**Data Analysis and Decision Making - II**
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**Lecture - 30**
**ELECTRE**

Welcome back my dear friends and dear students, a very good morning, good afternoon, good evening to all of you wherever you are, at whatever time. And this is the DADM II, which is Data Analysis and Decision Making II course under NPTEL MOOC series. And as you know this course is for 12 weeks which is spread over 60 lectures and the total duration in hours if you calculate is 30 hours.

And each week we have 5 lectures; so and which each lecture being for half an hour and after each week we have assignments to be solved by the students and as you know after this course there would be a and some examination. And my name is Raghu Nandan Sengupta from the IME Department, IIT Kanpur in India. So, this if you see the slide this is the DADM lecture number 30, which is the 6th week end of the 6th week classes; so, we will try to wrap up the ELECTRE process.

In the ELECTRE process initially we are discussed about general ELECTRE, then we are basically talking about epsilon ELECTRE. And in the epsilon ELECTRE if you remember I have been discussing that how you can make your decision choice in such a way that you have the discordance set, concordance set and the indifference set. And depending on the liking, disliking I will basically divide my whole choice; my means the decision maker choice can be divided into three mutually exclusive and exhaustive sets where disk.

Concordance means, when I take decision A k with respect to A l and j being the criteria. If I am willing to take that I get some positive points hence I will basically put that value of j into the concordance set or C and ultimate link comes into the those values that will be utilized to find out the matrix capital C.

And if I am able to take a decision A k with respect to A l, but in the case, the criteria j does not give me that benefit. So, those index value or indices values which does not support, A k will be utilized to calculate the D matrix which is the discordance matrix.

And in the epsilon method we added a extra set of decision making tool or a rule for decision making, where that in different set would also be covered where we are; whatever decision we take with respect to j.

If j means the criteria, if A k and A l does not give me any benefit by taking an A kl; A k I will put that j into the set which is I indifference indices values which will be utilized finally, to calculate the indifference matrix. And if we remember I did also mention in the last class which was in the 29th class that the indifference matrix or an inference set can be asymmetric values, in the sense that we can utilize the asymmetric penalty loss functions based on the civil engineering problem, the marketing problem and the electoral engine problem accordingly where we can being some concept of practicality into the decision process.

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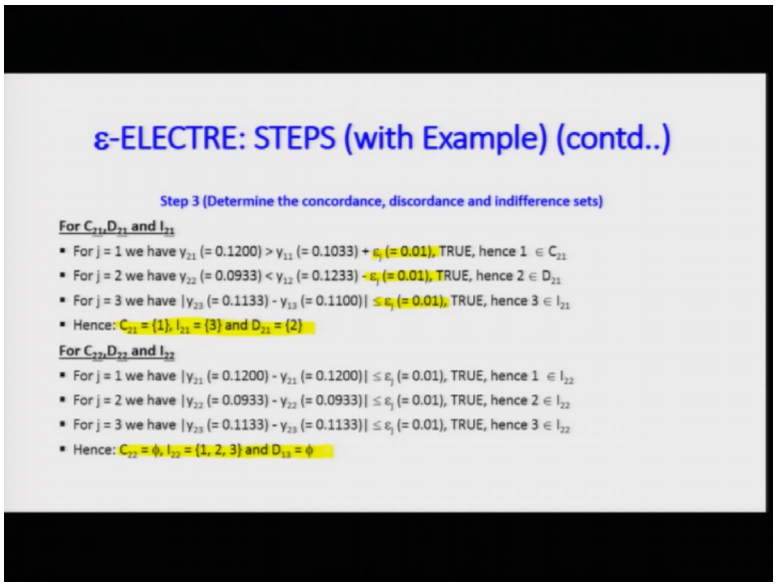


So, continuing our discussion if you remember we were doing the values calculating the values of C 1 1, D 1 1 and I 1 1; where C, D, I are basically the values in the concordance set, discordance set and the indifference set.

Similarly we will calculate C 1 2 after that we will consider C 1 2, D 1 2 and I 1 2. Then going to C 1 3, D 1 3 and I 1 3. So, we will go column wise or row wise. So, here we are trying to calculate C 2 1 and D 2 1 and I 2 1 corresponding to the fact that these are as I mentioned I am again repeating concordance concept, discordance concept, indifference concept.

So, we take again the values of epsilon; epsilon means the difference which you have as 0 point I will highlight it as 0.01 for all the cases. I am not changing it even though I did mention that you can consider depending on the j value the epsilon value can change, but I am not considering that. For j is equal to 1, we will have the values compared the y 2 1 with respect to y 1 1. And we will see that I 1 which is j is equal to on falls in the concordance set.

Similarly for j is equal to 2; 2 falls in the discordance set and j is equal to 3 falls in the indifference set. So, we have for C 2 1, D 2 1, I 2 1; the values as marked in the yellow highlight pointer.

Then when we go into C 2 2, D 2 2, I 2 2; obviously, here the comparison would be company amongst themselves. So; obviously, all of them would fall into the set of indifferent, because I am now indifferent whether I like it or not like it does not matter now because I am indifferent between trying to compare 1 with itself. Like comparing the 1 with 1 or 2 with 2 or 3 with 3, but here we are doing the 2 comma 2 cell values so; obviously, in that case and the null set would be C, null set would be D and the I set would have all the elements which is I j; I am j is equal to 1, j is equal to 2 and j is equal to 3.

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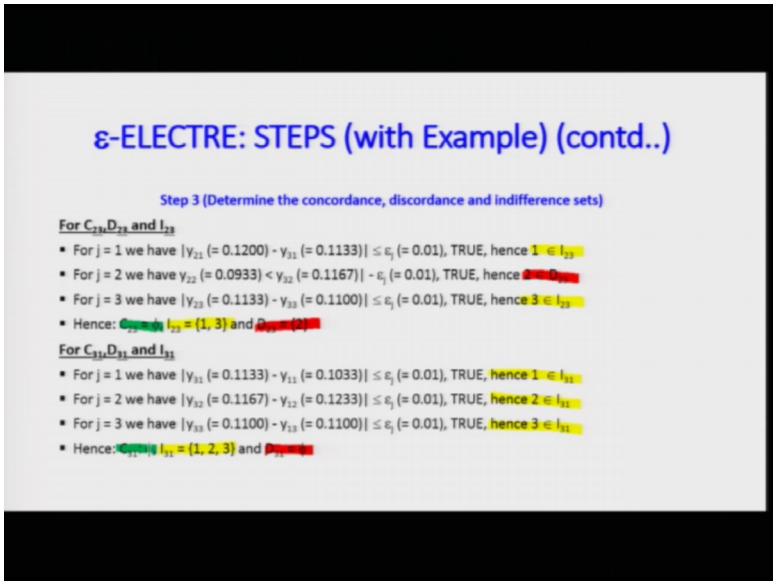


Now; obviously, I will go to the next level would be C 2 3, D 2 3, I 2 3; then we will go to C 3 1, D 3 1 and I 3 1. So, C 2 3, D 2 3 and I 2 3 would be the so called values in 2

comma 3 cell in the whole overall matrix, where each cell if you consider the 3 by 3 matrix over the cell would have elements corresponding to C D and I. So, the 1 comma 1 would be C 1 1, D 1 1, I 1 1, then 2 comma 2 would be C 2 2, D 2 2, I 2 2 and the 3 comma 3 cell, in the matrix would be C 3 3 D 3 3 and I 3 3. So, when you are considering 2 3; it will be 2 comma 3 cell position.

