Data Analysis and Decision Making - II
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Lecture - 27
ELECTRE

Welcome back my dear friends and dear students, a very good morning, good afternoon, good evening to all of you wherever you are. And as you know this is the DADM – II, which is Data Analysis and Decision Making – II course under the NPTEL MOOC series. And this total course is for 12 weeks, which is 30 hours being converted into 60 lectures. So, each lecture is for half an hour, and each week we have 5 lectures.

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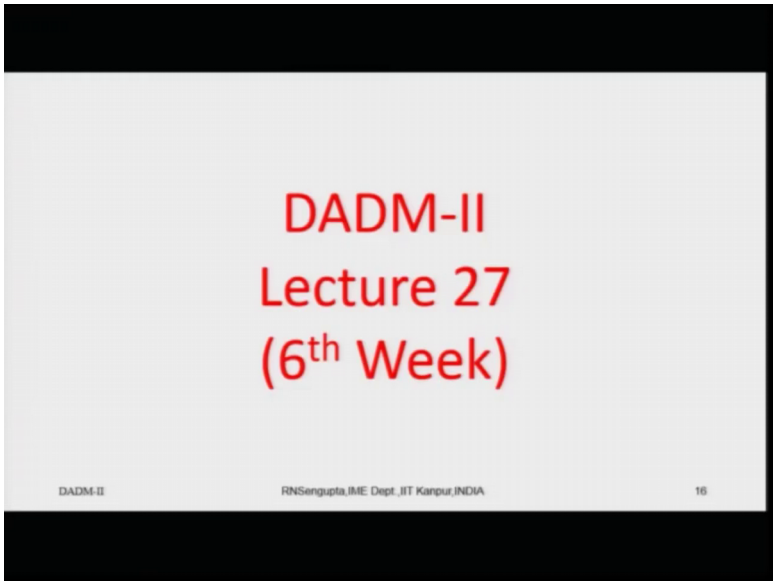

So, if you can see as you see in the slide, this is the 6th week, we are in the 27th lecture, which is the second class in the 6 week; and we have already completed 5 weeks.

Now, in the last fact 2 or 3 slides in on the 25th and then the whole of 26th lecture, we discussed about AHP. And we are going to sorry, we discuss our ELECTRE process initially it was AHP, and then in continue discussing the ELECTRE process. So, the main crab (Refer Time: 01:21) point which I was mentioning and that in the ELECTRE process. You are liking and disliking are basically converted into a outranking set, outranking can be both positive and negative in the sense, for liking you will basically have a concordance set, a positive set.

Positive in the sense not with positive points, but positive sets in the sense that if as you are taking the decision, you are accruing some positive benefits. And for the discordant set you are getting some negative points, not negative in the value sense, but negatives in the sense, that you would accuse some negative connotation for taking that decision.

And you will basically do a comparison for each and every criteria based on the; alternatives will be considered based on the criteria, and it can be done in the other way around also. Remember one thing in the AHP, we compared the criteria well we compared the alternatives, here may main focus would be to compare the alternatives based on the criteria and then you basically do the concordance and the discordance sets. So, we are considering the ELECTRE simple process and as I mentioned they were six methods, but we will only consider the first one.

So, in the step 2 you have the weights and if you remember the weights, which I told was basically some should be one and the each of these rows are only one element that means, the principle diagonal values are there, source it will be W 1 in the 1 comma 1 cell, W 2 in the 2 comma 2 cell, W 3 in the 3 comma 3 cell, so on and so forth. So, the principal diagonal values are W 1 to W n, when the sum of the which is 1 and of the diagonal elements are 0 as you can see from this weight matrix, which I am just pointing out in this slide.

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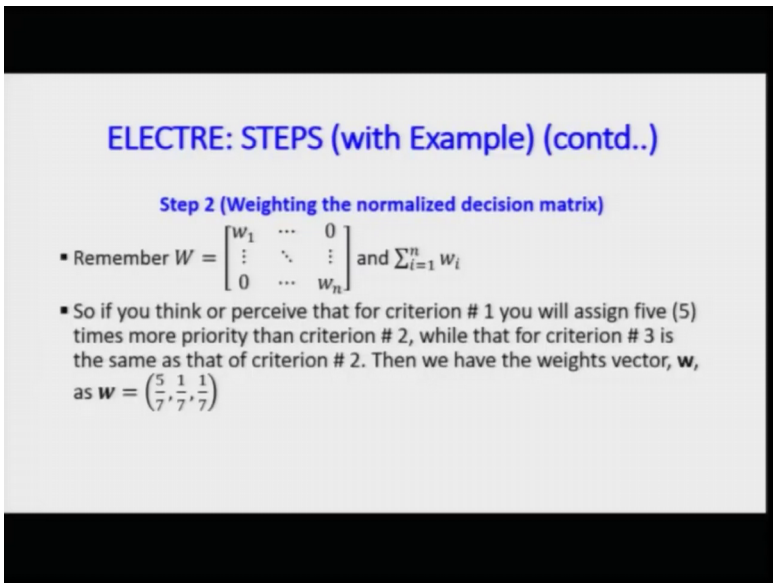

So, remember as it said that the weights W 1 to W n is 1. So, if you think or perceive that for the criteria 1, you will assign five times more priority than criteria 2, while that for criteria 3 is of the same value as criteria 2. Then the weights you are giving for 1 would be 5 times more than 2, and weight for 3 would be 1 is to 1 with respect to 2. So, the total weights is 7, you divided into the proportions of 5 by 7 is to 1 by 7 is to 1 by 7.

