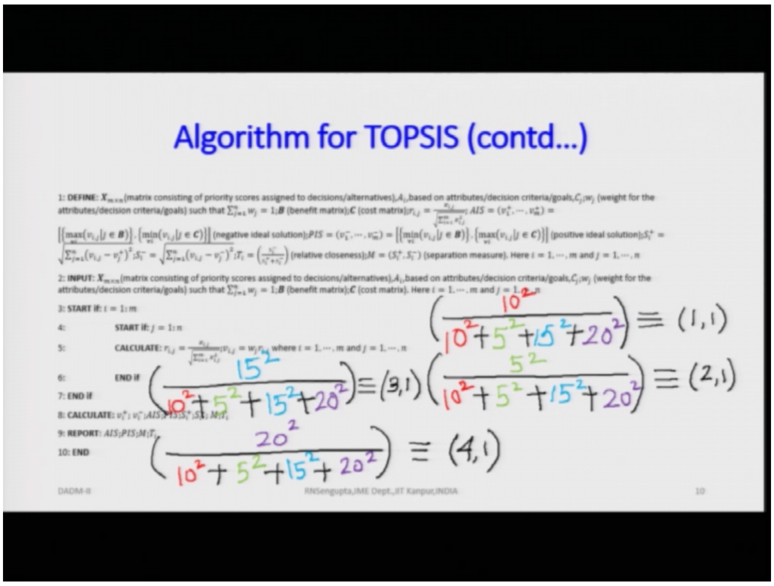

So, we will consider this set x, the metric consisting of priorities scores assigned to decisions and alternatives. So, m would be again I am repeating sorry if I am repeating too much, but it will make things much better for you, a much easier for you, m would be number of alternative and n being the number of criteria.

(Refer Time: 03:36) it will be based on the attributes and the decision criteria's which is C j, and W j would be the weights for the attributes and the criteria's which we have. So, W 1 would be basically the weight I am assigning to the criteria one, similarly W 2 for the second one, W 3 for the third one, similarly W n would be for the this 1, 2, 3, 4 till n are in the suffixes would be for the nth criteria.

Now, what it means that out of the total set off of criteria, I am giving say for example, if W suffix 1 weight is 0.15 out of 1 or 15 percent that means, 15 percent of the weightages for the criteria would be given to say for example the combination of all the alternative taking from 1 to m. Now, that 15 percent would basically be subdivided for the alternative 1, alternative 2, alternative 3 till alternative m, but we will be a singly subsume that such that we will have the overall weightages for each and every criteria, and then find out the breakages or the apportionment for each and every criteria for each and every alternative.

Now, we will basically find out using the ranking that means, distance ranking would be used. And the distance ranking is basically found out using r i, where I will try to find out that what is the ratio of the that value which is basically x i j or a i j or d i j depending on how I have been able to explain or how I have been able to write down the basically the priority matrix. And that will be divided by the square root of the distances at which I have.

Now, the distances can be either be calculated along the row or along the column or along the row. Then we will find out basically the scores based on the positive or the best idle co distance or best idol point which I mentioned as a PIS and the negative one basically being discussed by the NIS.

Once the distances are found out or the scores are found out based on the idle set of positive and negative, we will basically find out what is the overall ranking of the so called sets which are all formed positive in value and negative in value. So, now this sense of positive and negative, which is basically known as relative closeness and relative separation matrix or how close it is and how far it is something to do with the concept of concordance and discordance.

In concordance and discordance, we basically found out the level of liking and disliking based on the fact that for the criteria, whichever criteria it is jth one, how close or how far or what is the liking and the disliking factor which we have when we are trying to compare a k with a l.

Similarly, when you try to find out a k with respect to a l based on the distance matrix, we will find out those set of j's, which are have a close measure with respect to the positive idle solution. And you will also find out those set of j's for which we have the positive value, positive means the how close they are to the negative ideal solution. Then again when we compare the lth with the kth one, again we will basically have the closeness matrix, which is the how close they are with the positive and how close they are with the negative one. And basically based on that we will try to find out the sets of so called concordance and discordance and proceed accordingly.