So, corresponding to j is equal to 1 2 3 which are the criteria if you again compare, again considering the fact that we are taking epsilon as 0.01 which is just an assumption simplistic assumption. We have that the value ok; now here I will try to use the different colors like in the indifference set it would be yellow, in the concordance set it will be green and in the discordance set it will be red.

So, as it is indifference set 1 falls in indifference set, 3 falls in indifference set. So, 3 is 1 and 3. Now the correspondingly in discordance set 2 falls there so, 2 becomes red in color and the green one is a null set because C 2 3 is null; hence it falls in the null element is there in C 2 3.

When I go to 3 comma 1 cells that hence we have to consider the concordance discordance and the indifference sets as C 3 1, D 3 1, I 3 1; so, if we consider again j is equal to 1 2 3. So, j 3 are the criteria it can be more than that also; corresponding to that we have 1 2 3 all falling in the indifference set so; obviously, they would be yellow. So, the indifference set has all the elements 1, 2, 3 and the corresponding D set is null, corresponding C set is null. So, we are trying to now differentiate if you remember that a the real line trying to divide into three zones.

One in between zone D which will depend on the epsilon value would be the yellow one, where I indifferent. On to the right when I am looking from my side would be the green one which is concordance positive one and on to the left of that indifference zone would be the red one which is the discordance set. Now I go to the cell number 3 comma 2, in that overall matrix. So, again; so I will consider C 3 2 D 3 2 and I 3 2 and corresponding to j is equal to 1 2 3; when I calculate I am not going to the detail calculation you can basically check. And we are considering epsilon value as 0.01 for all the three cases; j is equal to 1, j is equal to 2, j is equal to 3 all that criteria you are not changing them.

So, based on that when we have; we technically would have j is equal to 1 is indifferent, j is equal to 3 is equal to indifferent. And if I come to the concordance values it is 2 is

equal to concordance set. So, correspondingly C 3 2 is element, only one element which is 2; that is the j is equal to 2. If I consider red it is which is the discordance is a null set and if I say consider yellow which is the indifference set would be 1 and 3, j is equal to 1 j is equal to 3.

Now, I will come to the last cell which is 3 comma 3; then you will have basically the concordance, discordance and the indifference concepts would be C 3 3 D 3 3 and I 3 3 the indices.Similarly for j is equal to 1 2 3 considering epsilon as 0.01 using the same calculation which I have shown; we will basically have all of them falling into the set C. So, they would be 1, 2, 3 falling into which my mistake I think, I have done it something wrong let me change it sorry just change it here.

So, they would be technically 1; it is my typo error sorry for that extremely sorry. So, once I have this, let me save it. So, I will again highlight the appropriate colors for the cell 3 comma 3. So, I will have corresponding to I. So, this is 1 is equal to 1 3, 2 is equal to 1 3; I is equal to 1 3 3 3; 3 is equal to in I 3 3, 2 is equal to I 3 3, 1 is equal to I 3 3. So, the indifference cell has all the elements 1, 2, 3 and the corresponding cells or the element in C 3 3 is null column in cell in D 3 3 is null. So, you are formulated the rule.

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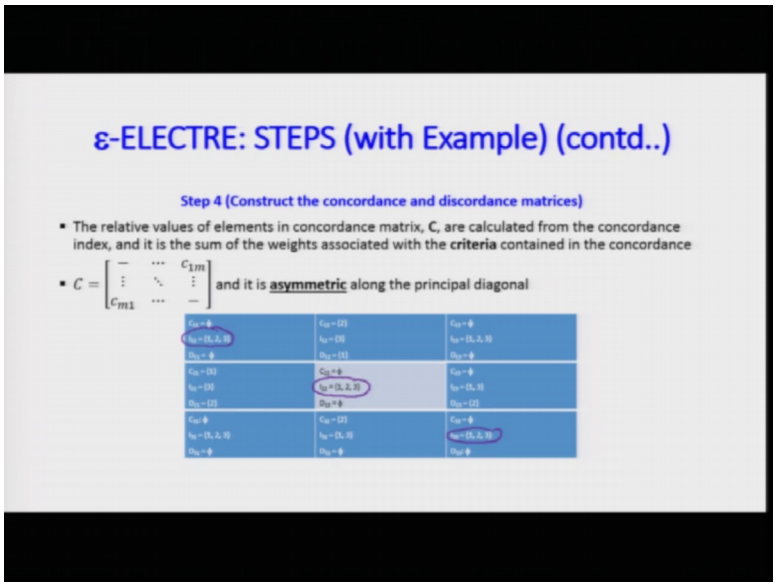


So, now if you see the relative structure of the construct which are formed so, 1 comma 1, 2 comma 2 and 3 comma 3; all the values I will use the yellow, I will use the say for example, the violet just marker.

So, the indifference set in the principal diagonal are all the elements. So, liking disliking and that is liking in concordance and disliking means t discordance sets are null. So, in the principal diagonal everything when you compare you are indifferent in the decision. And if you take the concepts of the off the diagonal element, then you will basically have the indifferent and the this a concordance set and the in this discordance set values based on the which you will proceed.

Remember that this actual value of C; this capital C which you have written is nothing to do with concordance, it is just a symbolic representation of that matrix C. So, this should technically be asymmetric in nature.

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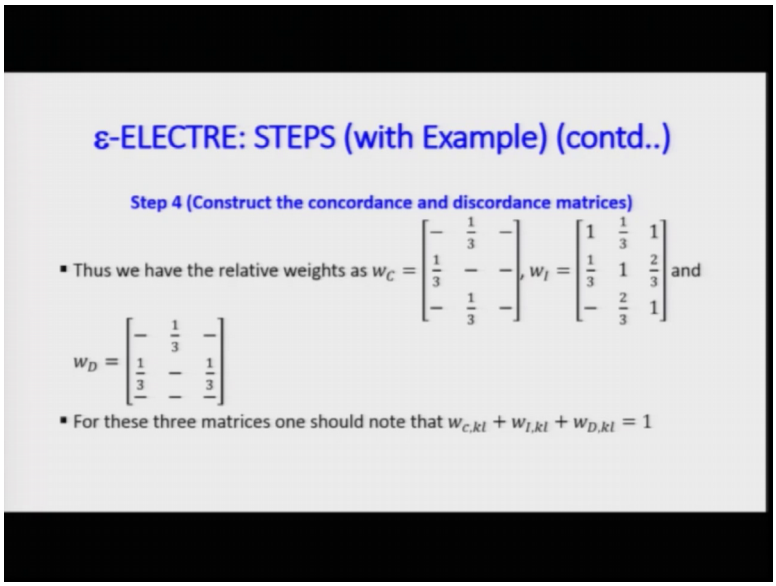


Now consider the weights are given. So, what I do is that or what we considered that? We consider different weights for the concordance set, different weights for the indifference set and different weights for the discordance set.

