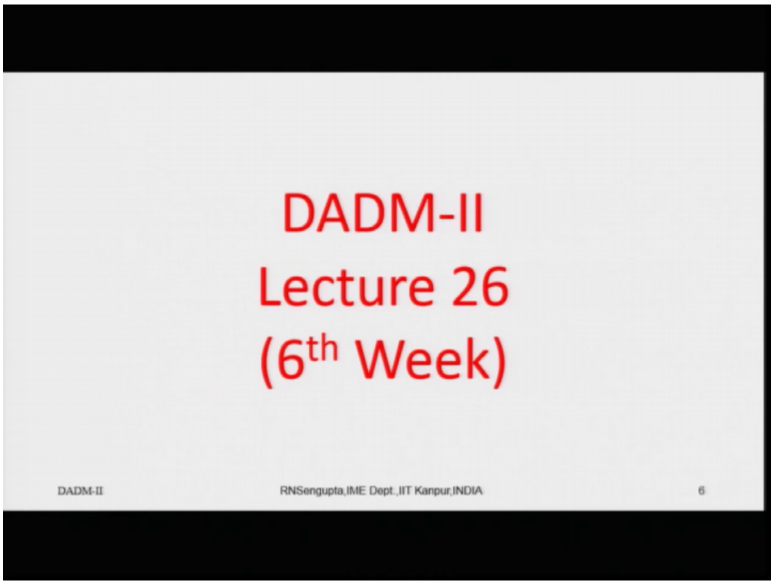
Data Analysis and Decision Making - II
Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture - 26
ELECTRE

Welcome back my dear friends and dear students, a very good morning, good afternoon, good evening, wherever you are. And once again welcome to this course of DADM which is Data Analysis and Decision Making 2 under the NPTEL, MOOC series. And as you know this course is total duration is 12 weeks which is 60 lectures that is 30 hours, each a class being for or lecture being for half an hour, and in a week we have 5 lectures.

(Refer Slide Time: 00:50)



So now as you can see from the slide, we are in the 26th lecture which is the 1st lecture in the 6th week. And my name is Raghu Nandan Sengupta from the IME department, IIT Kanpur in India. So, if you remember, we are considering the concept ofELECTRE process, and I did mention very briefly though. So, there are six different methods for ELECTRE, but we will consider the simplest one. I did also mention that this is multi-criteria decision making, where you make comparisons for decision based on alternatives taking two at a time, and you rank them.

And it is almost similar to AHP method, where you give scores in the AHP method. But, here you will basically find out two different criteria's. One set is that how much you like a decision with respect to the other. If you take the decision which you like, like means consider you have decision i and decision j, I am just naming them as i and j or alternatives which are or the decisions which are there based on criteria's.

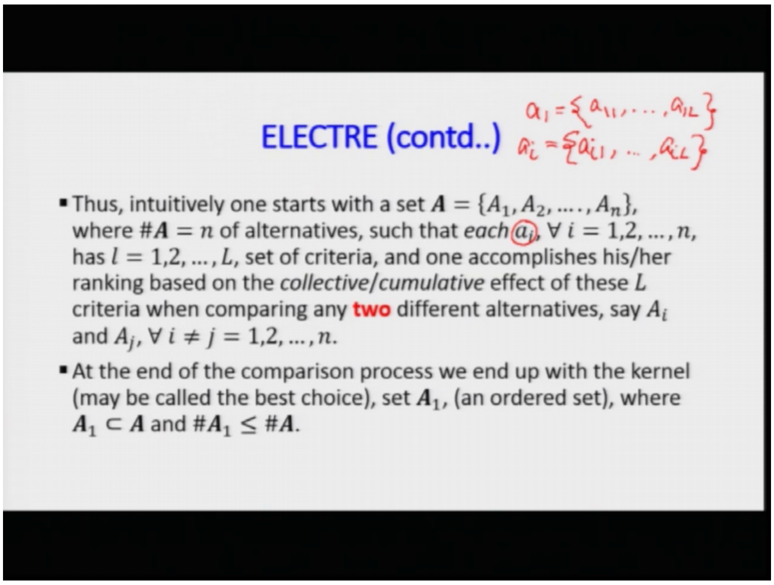
And if you take i with respect to j, you gets a positive benefit. And you basically put them in the set known as the concordance set or positive ranking. And if you take the decision which is non beneficial, you basically formulate the discordant set, and then also you have an negative out ranking. And combining them, you basically find out which decisions based basically best suits you considering the overall set of criteria which you have.