So, we will find out the relative closeness. Relative closeness would given be given by the ratios of positive divided by the sum the positive to negative one, similarly you can find out the negative scores also. We will start with for each and every jth one, as it changes from 1, 2, 3, 4 goes to n and we will find out the closeness factors or closeness distance as well as the negative distances, when we compare k to l or l to k. And we will basically calculate these closeness factors and rank them accordingly.

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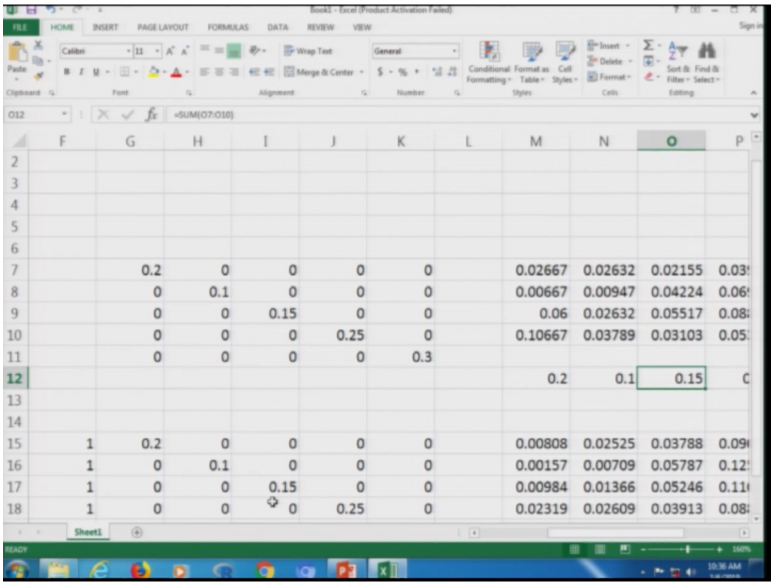

So, what I will do is that, first I will show you in the respect to the excel sheet and then come back to the PPT slide for better explanation. So, this is the same problem, so it would be easier for you. I will zoom in order for better explanation ok.

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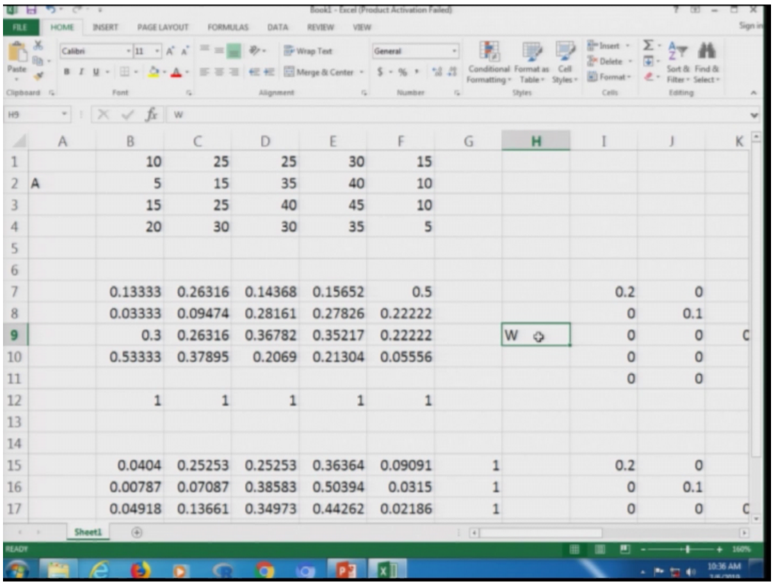

So, consider this matrix let me mention it, so it will be easy for all of us to understand. So, consider this as A and this is I am considered as there I should make a give a so this would be given as W, this will be given as W I should create yes, now it is fine ok.