So, weights are considered in a way such that, I will assign one third, one third, one third for the case for concordance; that means, the top rows values would be one third for the one cross two and corresponding with the diagonal element along the principal diagonal; the opposite mirror image which is two cross one will be one third. And if you consider here one third and the opposite elements dash, it means the liking disliking concept has been formulated accordingly.

Similarly, for the indifference one it is mirror; not a mirror image, the principal diagonal is 1. Now when I compare 1 2 to 1; I am indifferent when I compared 2 3 and 3 2 it is indifferent, but when I compare 1 3 and 3 1 then; obviously, this value of one and dash basically gives the indifference concept Martineau Starkly.

Similarly I have the discordance weights and based on that you what is important to note is that find out if you find out the sum of all the corresponding cells for this weighted or width matrix for CDI; it would be 1. And that can be, that will give you an idea that based on the weights you are doing the you have to done the normalization in the right way.

So, if we add up say for example, first element dash, second element 1, third element dash; so is 1 if we add up one third, one third, one third; it is 1. If you have dash dash 1; it is 1. So, if you add a one third, one third, one third it is 1; then if we add dash, 1, dash is 1. Then if we add dash two third, one third is 1. So, lastly going to the last row dash dash dash you have to check this; there must be some I will double checked that.

So, it should be definitely be 1 here also my mistake. So, I will I will verify that two third, one third. So, this should be; so if I add up 1, so now it should be dash 1 dash is 1; then one third, two third dash is 1 and dash 1 dash is 1. So, all these values comes out to one. So, in this principal diagonal value wise the symmetricity is maintained in the indifference part; not in the case for the discordance set and the concordance set values.

Now if you follow the rule, again we will just follow the same concept what we have done for the discordance fact like we give a range and try to my find out the maximum value based on all the combinations. So, if when I trying to compare will compare one cell value with respect to all the combinations which will be there along the row all the column depending on which way you have normalization you have taken place.

So, you will basically come find out the small c value small i value small d value, when you are comparing the k th and the l th this decisions based on the criteria j. So, you will basically compare them for all the criteria's and basically take the max value or the ratios accordingly.

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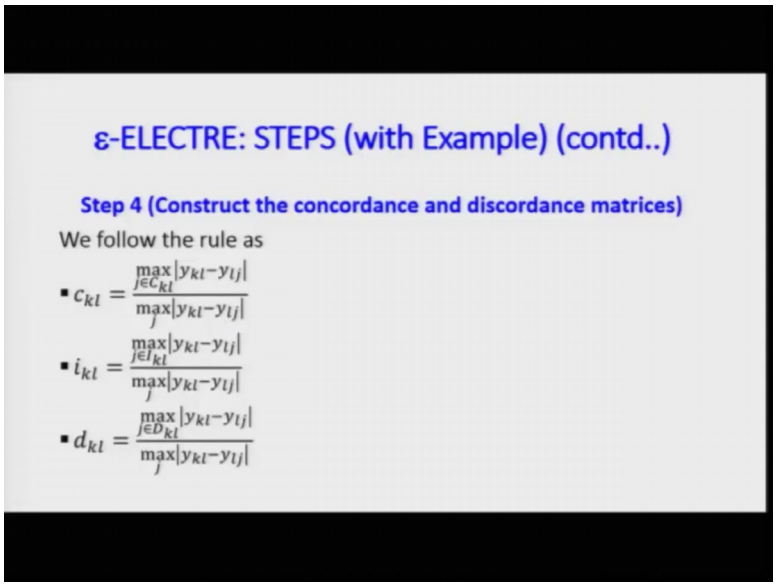


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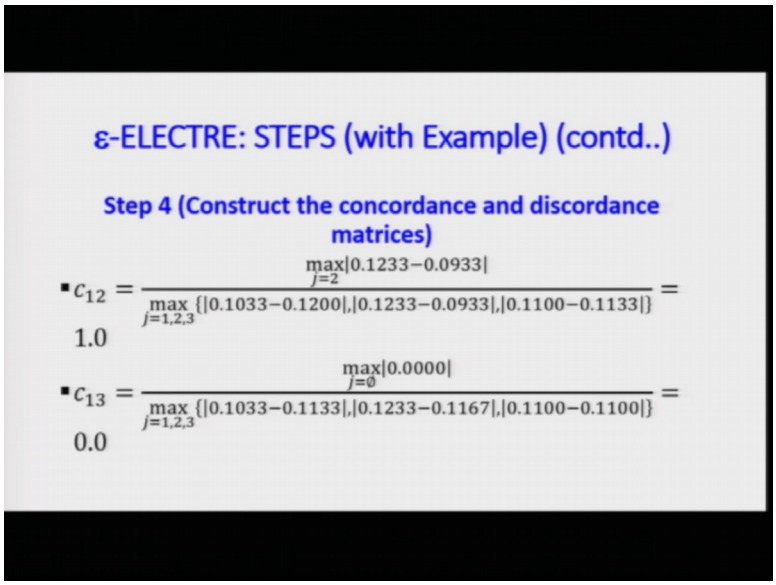


So, if I do that; so I will just go if you remember the values the cells let me write it down; so it is easy for you. So, this would be good point let me take these cells values. So, if I; so sorry I am not skipping, but I am trying to bring that slide in the picture. So, it will be easier for us to compare please bear with me because I thought initially, then because it was. So, I will remove this and this Y values would be utilize accordingly.

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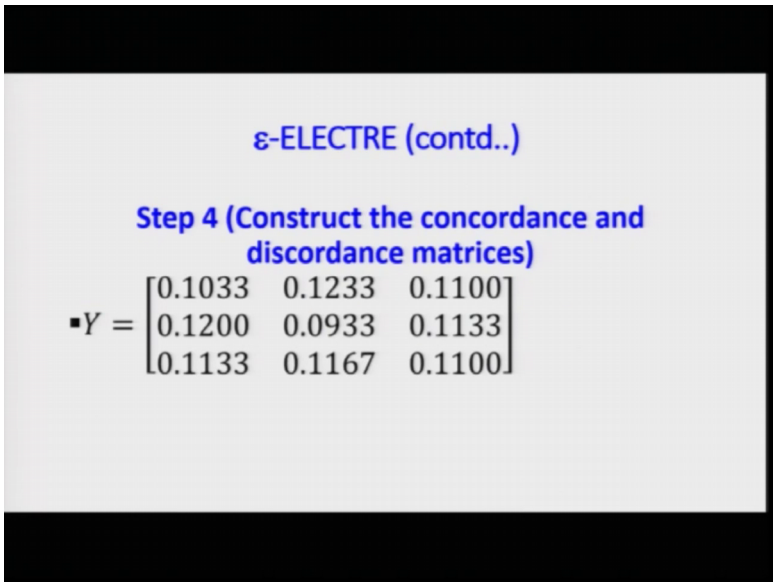


So, I will use this time and again. So, I am copying it for our own benefit.