So, with this, I will continue the discussion. And if you remember also I mentioned that will consider the outranking, whether in the positive sense or in a negative sense be to be of equal magnitude. In the sense, then if I have been 100 rupees or if I lose 100 rupees, my level of liking and disliking would be of the same magnitude, this is something to do with in a way the squared error loss function.

And there are some reasons for that because if you remember, I did mentioned beginning of the this DADM 2 lecture series that having a squared error loss is a keen to considering, the returns to be quadratic, the utility function to be quadratic and returns to be normally distributed such that there is a one to one correspondence there.

And we will try to follow that, and slowly relax and take the assumption that a symmetric loss function, where unequal penalization of outranking in the positive sense and the negative sense can be considered, and we can model that also accordingly. So, this slide, we have this was the ending slide for the last class which was the 25th class, but still I will continue discussing then.

(Refer Slide Time: 03:39)



So, thus intuitively one start with a set A where set A has A 1, A 2, so a 1, 2, 3, 4 are the suffixes. So, A 1, A 2, A 3 till A n, where the number small n is basically a number of alternatives or decisions which you have. So, thus each will basically have small a 1, small a 2, small a 3 corresponding to the fact that what is the weightages you are going to give to the alternate is based on the criteria.

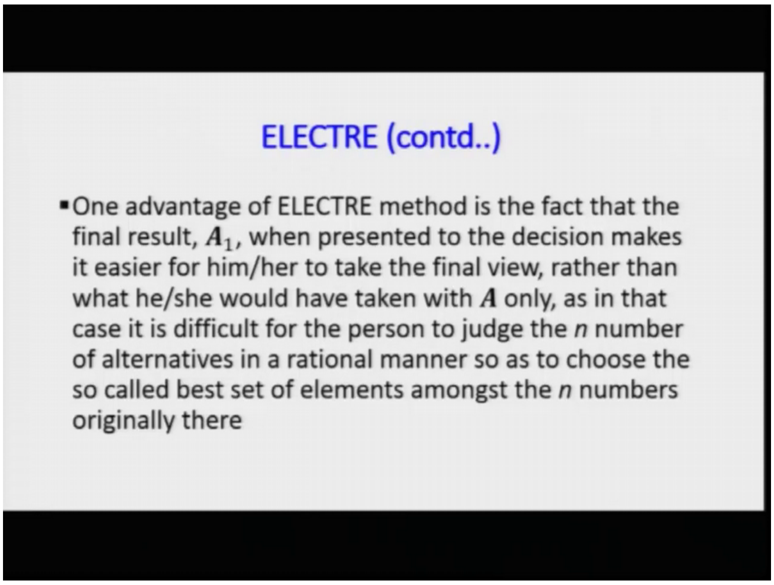
So, let me continue that, such that each A I, that is i is equal to 1, 2, 3, 4 till n has l number of set of criteria. So, obviously a 1 would be divided as so if you have a 1 from this, so a 1 technically would be a 1 1 till a 1 L, for all the L number of criteria's. Similarly, a i would be a i 1 till a i capital L and you can basically formulated accordingly.

Such that each a i, i is equal to 1 to n has small l, l basically nomenclature being from 1 to capital L number of criteria. And one accomplishes, one means the decision maker accomplishes his or her ranking based on the collective the cumulative effect of all these different alternatives criteria on the i th alternative. So, we will combine them, try to basically find out what is the collective effect and basically take the decision.