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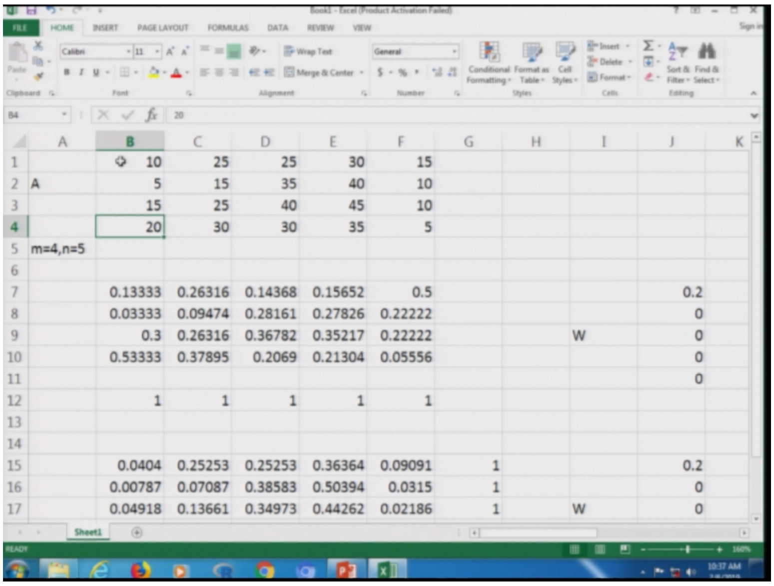

Now, let us go one by one. So, if I so here consider m, so let me write down, what is m? M is equal to 4, n is equal to 5.

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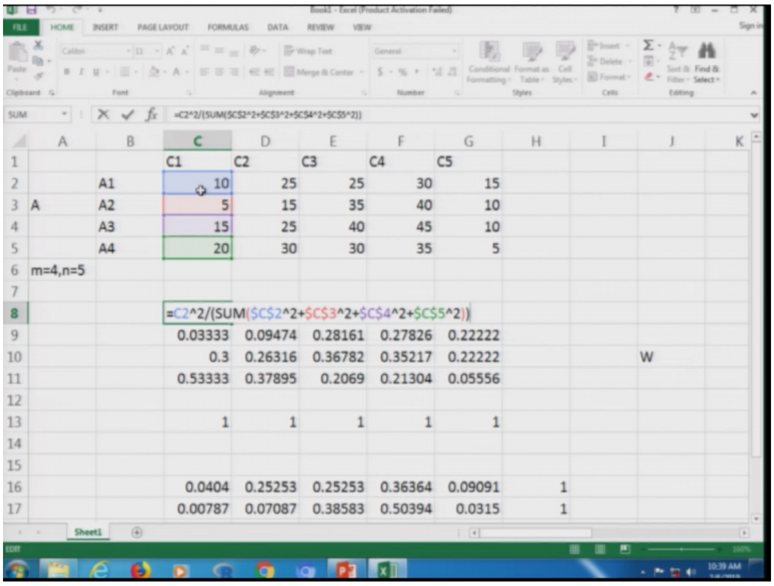

So, if you see 10, which I will mention let me mention it here, so these are technically so this m sorry my mistake I have taken m alone yes. So, if this means that the overall returns positive returns, what I am giving getting for alternative one from criteria one is 10 value 10 unit whatever it is, so that can be based on any utility function.

So, similarly when I consider the overall weightages or overall value, no I would not use the word weightages overall value which I am getting from utilising the criteria or finding of the effect of criteria to an alternative 1 is 25, similarly the effect of criteria 3 for alternative 1 is for 25, similarly for alternative 1 from the criteria 4 is 30 and finally from criteria 5 alternative 1 is 15.

Similarly, the corresponding values from C 1 to C 5 for alternative 2 or 5, 15, 35, 40 and 10. Similarly, the corresponding values from C 1 to C 5 for A 3 and A 4 are 15, 24 25, 40, 45, 10 and 20, 30, 35, 5.