So, we have these values of Y. So, it will be now much easier for us to denote yes. So, for all the students and the readers I will basically request them to have a look at this matrix which was Y; Y was mystically the normalized using the utility sense and normalizing along the row or the column; the x matrix multiplied the weights we had that y values. And based on that we are trying to proceed you have already found out the concordance and discordance concept.

So, giving the concordance and discordance I find out the cell values. So, c 1 1, then c 2 2 and c 3 3 the principal diagonal will be dash dash dash because concordance discordance concept does not come here. So, if I use these values; so, c 1 2 becomes 1, c 1 3 becomes 0 just pause and see. So, you will basically take the maximum of these values accordingly.

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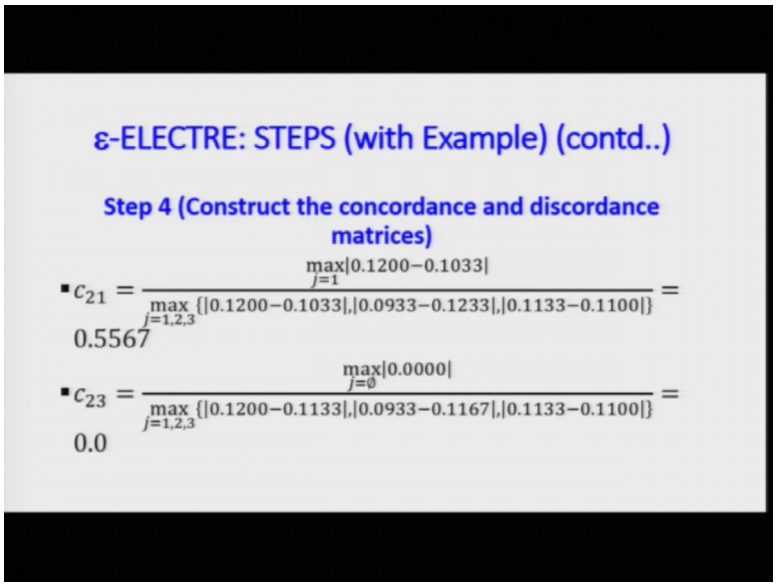


So, once we have that I will just proceed accordingly. So, then once c 2 1 will be 0.5567, c 2 3 would be 0.

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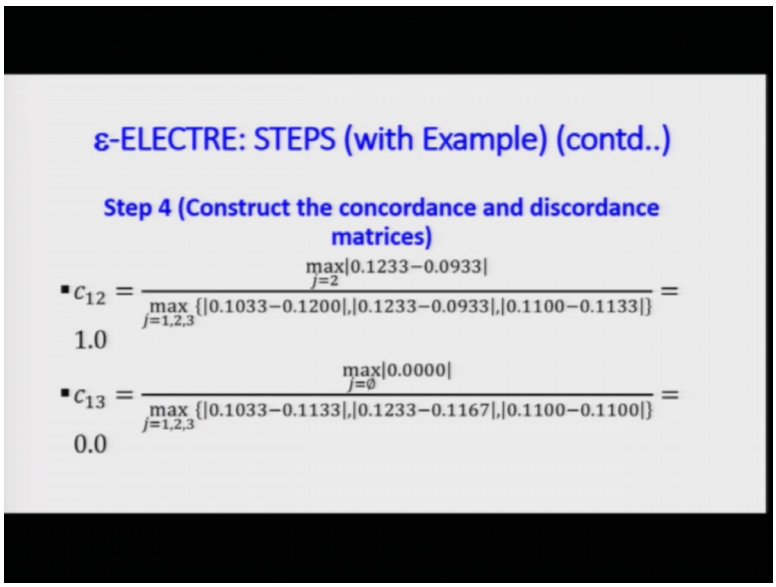


So, if you check here 1 2 and 1 3 are the placements of the cells which you are talking about 1 comma 2; to 1 comma 3; similarly when you come to 2 comma 1, 2 comma 2 were; obviously, will be dash 1 2 comma 3. So, corresponding values of c 2 comma 1 would be 0.5567 and use the same formula which we have just discussed that for the c

value, i value and the d value. The small c, i, d which I have just mentioned and utilizing the Y matrix normalized values.

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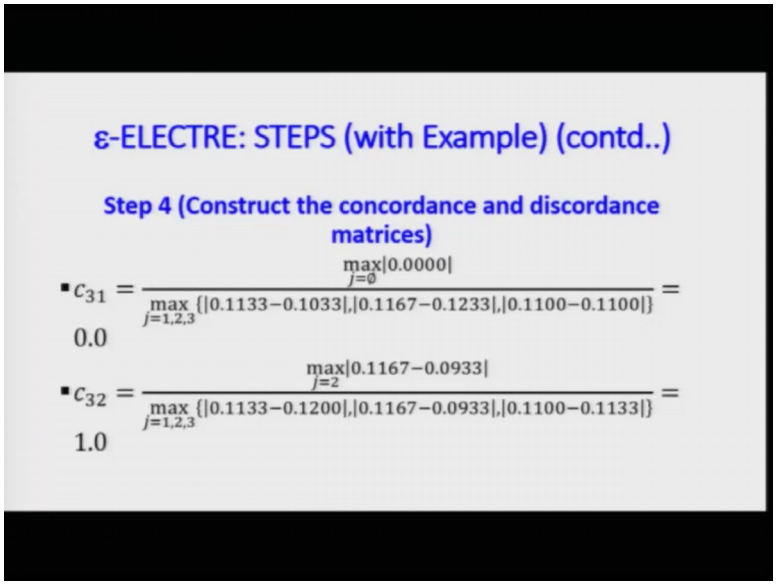


So, c 2 3 comes out to be 0, similarly 3 comma 1 which is c 3 1 would be 0, c 3 2 would be 1. Now you have found out the c value, similarly you have to find out the small i value, similarly to find out that d; small d values and the calculations would be given here I will just go through the final results.

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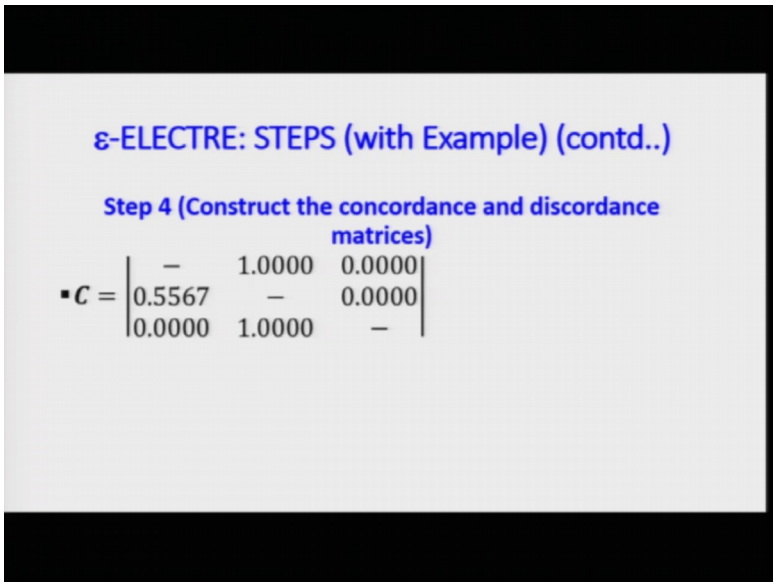

So, once you have that the c matrix comes out to be here this is basically the concordance so called matrix which you have the principal diagonal which I mentioned is dash and the off the diagonal elements have been given.