So, as you can see, the weight for the first one is 5 7, second one is 1 7 and third one is 1 7. So, obviously these are the principal diagonal values which you have, so in this case you have a 3 by 3 matrix.

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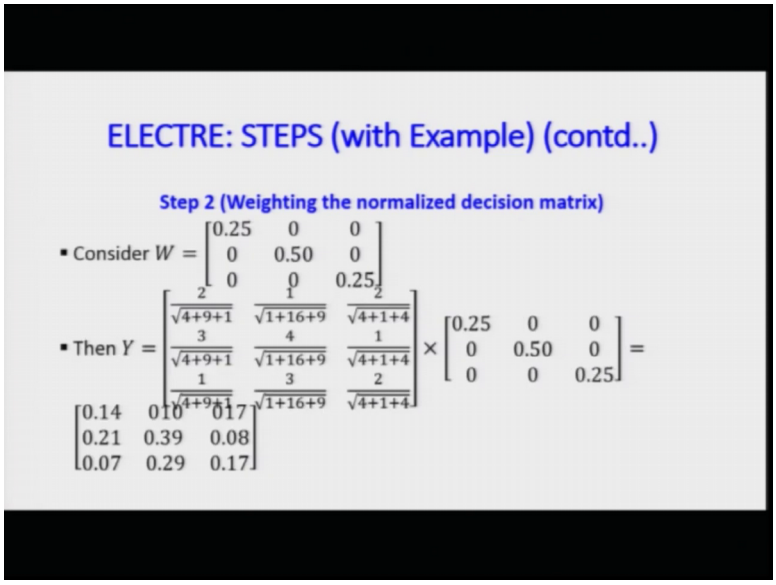

So, in this case we are just taking this example, we that that value which I told about 5 7, 1 7, 1 7 was an example that in words if it is mentioned, that you are giving 5 times the weight for 1 with respect to 2; and same weight you are giving for 3 with respect to 2 then the scores what I just mentioned.

But for this example, we are going to take a very simple case, where the weights would be divided in such a way. For criteria for the first one, it will be 25 percent that is 0.25 out of 1. For the second one is 50 percent, which is 0.5 out of 1 and for the third one is again 25 percent, which is 0.25 out of one. So, the weights if you consider for the second one is twice with respect to first and with respect to the third, and the weight for the first and the third are of the same consequence 1 is to 1.

So, if you consider the weights it is given as; W 1 is 0.25, W 2 is 0.5, W 3 is 0.5, sum is 1. Now, you are basically you have the matrix where the points were given. The points were if you remember, I am only reading the first row it is 2 1 1, second row was 3 4 1, third row was 1 3 2. And you are trying to basically normalize them by considering that those particular numbers or the values would be divided by the square root of the sum of the squares; that means, you are trying to utilize some concept of utility function. And utility function is quadratic in nature, so if you have that.

So, the values would be now another thing, as you had done in AHP, you remember that you can use this the sum as 1 on the normalization being done, along the rows or along the column. So, whatever process you follow, stick to that throughout the whole decision. So, when I basically do the normalization along the columns. So, the first one would be divided by the square root of 2 square it is 4, 3 square which is 9 and 1 square which is 1. So, you divide the first element Y, square root of 4 plus 9 plus 1, second element I am talking about the column, second element it would be divided again by square root of 4 plus 9 plus 1, third element which was one initially, it would be again divided by square root of 4 plus 9 plus 1.

Similarly, when I go to the second column and the third column the respective values are square root of 1 square plus 4 square plus 3 squares, which is 1 plus 16 plus 9 that is for the second column. And for the third column the respective values which will divide each and every element in the third column would be 2 square plus 1 square plus 2 square, which is 4 plus 1 plus 4.

So, once you have this, so this will give you the weighted normalized matrix multiplied by the weight which was capital W, once you multiply them. Remember the one important thing, which I did mention once or twice, I will again mentioned that, row versus column number should be such that matrix multiplication would be allowed.

So, in this case is 3 cross 3, in this case also 3 cross 3, so the end result will be 3 cross 3 matrix. And the values which you have would be along the rows, I am only reading along the rows is 0.14 0.1 0.17 which is the Y matrix, Y was basically a into W. The second row would be 0.21 0.39 0.08, and the third value would be 0.07 0.29 0.17, what are these values I am going to come to that later very soon. Just remember these values are given

and they give us some sort of preference values with respect to each of the on the alternative based on the criteria's.