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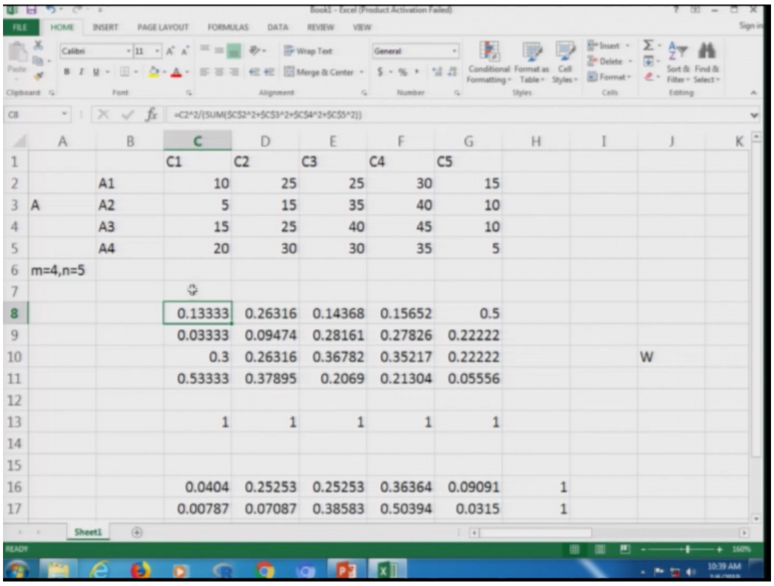

Now, we have to basically normalize then. Now, this normalising concept, I will use different functions accordingly. And then give you a picture, how it can be utilised. So, here I am doing the normalisation based on the fact that I am trying to find out normalisation along the particular column, so it will be 10 square divided by the sum of the squares of 10, 5, 15 and 20.

So, if you see the value here, you have C 2 which is basically 10, I find out the square of that. So, basically I will I can write it down if possible in that I will write it here, so the values are 10, 5, 15, 20. So, if I can write it down here, it is 10, 5, 15, 20.

So, the value which will find out for the first cell, I will use the red colour, it can be one of those utilities can be 10 square plus 5 square plus 15 square plus 20 square, so this will be the value for 1 comma 1. Similarly, when I go to one comma if I go if I am going along the column or along the row, so that is how we will get.

So, let me check that how am I going here. So, if I am here I am going along the column, so this would be 5 square no let me use a different colour in this denominator should be easy for all of us to. I will use with your permission, I will erase it, so it is much easier for me to write.

So, I will use the first the black the bracket remains, there would be 1 element, 2 element, 3 element, 4 element. So, will use the colours accordingly, so it will be 10, 5, 15, 20, so 10, so this is square 5, 15, 20. So, this is basically the value which will have the 1 comma 1. If I come to 2 comma 1, the values again I will write it first value, second value, then third value, fourth value. So, I will use the colour schemes same, so this will be the second value.

If I go, I will write the third one, so this is basically corresponding to 1 comma 1, this is corresponding to 2 comma 1, I can reverse according to lower column that does not matter. Let me write the value for these are the first, second. If you have understood, it please bear with me let me solve with, so this is 3 comma 1 and there would be 4 comma 1.

So, I am writing the value. So, first was red, I will use the value red here red here, then was green, then was blue, so blue is being utilised for the 3 comma 1, so obviously this will come here. And violet was being used for the last cell, so it would be so I will have these values accordingly.

And similarly, I can find out the second column, third column accordingly. So, let us shift to so this comes out 0.13, 0.03, so these are the values, just and highlight check. If you see the screen the values in I am finding out the square divide by sum of the squares (Refer Time: 17:35), 2 comma 1, 3 comma 1 and 4 comma 1.

Similarly, 1 comma 2, 2 comma 2, 3 comma 2, 4 comma 2, 3 comma 1, 1 comma 3 sorry 1 comma 1, 2 comma 3, 3 comma 3, 3 comma 4, this is 1 comma 4, 2 comma 4, 3 comma 4, 4 comma 4, finally 1 comma 5. Just check what is there in the numerator and the denominator. This is 2 comma 5, this is 3 comma 5, this is 4 comma 5.