So, only part is to require is if you compare 0.5567 with 1; it means that if I take 1 with respect to 2, I get much more benefit than when I take 2 with respect to 1. That means, for the conglomeration of the or the collective positive points which I get for choosing 1 with respect to 2 for all the criterias taken together I get a much higher benefit.

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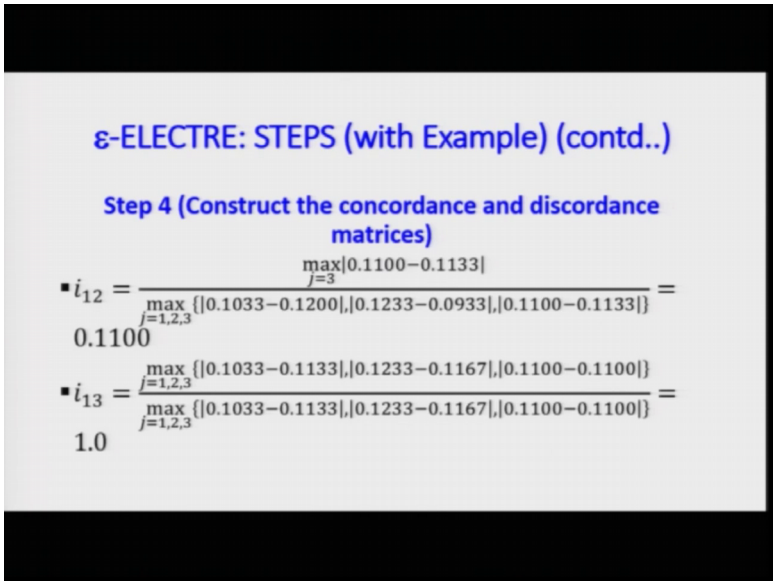


Similarly, when I come to i 1 1, i 2 2 and i 3 3 which is the principal diagonal in the capital I matrix; it this is the indifference matrix it will be dash and the off the diagonal elements are calculated here. So, i 1 2 is 0.11, i 1 3 is 1.

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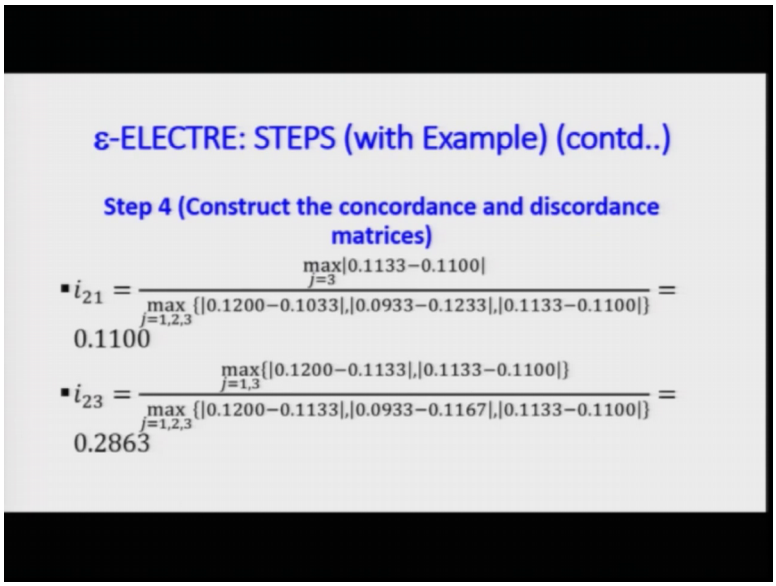


Similarly when I go to i 2 1 and i 2 3; the corresponding values are 0.11 and 0.2836. I am going a little bit fast we check the formula they just a deputation just put them and solve it accordingly. Similarly, i 3 1 and i 3 2 values are 1 and 0.2863; so based on this when I find out the indifferent matrix, the value comes out to be like this.

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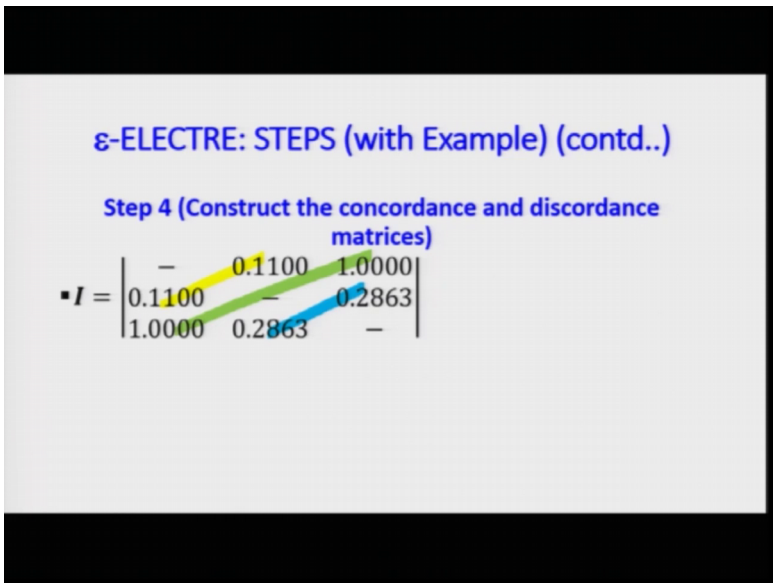


So, this would be a symmetric matrix because in the indifferent matrix here the concept of symmetry would hold true; not for concordance and discordance. Concordance is liking and disliking concepts are little bit different. So, this is and use the highlighter this

is symmetric; this is symmetric and this is symmetric. So, that was not the case when we consider the C matrix and we will also see may not be may not hold true for the D matrix.