So, if I continue reading it, it says that one accomplishes his or her ranking based on the collective cumulative effect of this L criteria, when comparing any two different alternatives, say for example A i and A j, i not equal to j and i and j being equal to 1 to n. Because, if i and j are same that means, you are comparing two alternatives which are the same with respect to the criteria, so obviously they would not make much sense. So, is almost equal to the concept that you are taking the principle diagonal for the AHP, where the numbers you have assigned is one that means, it is difficult to us take decision i with respect to i, if i and j are the same.

And the end of the comparison process, we end up with a with a best choice set which is capital A 1, where capital A 1 would basically be a subset of A, bold A would be the set of all the A is which you have considered. So, we will basically rank them according to ordered them like A 3 can be better than A 4, A 4 can be better than A 10, A ten can be better than A 1; so, based on the collective decision of the criteria's.

(Refer Slide Time: 06:38)

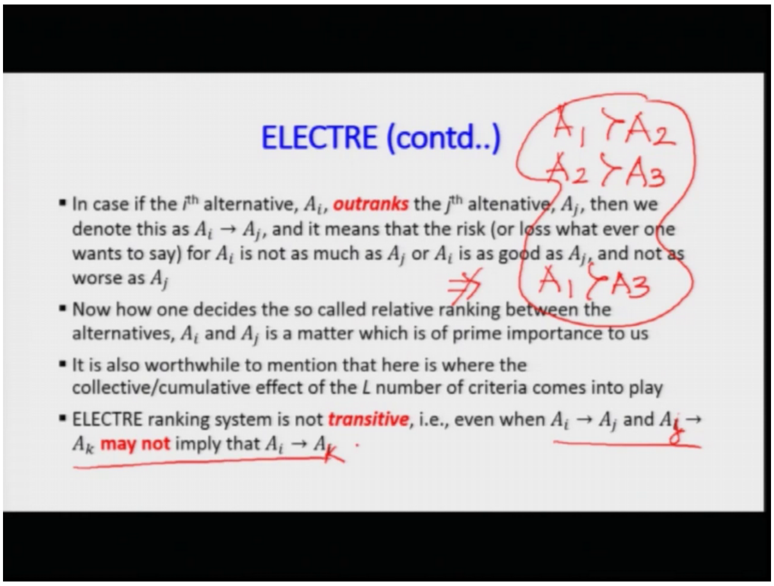


One advantage of the ELECTRE process on the method is the fact that the final result which is A 1, when presented to the decision maker it is easy for him or her to take the final view, rather than what he or she would have taken with A only. So, you basically give him or her, the choice set based on the criteria such that is easier for him or her to take the decision.

So, as in that process it in case, it is difficult for the person to judge the n number of alternatives in a rational method, considering n can be large. If we remember in the AHP method I did mention, that trying to basically find out the ranking process considering n is very large, and you are doing going to do N C 2 combinations may be confusing.

Because, if we are trying to compared on the same criteria for all the alternatives time and again, like say for example style. And you have basically 20 cars to compare 20 C 2. So, style when you are going to compare, obviously this consists of consistency will be violated, because then it becomes very difficult to make a rational choice. So, you want to basically may choose alternatives in a rational manner so as to choose the so called best set of elements amongst the n number of originally set which is already there.

(Refer Slide Time: 07:56)



Now, how does the process work? In case if the i th alternative, now I am considering the alternatives not in the criteria. So, with the i th alternative, A i, outranks, the j th alternative A j, then we say that A i is better than A j. Now, the question may come that, if there is only one criteria, then it is very easy. I only consider one criteria, find out A i better than A j, and my job is finished.

What if say for example, if there are two criteria , and I am considering the alternatives i and j with respect to the first criteria A i is better than A j, and with respect to the second criteria A j is better than A i. So, how do we do that, I will come to that later. So, it means that the risk or loss whatever one wants to say, risk or loss in the monetary sense we will say that for i i A i is not as much as that A j.

So, one whenever we are saying that A i is better than A j, we are thinking from the lost perspective that the loss of A i is just equal to A j or is less. So, obviously, we like to take A i as the best alternative with respect to A j. So, we will say that an m and if I continue reading; it means, that in and it means that the risk or loss whatever we want to say for A i is not much as much as A j or A i is as good as the A j and not as worse as A j.