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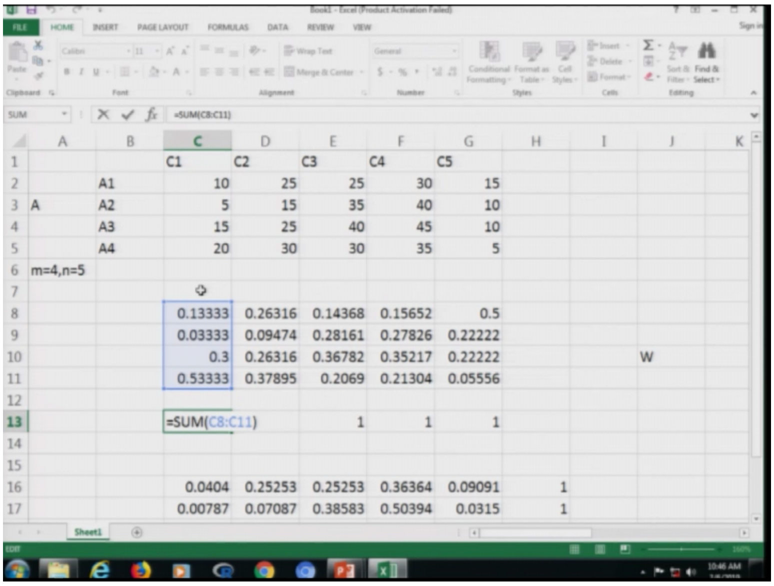

Now, interesting fact is here. If you see the knowledge can be done along the column, so if you check the sum along the respective columns are all one, this is 1, this is 1, this is 3rd column is 1, 4th column is 1, so in the 5th column is 1. Now, there are two questions. So, let us so both of them. What if we do the normalisation along the rows, so here it is. So, if you see I am taking 10 square divided by 10 square in some 10 square plus 25 square plus 25 square plus 30 square plus 15 square.

Similarly, when I go to this cell, this will be 25 square and in the denominator everything remains the same 10 square, 25 square, 25 square, 30 square, 15 square. Similarly, denominator remains the same. And in the numerators 25 square denominator remains the same. Same means the first square of each and element in the first row. And in the numerator you are 30 square finally, you have 15 square divided by that the sum of squares for the first row.

Similarly, if I go to the second row, it will be numerators respectively will be 5 square, then 15 square cell wise, I am not coming to the denominator 35 square, 40 square, 10 square. And the denominator as it you are trying to normalise along the rows which is the second row, all the values at the denominator will be same for this 5 cells and their 5 square 15 plus 15 square plus 35 square plus 40 square plus 100 10 square.

Similarly, I go to the I should so it will be now 15 square in the denominator 15 square plus 25 square plus 40 square plus 45 plus 10 square. And in the numerators respective cell values are 15 square, then 25 square, then 40 square, then 45 square, then 10 square. Finally, it is denominator again same 25 square, same in the sense we are going through the that last row 20 square plus 30 square plus 30 square plus 35 square plus 5 square. And each cell corresponding numerators are 20 square, 30 square, 30 square, 35 square, 5 square respectively.

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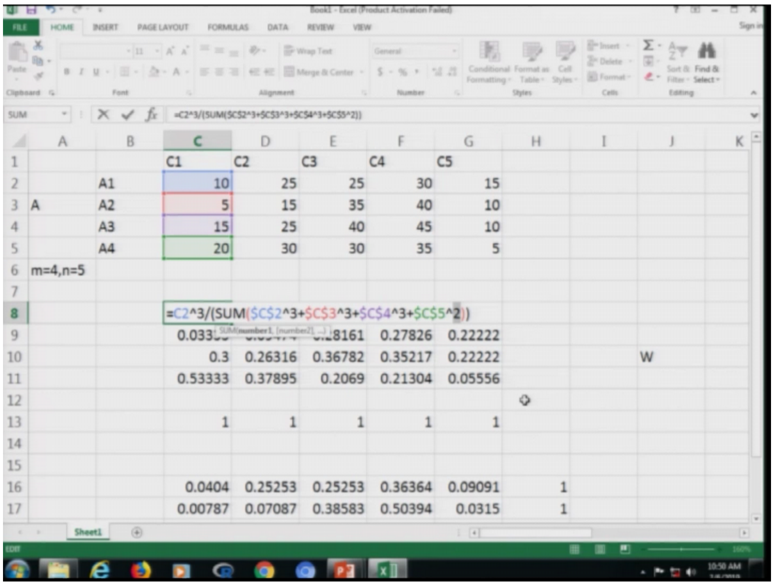

Now, again another interesting fact. If you remember, I am mentioned that in the AHP method. The some normalization along the rows is 1. So, now the question comes, what if we use a normalisation along the column, but not this utility function. So, let us see that is so I am using some theoretical values, see theoretical utility function cubic one. So, let me use the cubic one.