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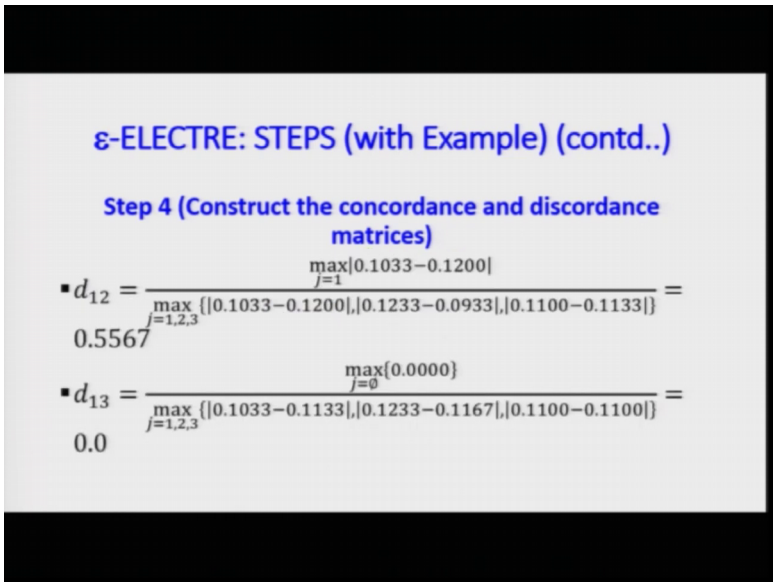


Similarly, when I come to the D matrix which is the discordance actual matrix, which you have. So, similarly d 1 1, d 2 2 and d 3 3 the principal diagonal will be dash values because you are not comparing them and the off the diagonal values are given as d 1 2 is 0.5567, D 1 3 is 0.

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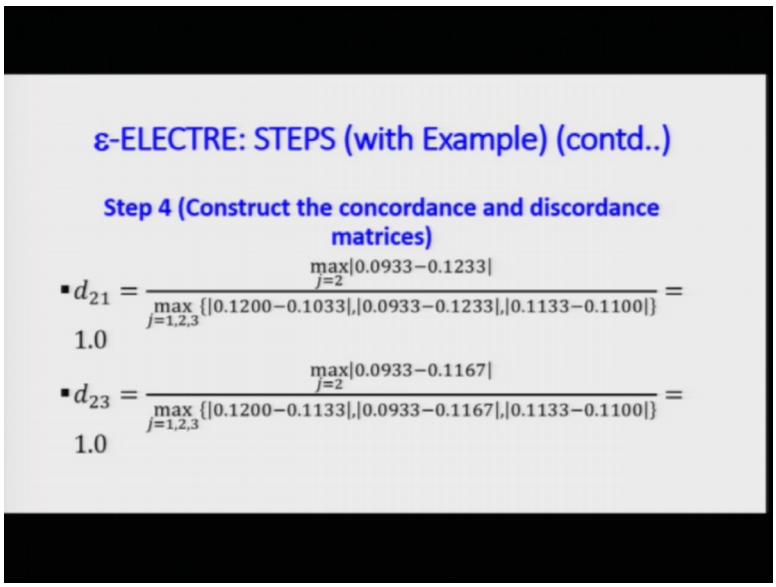

When I go into d 2 1 and d 2 3; the values are one respectively in both the cases.

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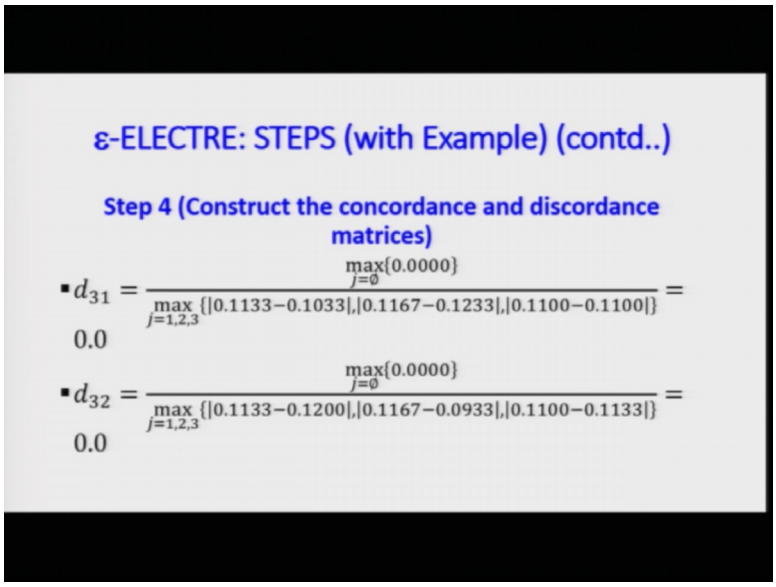


And when I go to d 3 1 and d 3 2 both the values are 0.

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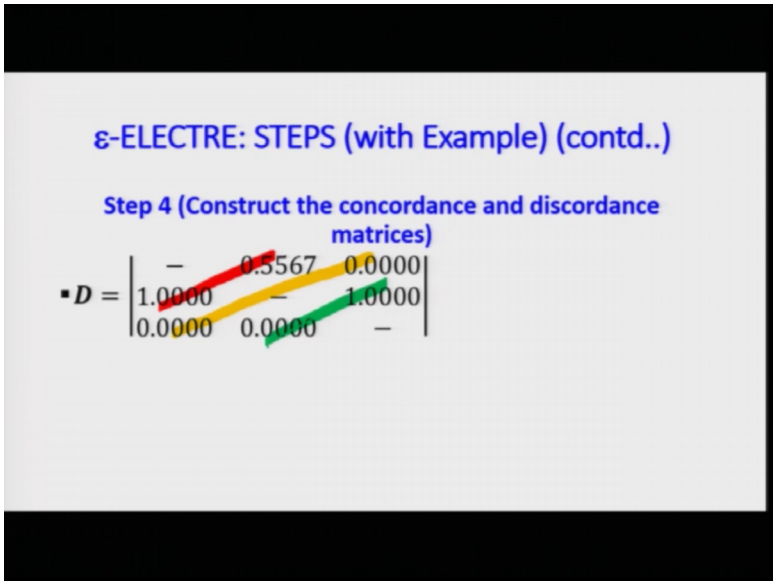


So, utilizing this when I have the mid D matrix; so you check, so the principal diagonal values are not symmetric and this is very important. This is symmetric that is coincident, but this is also not symmetric oh I should use a different color. So, this is also not symmetric and this try to I am using a different color in order to note down. So, this is also this is symmetric, but the other elements are not symmetric.

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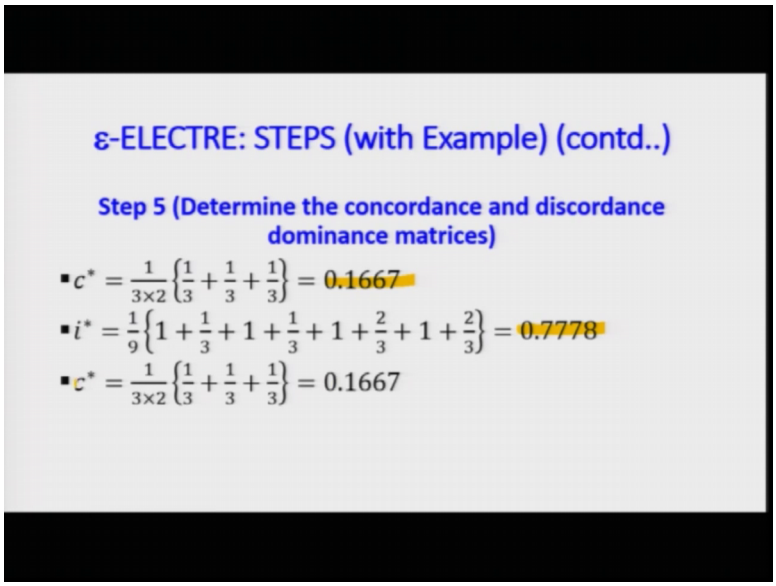


Now, I need to find out the c c star i star and d star values. So, these are the average values which I find out; the formulas which I use depending on 1 by m into m minus 1, the values which you have; the values which you calculate for the concordance discordance, dominance matrix and the indifference matrix which you want to find out; c star comes out to be 0.1667, i star 0.7778 and again this should be d star. So, this value comes out the d star also comes out to be 0.1667.