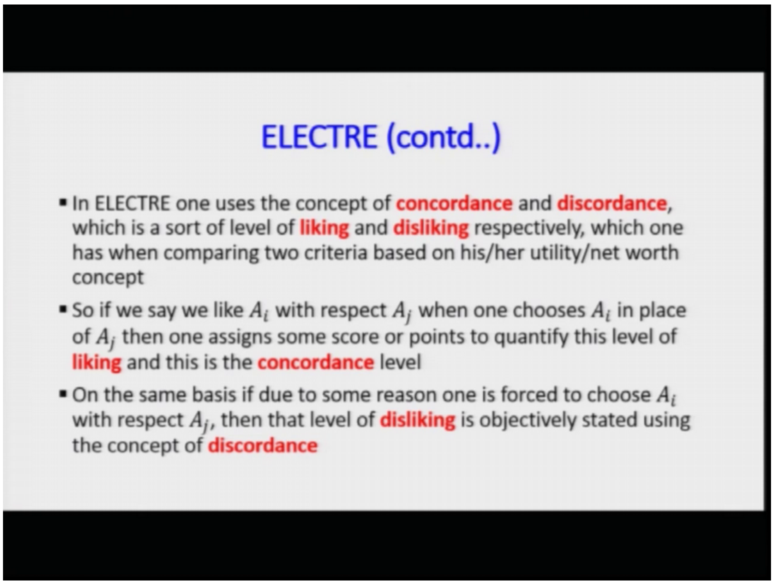
So, whenever we are comparing if say for example, there are two alternatives. So, say for example, A 1 and A 2, and the criteria's are two in number, the first and the second. Considered on with respect to the first A 1 is better or just equal to. And in the case of the second criteria A 1 is just equal to A 2. So, even if you one of the criteria; on based on the one of the criteria A 1 is better, so we will definitely take A 1 and proceed. And that question that if they are conflicting criteria, how we do that?

Now, how one decides the so called relative ranking between the alternatives, A i and A j is a matter of prime importance to us, and we will basically consider that accordingly. It is also worthwhile to mention that here is where the collective or cumulative effect of L number also criteria come into play, such that we are able to consider all of them at the same time or individually, and find on the collective total effect of all the L number of criteria's, when we are trying to consider the alternative 1 and 2 or i and j.

So, ELECTRE ranking system is non transitive that means, if say for example, A i is better than A j, and A l is better than a A k. We may not imply that say for example, we should say there is one error, this should be changed. So, if A i, so what we actually mean is if A 1 is greater or better not greater, better than A 2, 1 and 2 are just arbitrarily, A 2 is better than A 3, it does not imply that A 1 is better than A 3.

So, you have to basically consider that, when considering the method of ELECTRE process, I will change that accordingly. So, obviously it means that 1 is better than 2, 2 is better than 3, he does not obviously, always mean that 1 is better than 3.

(Refer Slide Time: 11:53)



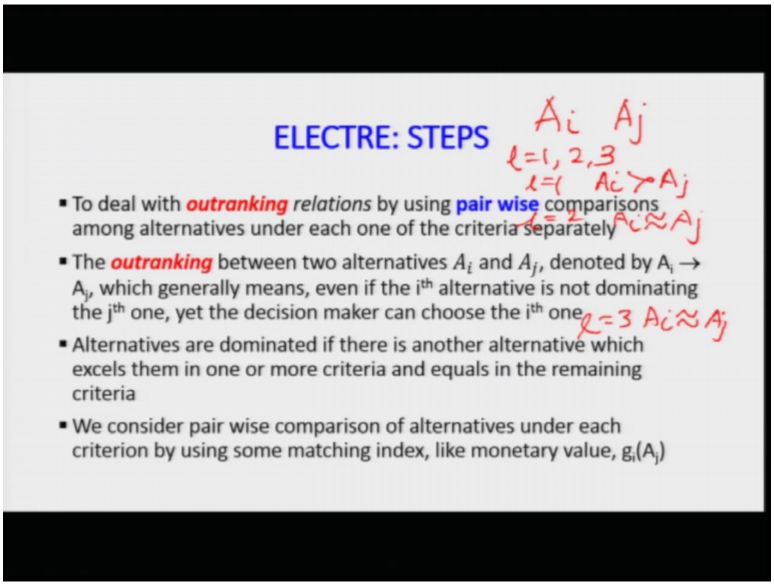
In ELECTRE one uses the concept of concordance and discordance set or concept, which is a sort of level of liking and disliking, concordance means I like. So, hence I give positive weights or positive points, discordance set is basically, I dislike, I means the decision maker dislike. So, hence he gives a level of points, which gives the level of discordance or disliking, so which one has to use this concordance and discordance set has to one has to use based on the criteria's between any two alternatives.