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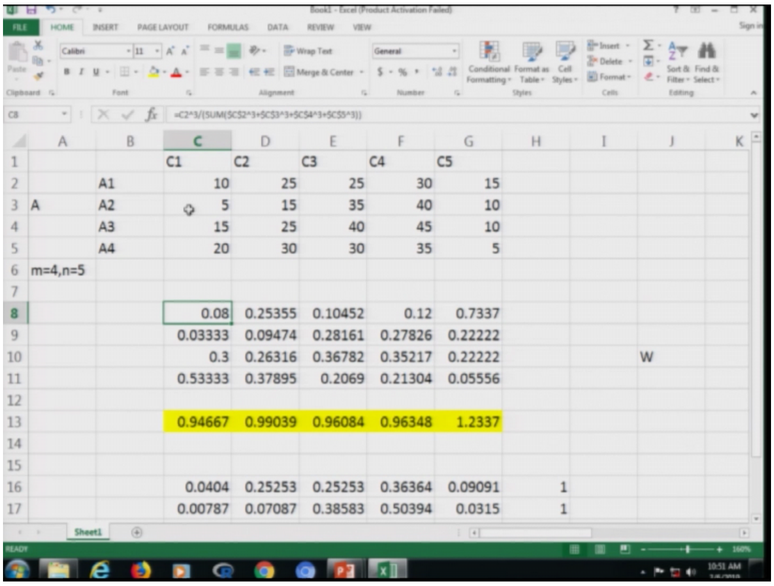

So, some obviously, keep looking at the some values, these are important, we will find it out very soon. So, now it is changed, because I am changing the utility function value. So, this was cubic. Similarly, I am going a little bit slow, so I can basically then speed in up the whole process of the calculation. So, I will take the cubic function normalization being done on the cubic so called concept.

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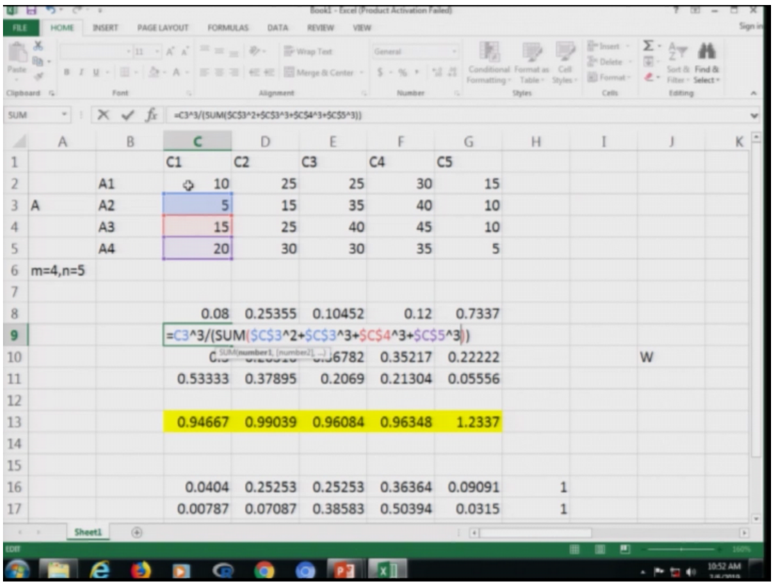

So, the first row is being cal row is being almost completed, after we have done. So, let me check with this cubic everywhere. So, now if I cal copy, what will happen? So, basically I find replace 3, 3, now this will 3, 3, 3.