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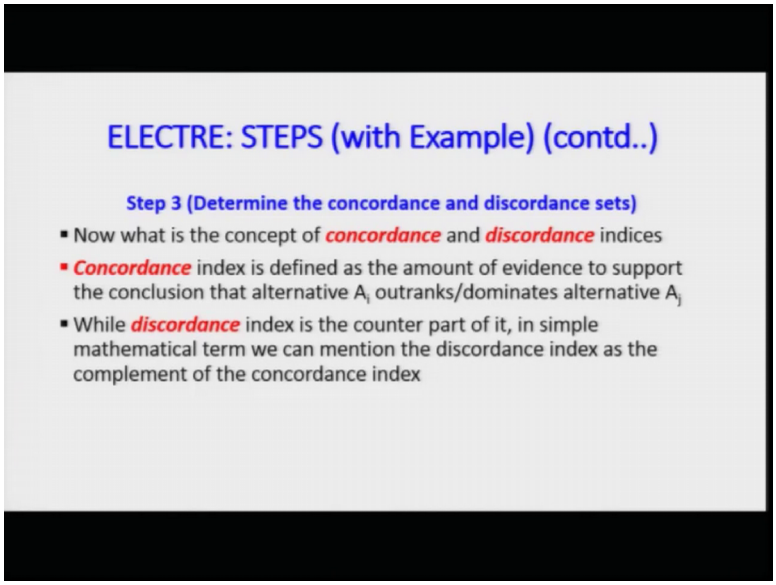

And we are going to further proceed with that. So, in step 3; so you have completed step 2 and step 2 you have initially normalized vector x multiplied by W, you get the matrix. Matrix x was normalized multiplied by matrix W of the weights you get the Y matrix. In the third step, you determine the concordance and discordance sets. Now, you are going to divide the liking and the disliking set in such a way that combining them you can get a overall the relative ranking.

Now, what is the concept of concordance and discordance sets? Concordance index or sets are basic based on which you will have the sets indices will give you, the way how you take the values and put them either in the concordance set or in the discordance set. So, concordance index a value is defined as the amount of evidence to staff support the conclusion, listen to this carefully is amount of evidence to support the conclusion that alternative A i would outrank or dominate that the alternative A j. So, to what degree it will dominate will be given by the concordance set or concordance values, once you have those values indices you go to for the concordance set.

While on the other hand discordance index is the counterpart of the concordance set. In very simple terms, it mentions that discordance index as the complement of the cordon set, which will basically support the conclusion that the alternative A i does not outrank,

does not dominate the alternative A j. So, to what degree liking and disliking is there, such that you have been able to take A i. So, A i if you like you put it in the concordance set, depend on the (Refer Time: 10:01) base index. If A i you are going to take it, whatever the reason is, but you do not like that based on the discordance index, you put it in the discordance set.

Then again when you come to A j with respect to A i, you will also have the concordance index for comparing of A j with respect to A i; again you have the discordance index for comparing A j with respect to A i. Now, one thing should be mentioned, that if say for example, the concordance point of taking or point means the values, concordance values, which you have for taking A i with respect to A j; it does not mean the level of discordance index would be of the same value.

When you are told to take A j with respect to A i; that means, they are not symmetric in the sense, liking of A i with respect to A j does not give me the same level of dislike, which I have for A j with respect to A i. But even if that is the case we will consider the equidistance concept, which I will come to that within few minutes.

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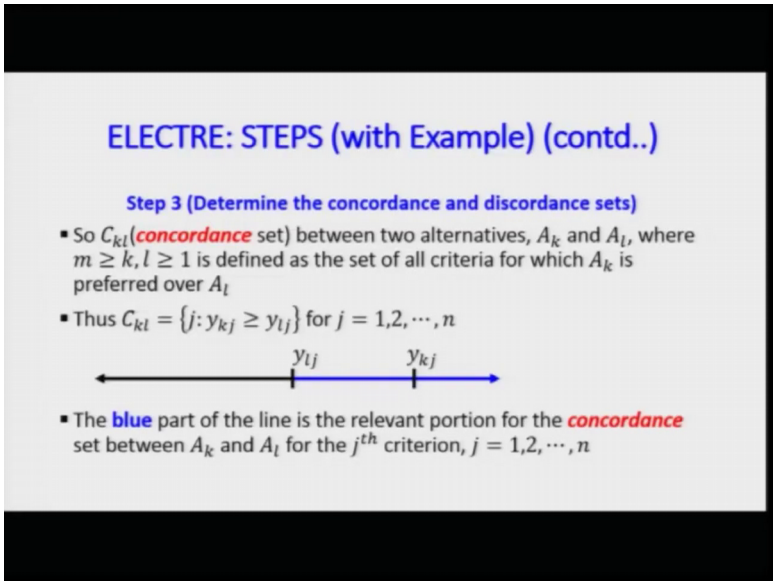

So, continuing the third step which is determine the concordance and discordance set. So, you will basically have the concordance set between two alternatives A k and A l. So, this formula looks a little bit tricky it is not, once I basically have the values for the decision the problem which you are solving, it will become clear. So, for two alternatives

A k and A l, where you want to basically have k and l any number between 1 to m is defined as the set of criteria's for which A k is preferred to A l. And what I will have? I will have those values of j.