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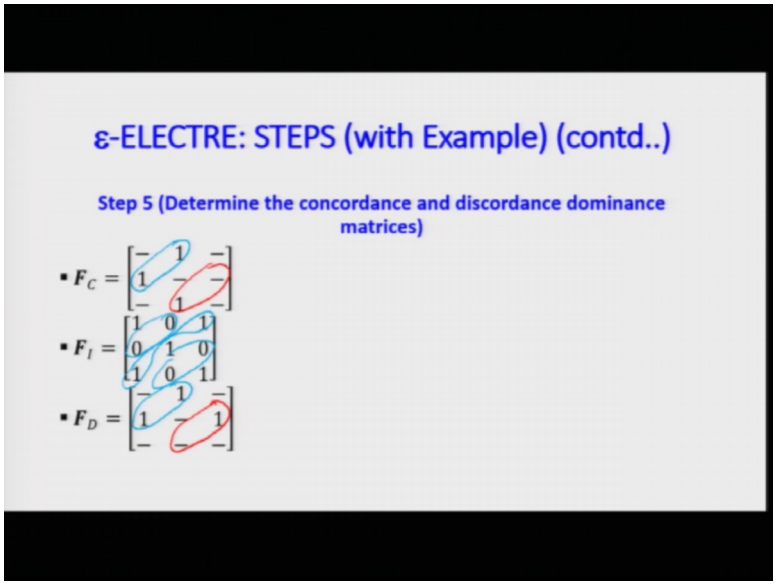

Similarly, on based on this I need to find out the capital F matrix, capital G matrix and then multiply the corresponding elements for F and G, then find it out.

So, the concordance concept, discordance concept when the indifference concept for the liking in disliking are if you remember in the general ELECTRE process what basically subsumed under the matrix F which is a set when I take the positive decision. G was the set when I take the negative decision and then I try to compare them and find out that if I multiply those values whether they are positive or 0.

So, once I have the F set for C I and D; so the values are coming out to be these remember for the principal diagonal they may be definitely symmetric for the I indifference concept, but not for the D and C. So, if you check; so I will use the red this color scheme has nothing to do. So, this is symmetric and here in D this is symmetric.

If I consider this one is asymmetric in the D set or this D matrix this is asymmetric, but if I consider the values for I symmetric, symmetric, symmetric. So, I would continue remaining the symmetric one and not for the D or C; it may be a just a coincidence if C and D are asymmetric, but it would not be.

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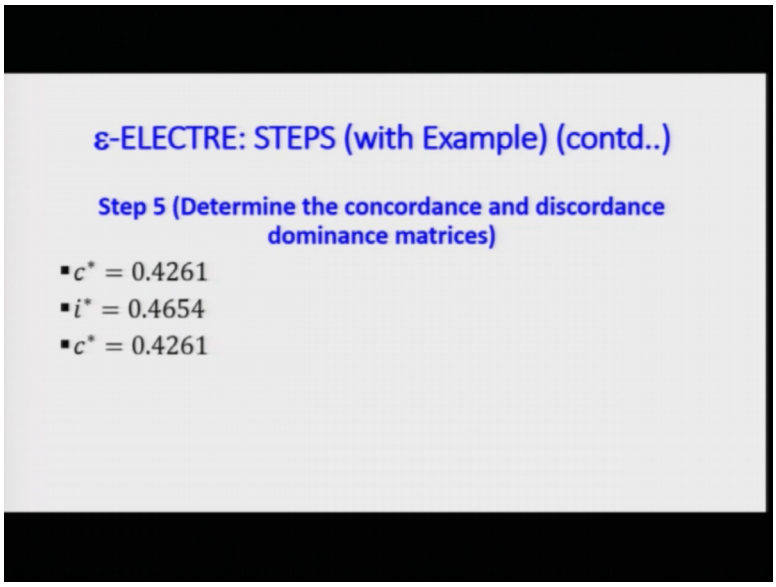


So, when I go and try to find out the g capital matrix when I am taking the other decision; so what is the dis benefit?

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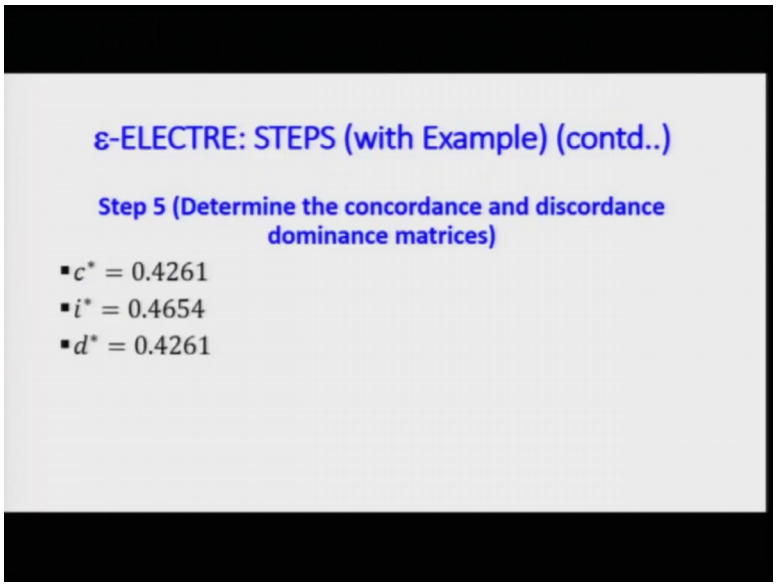


So, correspondingly again the values of c, i and d; concordance average values, discordance average values, indifference average values they come out to be 0.4261, 0.4654 and 0.4261 again.