So, it does not mean that, if I give a concordance score between two alternatives, it does not mean that I am not going to and discordance score. So, if you remember I did mention that if I a, A 1 and A 2 their alternatives, and I like A 1 or the decision maker likes A 1. So, in the case A 1 is chosen, then obviously there is a positive point. But, in case say for example, due to some situation you are forced to choose A 2, then in that case you will basically give a level of disliking such that you basically make a very rational decision that liking and disliking would be given as positive negative points, so that collectively we are able to balance that and take a rational decision.

So, if we say that we like A i with respect to A j, when one chooses A i in place of A j, then one assigns some score or points to the quantify this level of liking and this is known as the concordance set or level. On the same basis as I said, if due to some reason one is force to choose A i with respect to A j, it is just the reverse not to choose A i with respect to A j, then the level of disliking is objectively stated using the concept of discordance.

Now, consider this, there may be some liking and disliking. And this liking comes out more prominent, so hence it will become a concordance set. And in the disliking comes out to be more important, then it will be a discordance set. And we will try to take a collective decision based on both of them.

(Refer Slide Time: 13:59)



To deal with the outranking relationship by using pairwise comparison that means, I am trying to analyze A i with A j among alternatives. So, such that under each of the criteria, we are able to do it separately. So, say for example, for the L is equal to 1, we rank them A 1 and A 2; for L is equal to 2, with they rank them between A 1 and A 2 and continue doing that such that we take a collective score based on the concordance and the discordance concept.

The outranking between two alternatives A i and A j, denoted by A i is better than A j which generally means, even if the i th alternatives is not dominating the j th one, yet the decision maker can choose the i th one, because in that case the effect which the person has on trying to choose A i is better than A j. So, say for example, if I am considering that, the A i and A j's are ranked in such a way for the criteria's. I am not getting any benefit for choosing any of the alternatives the first and second based on the fact say for example, I am considering the criteria's.

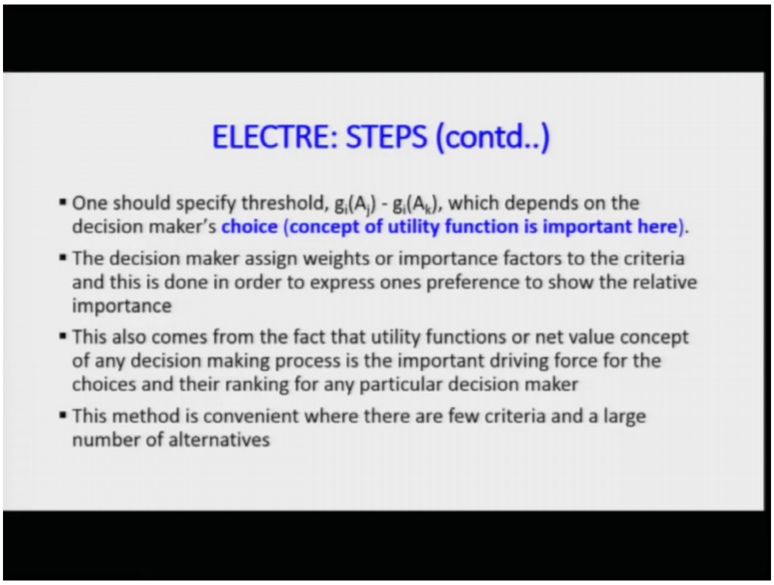
But, still I may be interested to take A i with respect to A j, in spite of the fact that the alternatives which are there or say for it is criteria's which are there are not helping, but yet I will basically assign some concordance set or concordant points such that in the long run, when I take a decision of A i with respect to A j, I will get a positive benefit.