So, I am remember m was the index and n was the index for two different things. One was for the case of the number of alternatives and another was basically the case for the number of criteria. So, C kl would be given a value. So, if it is concordance that j th value would come into this concordance set; C is now the concordance set values which I have where the value of y kj depending on j is equal to 1, 2, 3, 4 till n, where it will dominate the value of y lj that means, each row or each column you take, you compare that values with respect to that fixed value which you are going to compare.

And then basically place that decision either in the concordance set on the discordance set. So, we are only going to consider the concordance set; that means, again I am repeating, you will take ones value in the cell, which is basically the overall score you are trying to assign to any one decision based on the alternatives on the criteria. Now, in the second step, what you will do is the; second step; mean s, in step 3 combine one what you are considering? You will compare each and every cell value; either along the column or along the row with respect to that fixed value.

And if it is a concordance value, you will put those based on the concordance index, you will place those j's into the concordance set and if it is a discordance index is predominant, then you will put that value of j into the discordance set. So, and you will compare in this way and I have basically have the concordance overall set, matrix and the discordance overall set or the matrix.

The blue line, which is showing here is part of the line is relevant portion of the concordance set. So, you will basically; it can go on more on to the right, there is no issues. What you want to find out is the moment, it is on to the right. Let me use the blue color one, so the moment it moves some delta value to the right, it FALSE under the concordance index and the concordance set.

Higher the value is higher the liking is. And in the similar way if I go onto the left, so obviously the I will mark with the red color, as I had done for the decision trees if you remember. Positive input would be marked as blue color, negative input will be marked as red color that means, cash flow going out of your pocket. So, moment is on the left it

is red, further you go on to the left it will basically be a high value in the discordance set which you have.

So let me read it, the blue part of the line is the relevant portion for the concordance set between A k and A l the alternatives for any of the criteria j; that means, you are going to take two alternatives, take each one of this criteria and compare where the criteria stands with respect to A k and A j or A1 and A 2 and then, basically put it in the C set or in the D set. D set I am going to come to that within few minutes.

Now, come to yes, this is what I said. Now, what is the discordance set and based on that the discordance index we have the discordance set? The discordance set between two alternatives, again the same alternatives A k and A l would be marked by those criteria's between all the sets of combinations we are going to take, will be defined at the set of all the criteria's for A k is not preferred with A l. In the first case A k was preferred with respect to A l that is why, we put in C set. And in the case when A k is not preferred with respect to A l we will put them in the D set.

So, as I said the red line which is on through the left, further you go higher the value of index is discordance index it and it will definitely come in the D set. D and C sets which I am mentioning would be converted into a matrix; so obviously, they would be given as bold. Even though it may not be very specific in this slides, but they are basically matrices once you basically have them.

Further, you go on to the left higher the values and they are in the discordance sets. So, it means that if y kl is less than y lj as j changes from 1, 2, 3, 4 till n, you put those values of j in the discordance set based on the discordance index. The red part of the line is the relevant portion of the discordance set between A k and A l, which are the alternatives for each and every criteria j from 1 to n.

Again what I will do? I will compare two different alternatives take any one of this arbitrary criteria and compare that criteria with respect to these alternatives and then basically put those alternatives in this discordance set; that means, I am comparing the two alternatives based on the criteria each and every time. So, if in criteria 1, the alternative first one is better, it will go into the concordance set; if it is not better depending the score, it will go into the discordance set, we will continue in this way.

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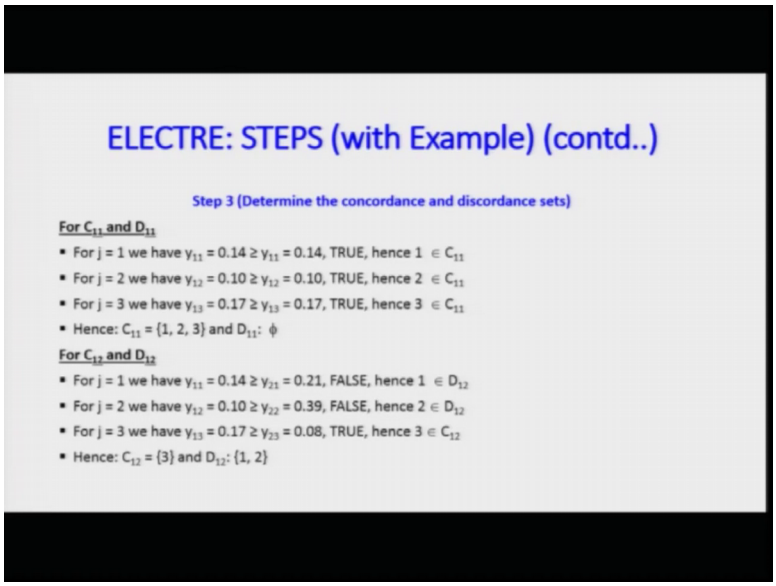

Now, I would beg to be excused let me go to the values. So, make a note of these values.