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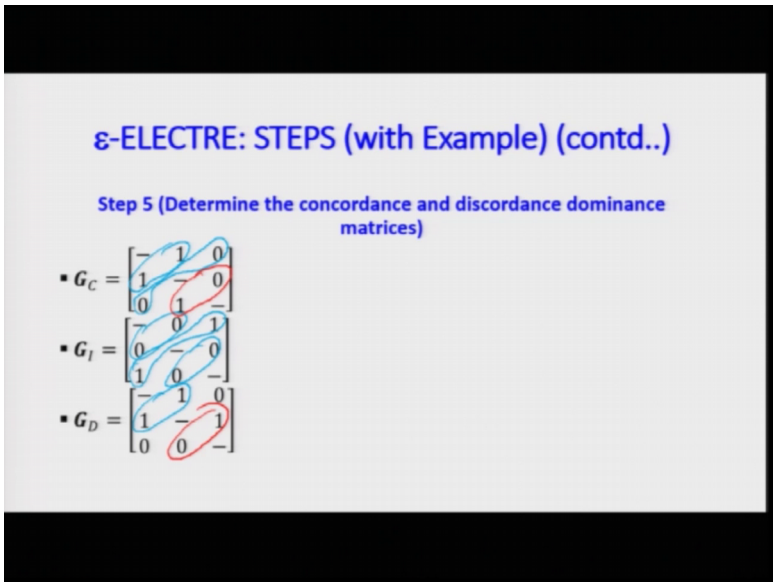


So, based on that once I formulate; the G matrix for C I and D again you will see, this values which I have; I will use the color for blue. So, this is symmetric which is a; which is a fine this is symmetric; for I matrix everything is symmetric remember. And this is

also symmetric here, but for C and D as there is symmetricity and asymmetricity; these are the case.

And once you have them, you will basically want to find out the combined effect. Like if I have taken k, when I try to find out the benefit, I find I would break down the benefit to C set, I set, D set and find out the total benefit and. Then try to find out the so called dis benefit, again I am basically divided into C, D and I combine them and basically at the end of the day I have an F matrix for C, D, I, a G matrix for cdi and it can multiply the corresponding cells of C with C, C with D, C with I.

Then next would be in order to find out the ranking, then I compare I with C, I with I and I with D and finally, I will compare the concept of D; D with C, D with I and D with D.

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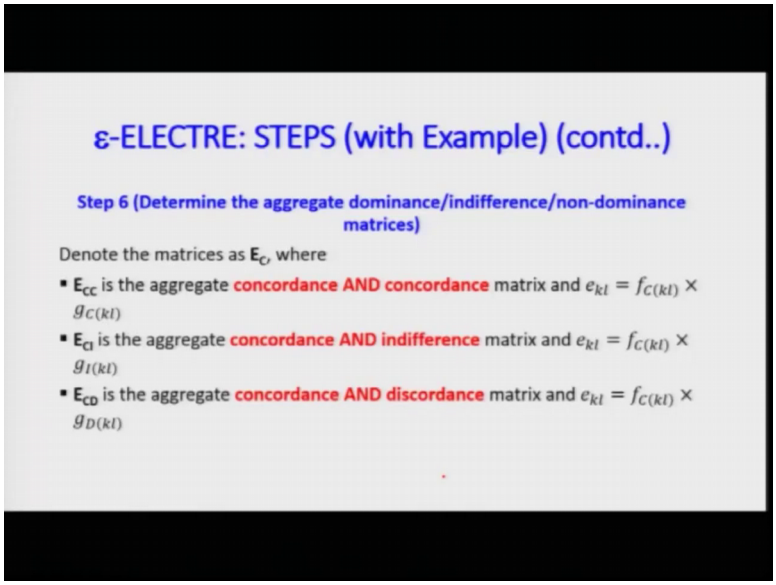


So, once I have this. So, I will try to find need to find out the E matrix only remember the subscripts. I have not drawn gone with the detailed calculation; we need to find out E CC; that means, concordance.

E CI concordance in indifferent ECD concordance and this concept of discordance; so I will multiply the corresponding values for the C set with the C set transferring to f and g, then C set with I set for f and g and C set for D set with f and g; f and g will remain the same only the subscripts are changing.

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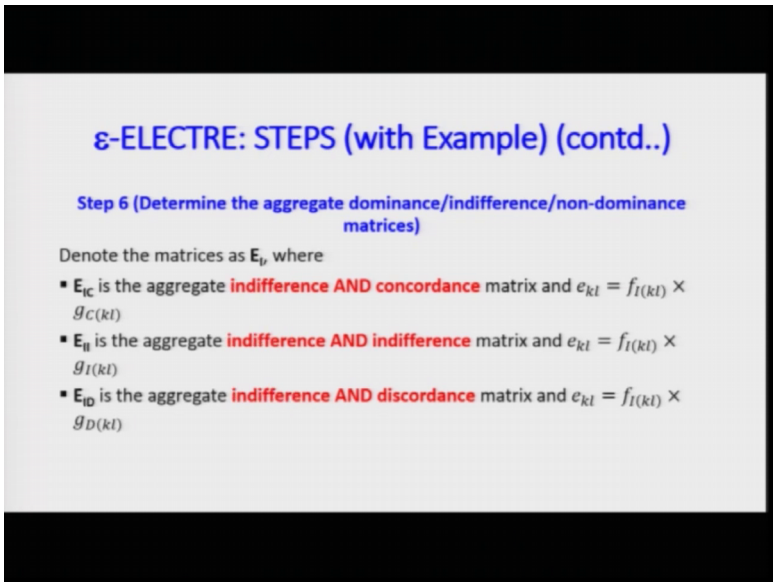


When I go to the indifference set again I will be have basically I C, I I and I D; that is indifferent concordance, indifferent different, indifferent discordance the value is multiplied f and g, but the subscripts are I C, I I and I D.

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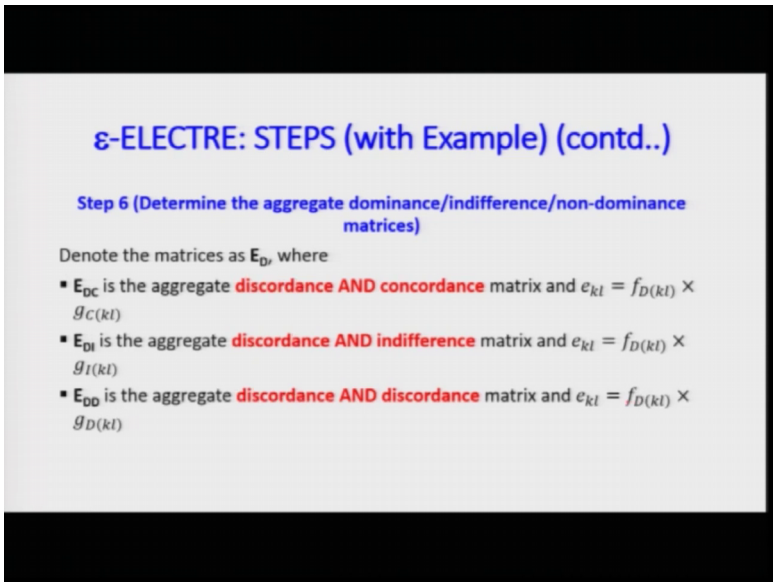


And finally, when I would find out the discordance so called overall matrix, it will be E DC, DI, DD which discordance concordance, discordance indifferent, discordance discordance.

The values multiplied would be from the f and g the matrix, the corresponding subscripts would be DC, DI and DD. So, with this I will end the 30th lecture which has to do with ELECTRE process both the ELECTRE and normal process then and the epsilon ELECTRE. And we will consider the other topics of topsis and other matters in the subsequent class starting the next week have a nice day and.

Thank you very much for your attention.