Alternatives are dominated, if there is another alternative which excels them in one or more criteria and equals in the remaining criteria. So, say for example, I have A i, and A j, these are the alternatives. And l is equal to say for example 1, 2, 3. Now, consider with respect to l is equal to 1, A i is greater than A j; with respect to 2, A i is equally disposed with A j; and respect to 3, A i is equally disposed with respect to A j. Then obviously, there is one concordance point, hence will be interested to take A i, because for thus l is equal to 2 and 3, they are equally disposed in the sense the alternatives.

We consider pair wise comparison or alternatives under each criteria by using some matching index like monetary value and we will give that function as g i A j; that means, for each of this criteria, we take the comparison of the alternatives compare them, and assign scores accordingly positive or negative whatever it is.

So, once you specify the threshold, so what is the difference of the threshold? So, if I take a functional mapping or the difference of the scores between A i and A j for the criteria 1, criteria 2, criteria 3. So, obviously, if the difference is almost negligible, so now then in that case I am indifferent. In case if they are not, so obviously I will be tempted to take the decision 1 with respect to 2 that means, alternative i with respect to j.

(Refer Slide Time: 17:23)

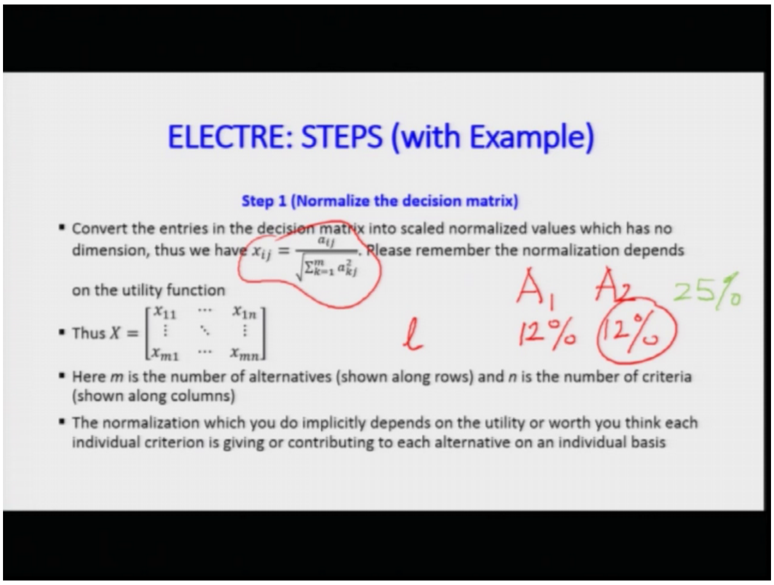


Hence the threshold which is given by the functional form of g i A j minus g i A k, it will depend on the decision makers choice the concept of utility function would be very important here and we will consider that later on. The decision maker assigns weights or important factors to the criteria, and that is done in order to express one preference to show relative importance of one alternative with respect to the other, and take decision accordingly.

This also comes from the fact of outranking liking and disliking comes from the fact, that the utility function or net value concept of any decision maker or making process is important driving force or factor for the choices, and they are ranking for any particular decision maker would always hold, because the decision maker is trying to utilize the concept of ranking of liking and disliking in such a sense that he or she will give some positive and negative points to this out ranking method of liking and disliking and then take the decision.

This method is convenient when there are few criteria and a large number of alternatives. So, trying to compare the alternatives with few criteria's, this will definitely be advice, because you remember same thing was said of the AHP. Because, if the number of different comparison increases, then there is a chance that the that the rationality or criticality index may be violated. So, we will basically consider this algorithm in a very simple step like step 1, 2, 3, 4, and then also solve a very simple theoretical problem.