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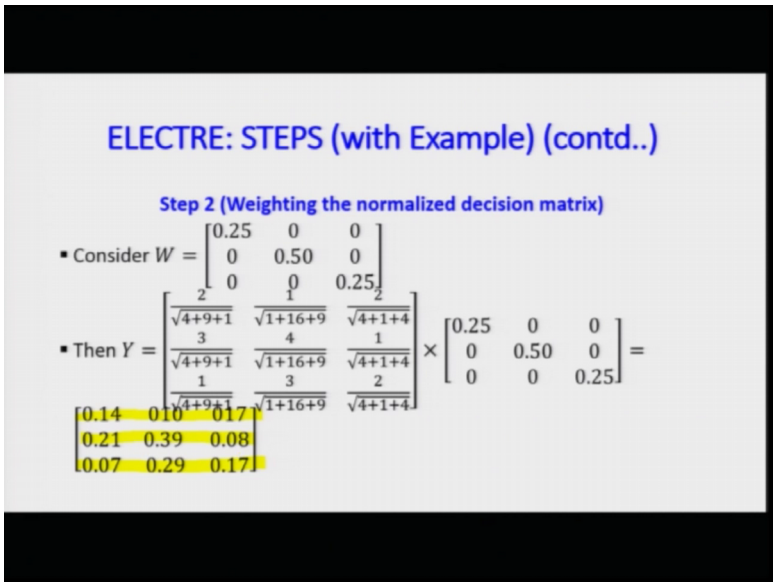

So, the values are this, please make a note, again I mentioning 0.14, I am calling along the first row 0.14 0.10 0.17, second row is 0.21 0.39 0.08, third row is 0.07 0.29 0.17. So, these values I will basically consider to formulate the concordance and the discordance set based on the indices of C and D.

So now I will basically have the values. Say for example, when I am basically comparing for j is equal to 1, so I will go if j is 1, 2, 3; so in that case Y 1 1 was 0.14. So, Y 1 1 is

greater than I am going to compare itself and then basically change the values. So, Y 1 1 which is 0.14 is greater than an equal to also 0.14, which is TRUE, hence j 1 FALSE in C 1 1 value in the element concordance.

Now consider 2, j is equal to 2. Y 1 2 which is 0.11. So, I am only considering C 1 1, which is comparison of itself. 0.11 is obviously greater than equal to 0.10 which is TRUE. So, second, when I am comparing the second to second FALSE in C 1 1, concordance. Similar for j 3, 0.17 is greater than equal to 0.17 which is TRUE, because I am only considering Y 1, Y 2, Y 3 along the first column, which is TRUE. Hence, j 3 those are the criteria's FALSE in to C 1 1 concordance set.

So, hence now I have the concordance set would have 1, 2, 3 and the discordance set would be a null set. So, when you are comparing itself, I will have all the concordance set. Which is in the way, if you remember the principle diagonal of the AHP all over one; that means, I am comparing first to first, second to second, third to third so, obviously this score would be 1, that is why in the concordance set. This is just a simile between AHP, I am trying to draw something with AHP an ELECTRE.

Now, come to C 1 2 and D 1 2. Now, the comparison would now would basically be placed again for j 1, 2, 3, but I will compare Y 1 1, Y 1 2 and Y 1 3 and the comparisons would be done with Y 2 1, Y 2 2 and Y 2 3. So, let us go through it, Y 1 1 is 0.14 is it greater than point 0.21 answer is FALSE, hence the first j is equal to 1 FALSE in D 1 2, which is the discordance value. j 2 y 1 2 is 0.10, Y 2 it is 0.39, y 3 9 is in the 2 comma 2 cell, which is FALSE, hence 2 j is equal to 2 FALSE in the discordance set.

For j three, Y 1 3 which is 0.17 is greater than equal to 0.08 which is TRUE, hence 3 FALSE in C 1 2. So now, when I complete the second stage, concordance set has that values 3 and discordance set has this value 1 and 2.

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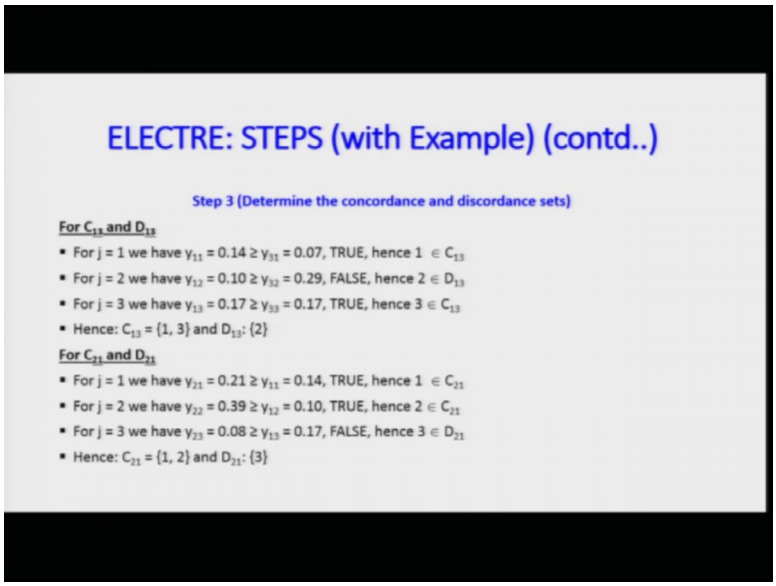

So, let me go to C 1 3 and D 1 3 comparison. So, for j is equal to 1, I would not read it in details for I will just mention the important points. For j is equal to 1, j 2, j 3, the values are TRUE, FALSE, TRUE. So, please I will request as I showed that write the values which were there in the Y matrix and then compare.