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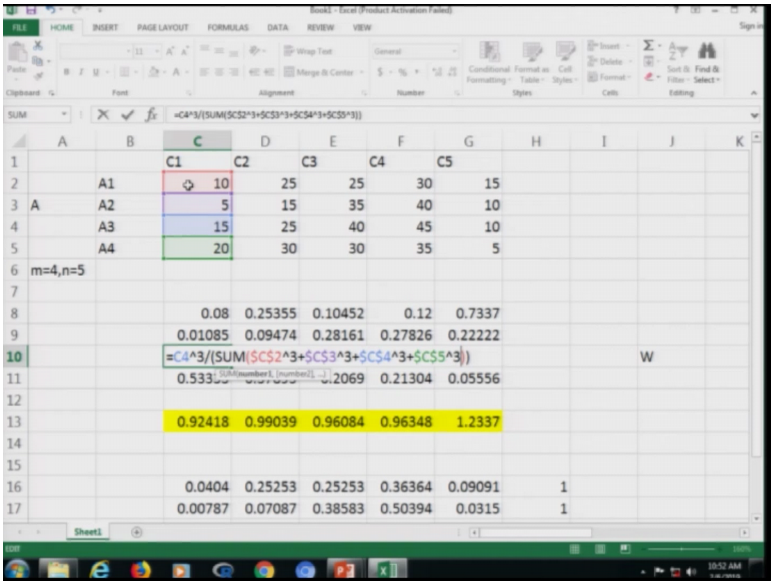

So, I will I am now going through the row wise column wise 3, 3, 3 sorry this should be 3.

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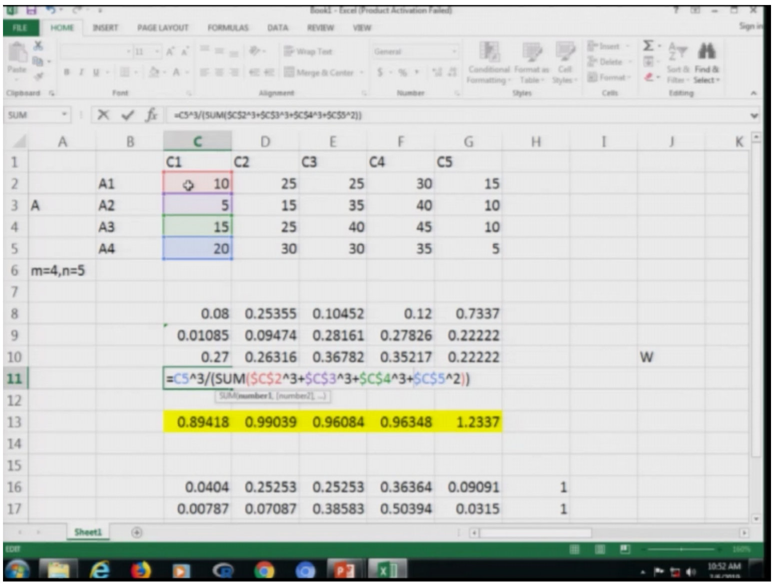

Now, you if it is normalize along the column, so some which is the yellow one 0.89418 should come out to one, so let us check. If it is not, then we must have done some calculation mistake, but let me check.

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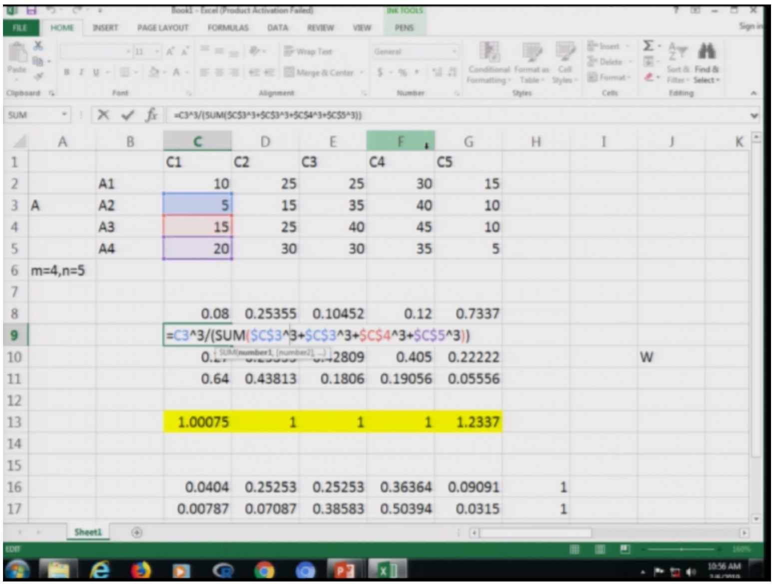