(Refer Slide Time: 19:08)



So, what for the step-1 is normalized the decision matrix, and what we do? So, we have a set X, so X is basically in the scores which have been assigned with respect to some monetary value, so we will convert the entries in the decision matrix called the scaled normalized values which has no dimension. So, thus we will consider that the matrix A, which we have where the cells are A 1, if you consider along the rows, it is A 1 1, A 1 2, A 1 3, A 1 4 so on and so forth.

So, we will normalize them considering either to the row or through the column. This is the important fact which had mentioned time and again in AHP that you normalize using the utility function 0.1, and you also ensure the normalization can be done along the row or along the column, but remember the fact after normalization, the sum should be equal to 1, such the you have basically have the relative ranking system in such a way that it is better for you to compare.

So, here we take x i j as the ratio of a i j divided by the square root of the sum of the squares of the a i's, a ij's corresponding to the sum for the row or the column. So, please remember the normalization depends on the utility function. So, thus the square concept, which you have used has something to do with the utility function which is quadratic which means that the returns which you are going to take is normal. So, this is just a simplistic. One it can be logarithmic, it can be power utility function, it can be hara function, whatever you want to take.

But, remember that as I had mentioned that in AHP, once you basically decide on the utility function for the decision maker, do not change the utility function or the decision maker, when you are trying to compare different type of alternatives or different type of criteria based for the same alternatives. Thus, when the normalized vector is obtained from a, so you have x as the as the matrix. It can be a vector depending on how many alternatives and this criteria is if you have.

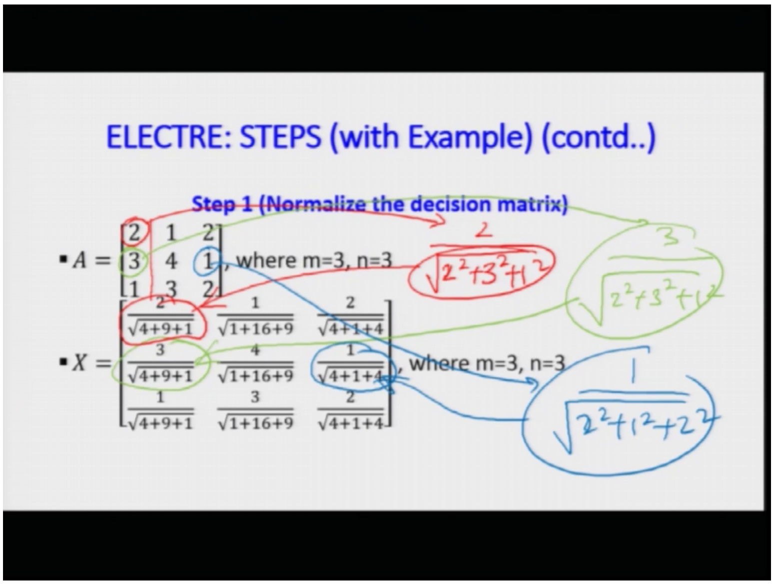
So, X is basically the matrix which has the cell values, if we consider the topmost row, it will be x 1 1, x 1 2 so on and so forth, till x 1 n. And the last row would be x m 1 till x m n, and remember what is m and n are basically the numbers of decisions on alternatives and the number of criteria. Here m as I just mentioned here m which is the third bullet point it mentions, here m is the number of alternatives, shown along the rows and n is the number of criteria which is basically shown along the columns.

The normalization which you do or which the decision maker does, implicitly depends on the utility or worth, he or she or you think each of the individual criteria is given giving or contributing to each alternative on an individual basis. So, once you normalize the scores which you get, say for example the score for one of the criteria, for one of the alternatives is 0.12, which means the total weightages or the overall effect of that criteria on that alternative, when you are trying to compare the alternatives would be of 12 percent worth on a scale of 100.