So, once you have this 1 will go into concordance set, 2 will go into the the discordance set sorry and 3 would go into that the concordance set. So, the concordance sets are 1 and 3, discordance sets is 2. When I compare C 2 1 and D 2 1 for j is equal to 1, 2 and 3; the values are TRUE, TRUE, FALSE and hence 1 and 2 FALSE in the discordance set and 3 FALSE in that on 1 and 2 FALSE in the concordance set sorry, and 3 FALSE in the discordance set. So, the C values are 1 and 2, D values are 3 only, because there are 3 elements.

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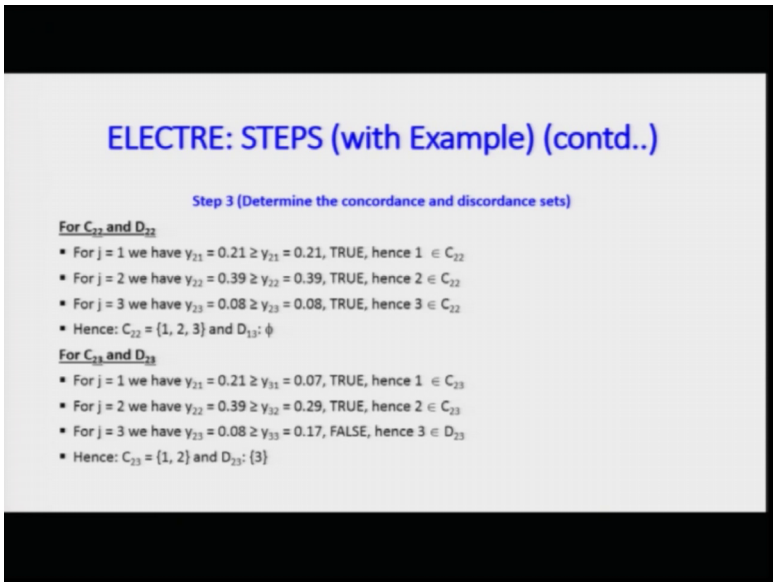

Then I go to C 2 2 and D 2 2, again I am comparing j is equal to 1, 2, 3, the values are TRUE, TRUE, TRUE; because that would be obviously for C 2 2, D 2 2 or C 1 1, D1 1 and C 3 3, D 3 3, everything will fall in the concordance set. Hence, as usual 1, 2, 3 goes into the concordance set, and D which is the discordance set is a null set. When I come to C 2 3 and D 2 3 for values of j is equal to 1, 2, 3, we have TRUE, TRUE, FALSE; hence they fall in two concordance, concordance and discordance set. So, the values of the concordance set is 1 and 2, discordance set is 3. So, we will go one step at a time like this.

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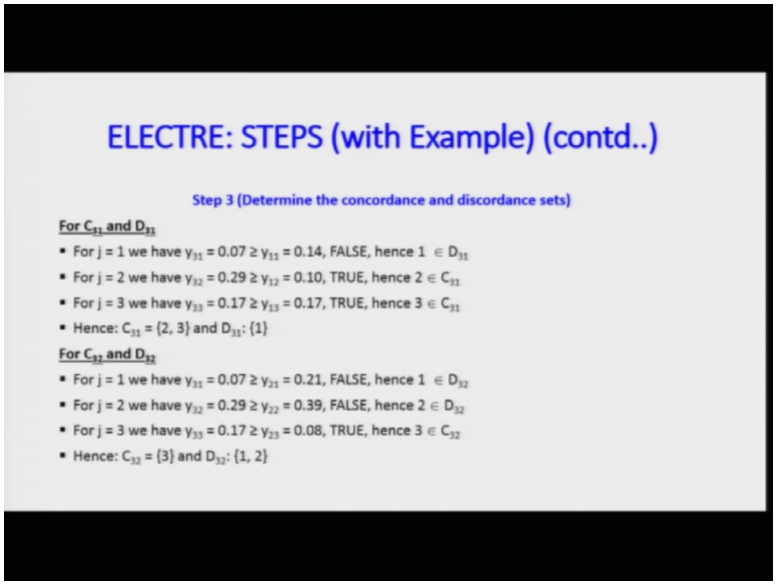

Now, the last leg of its calculations would be C 3 1, D 3 1, C 3 2, D 3 2 and then finally, it will be C 3 3 and D 3 3, which will come in the last slide, which will just be after this. So, again I am going to take each and every criteria j is equal to 1, 2, 3, the values when I compared are FALSE, TRUE, TRUE. So, the j is equal to 1 goes into the D set and j is equal to 2 and 3 goes into C set. Similarly when I go to C 3 2 and D 3 2 for values of 1 2 3, I am just repeating the important points.