So, there is some errors let me check ok, this should be cube sorry. So, let me check, it is cube here, cube here, cube here, cube here, cube here, the first (Refer Time: 25:25) is done. Second one cube, cube, cube, cube, cube done cube, cube, cube, cube, cube done cube. So, there will be some issue, let me check.

Let me check the second column, I think we should be able to find out cube, cube, cube, cube, cube done. This is cube, cube, cube, cube, cube, then we are doing it for the cube, cube, cube means, I am trying to find out the cubic function.

So, finally cube, cube, cube, cube, so this value is 1, so obviously there some error. I will come back to that later, first let me do for the this is all cube done. So, it is taking a little bit long, but you will understand the concept of normalisation along the row and the column and the concept of utility.

I mean now using meaning the column wise normalization, but I am changing the utility function, quadratic to some sort of cubic form. So, then I am taking the cube here, last cell in the 3rd column, T 2, 3 for great this is also one. So, this is done.

I will spend few minutes of the last moment for this class, this is cube everything is done. So, do this is also done. So, the last cell let us see whether the 4th column is also one, because we are doing noun normalisation, this first F 2, F 3, this should be 3, this should be 3, so this is also done.

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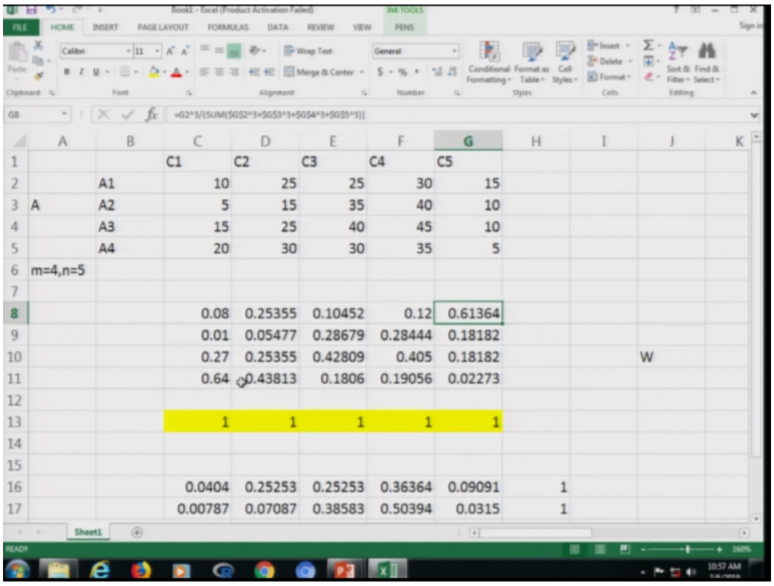

So, now let us come back here, you must (Refer Time: 28:38) some mistake ok, we omitted this one. So, it should be C 2 also ok. Now, I should be one yes, it is solved. And let me so we have few minutes, let us wrap up this with the last calculation. So, this is cube, this is cube for the 4th columns 2nd cell, so the 4th column 3rd cell, I am going cell by cell column wise. So, it will come out to be one, because I am not going normalisation longer column.

Finally, the last cell left to give us the answer and prove what I have been talking about ok. Again I think is this a cube, cube, cube, cube I did some mistake wait. Let me check, whether I am taking all the values. So, there is some error in cube, cube yes, so this is done.

So, all the values are 1 and then we can expand it. So, I will continue discussing this problem in more details and use the concept of closeness concept of distance from the ideal and the non-ideal one. And also use the concept of say for example, epsilon, Electra which I have done try to, bring that in the concept of TOPSIS method also. So, with this, I will close the 2nd lecture for the 7th week. And hope you have understood any coots questions, obviously it will be answered in the forum as usual.

And have a nice day and thank you very much for your attention.