This main calculations which you do, greater than equal to and all these things, please, please check it with the main matrix of Y which you have. For j is equal to 1, 2, 3, the values are the truth values are FALSE FALSE TRUE, hence they would be in the D set, D set and C set. So, C set would have 3 only and D set would have 1 and 2. Finally, for C 3 3, D 3 3 it is very logically very simple, all of them would be TRUE. So, j is equal to 1, 2, 3 would go into the C set and D set would be a null set.

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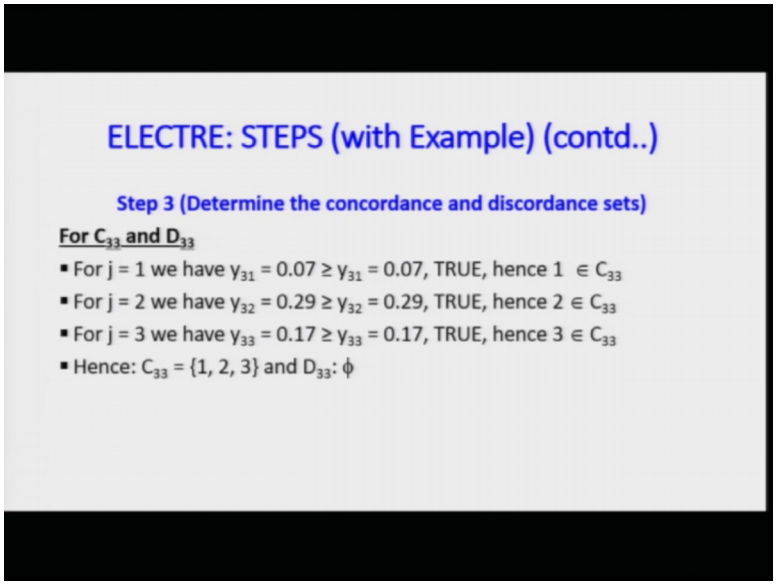

So, once this is formulated, you are sure that C 1 1, D 1 1, C 1 2, D 1 2; C 1 1, D 1 1, then you go to C 1 2, D D 1 2, then C 1 3, D 1 3, then you go to C 2 1, C 2 2, C 2 3; correspondingly you go to D 2 1, D 2 2, D 2 3 and finally going into the values of C 3 1, C 3 2, C 3 3, you go into D 3 1, D 3 2, D 3 3, because there is a 3 by 3 matrix, once you have that, the concordance set and the discordance sets would be formulated.

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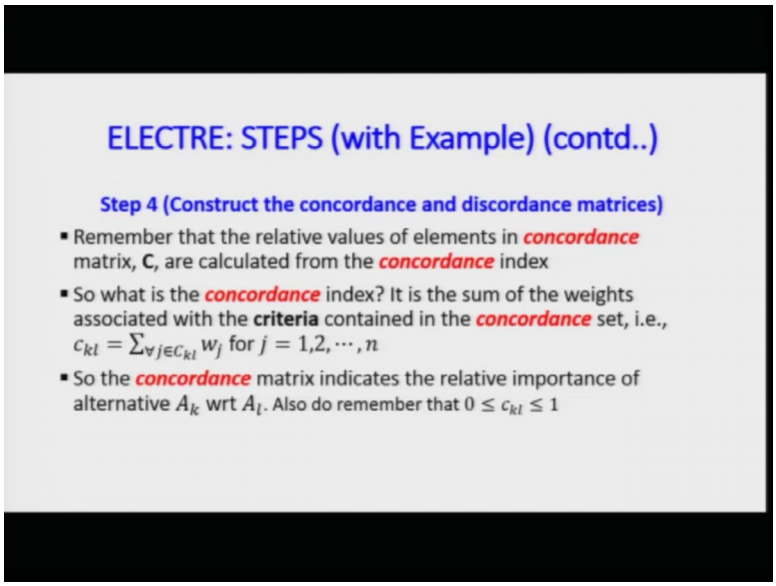

Now, you will basically need to find out the concordance matrix and discordance matrix based on the indices. Now, remember that the relative values of the elements in the concordance set C, C these is matrix is that is why it is a bold, are calculated based on the concordance index. So, what is the concordance index? Is basically some of the weights associated with the criteria contained in the concordance set, which will basically the sum of all the values of C which you are getting for j is equal to 1 till n, j remember is the criteria.

So, the concordance matrix indicates the relative importance which you are going to place to A k with respect to A l based on the values of C; that means, you will have some sets of criteria's which will support A k, some sets of criteria would not support A k. So, hence those support means you are liking; so, for those sets which you support, they fall into the C matrix and those which you do not support should technically go into the D matrix, but we would not go into the D matrix calculation immediately, we are only considering the concordance at based on the concordance index.

So, you are trying to basically divide the overall set of criteria into two sets liking, disliking; but as I have only considered or in the set of calculation is only to compare the concordance values hence we will take the liking one and put them in the C set, for the D will basically do another set of calculations separately.

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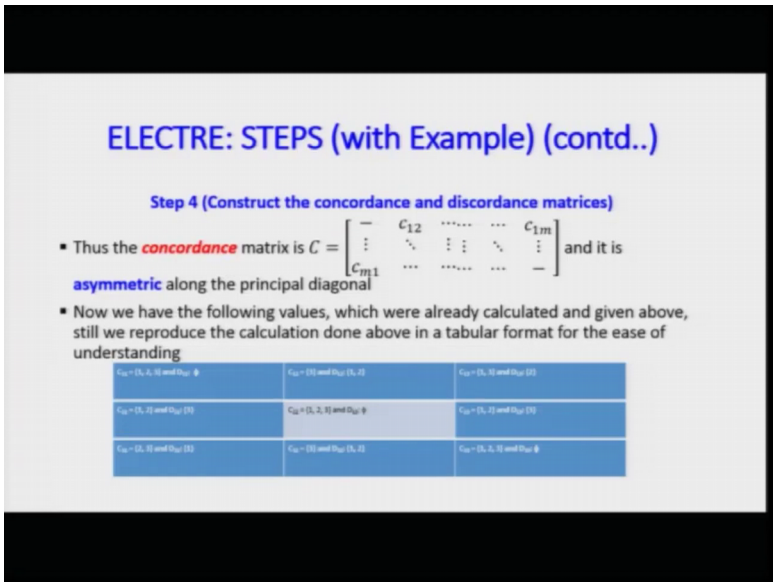

So, the concordance a matrix again will be of size m cross m. Now, remember what is m? m is the number of decisions which you have, alternatives which you have; that means, you have been comparing the alternatives amongst themselves taking two at a time based on each and every criteria, again I am mentioning and dividing them into two sets, we are only going to consider on the C indices.

Now, this would be asymmetric, because the scores which will have when you assign, the important levels between the alternatives will change keep changing depending on the criteria which you have. Now, so they would be symmetric. So, if I read the second bullet point, so now we have the following values which were already calculated and given above, still we produce the this same calculation in this matrix so.

If you remember the first cell value here, would basically which is 1 comma 1 will have all the C values as 1, 2, 3, D is null set. Similarly, the second cell TRUE, all the values are C set and the D set is null. And similarly in the 3 comma 3, all the values are in C set, D is basically null set. And the of the diagonal element, I will just read the numbers which is 3, 1, 2, 3 being C D 1 2 being D. Similarly, the values are 1, 3, 2 where the first two values 1, 3 are C, D is 2. If I go into the second row, it is C is 1 2, D is 3. In the element 3 comma 2, it is 1 2 it is C, D is 3. In 3 comma 1, it is 2 [and] and 3 are C, D is 1 and in the cell 3 comma 2 is 3 is C and 1 2 are D.

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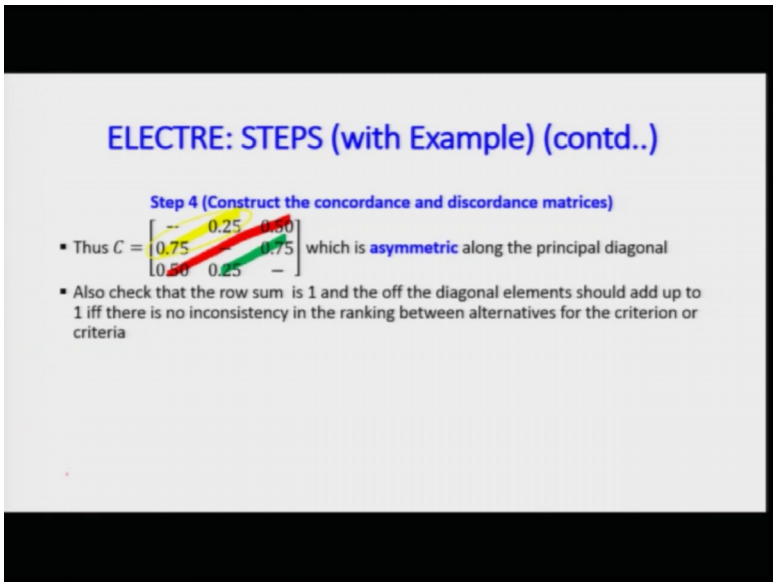

So, once I have this, step 4 is construct the concordance and the discordance matrix. So, I will only concentrate on the concordance matrix. So, this should be a bold one, sorry for that. So, if you remember the values are like this, if you consider the first row, second row and third row the principle diagonal dash, dash, dash; value means you are comparing against themselves, so it does not make sense.

So, it means it is as asymmetric one, because the values which you compare along the principal diagonals are not symmetric. So, it means that when you are trying to compare the first with the second, you get a benefit of 0.25; when you are considering the second with the first you get a benefit of 0.75, what are those values? They are the relative scores. So, this is the asymmetric matrix along the principal diagonal.

One thing is very interesting is that the sum of the values, which you should have of the off the diagonal element should be 1. So, when you are comparing comparison 0.75 0.25 is 1, then when you are comparing 0.5 and 0.5 is 1, then when you are comparing 0.25 and 0.75 is 1 that should be TRUE, because your comparing in such a way, that you are trying to give in the concordance value the same way ok. So, with this I will end this 27th lecture and continue more discussion about the method of ELECTRE process for the discordance set, considering symmetric loss to be there and continue more in the discussion of asymmetric loss later on.

So, have a nice day and thank you very much.