Now, obviously the question may arise, that what if the weightages of this criteria, changes for a per alternatives. Like say for example, for the j i th one, I give a weight of 12 percent for the for the alternatives which you are going to consider. So, it will be say for example, A 1, A 2 are the alternatives and l is the criteria. So, say for example, the normalized basis 12 percent. So, we will continue considering 12 percent for A 2 also.

But, if the question arises, what if we take 25 percent here, and 12 percent here, that concept we are not going to consider in our problem, because we will consider there is a changing weightages, you are going to assign to the criteria's, but we are not going to consider that. So, the last point mentioned the normalization, which you do implicitly depends on the utility or what you think, each individual criteria is given or contributing to each alternative on an individual basis.

(Refer Slide Time: 23:37)

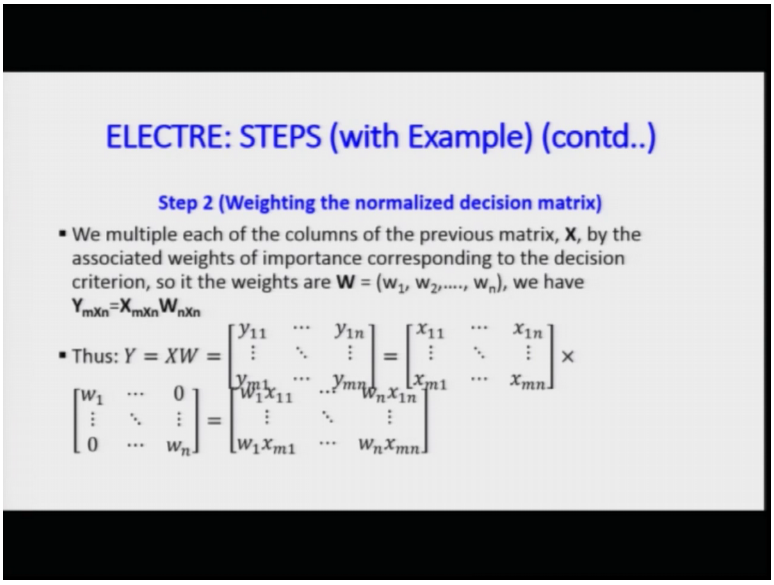


So, consider this, so I have the matrix A which is the so called non-normalized matrix, the points which are given without any units as per the figure with the values which are given along the rows is 2 1 2, 3 4 1, 1 3 2, here m is 3, n is 3, it can m and n n can be different also, but I am taking in a simplistic problem.

So, once you normalize using the quadratic concept x becomes the first cell is the values which I will take, and if I am doing it along the column, so it will be 2. So, this one which is coming, and this will be the sum of the squares of these, so square root of 2 square plus 3 square plus 1 square, so this value which you see is here. Now, say for example, when I do it for the second one, which is 3. So, 3 and in that case you have 2 square plus 3 square plus 1 square, this value is here.

Similarly, so this one was considered, this one was considered, let I consider this, so it becomes 1 divided by 2 square plus 1 square plus 2 square the value is here, this is the one, this is the one, I am just marking it with the color scheme. So, it is easier for you to understand, where and how the calculations have been done, so you can find it accordingly.

(Refer Slide Time: 25:38)



So, you normalize the decision matrix; now, we multiply each of the column of the previous matrix X, which you have observed by the associated weights you are going to assign level of importance you going to assign to the decision. So, consider the weights are given as W 1 for the first until W n for the second; for the last one.

And here if you remember I had mentioned just a few minutes back, that if I consider the alternatives, and we are also going to consider the fact which is a simplistic assumption that the weights; weights which are there for the alternatives for any of the criteria would be of equal weightages. So, if it is 12 percent it remains 12 percent, if it is 25 percent it remains 25 percent, we are not going to change that, and that is why the weights are there. So, W 1 remains at W 1 for all the things.

Remember one thing, the first row, the first cell value is W 1, others are 0 that is why the weights are same. For the second row, the second cell which is 2 comma 2 is W 2 rest values are 0. For the third row, the third cell with this 3 comma 3 is W 3 rest are 0. So that is why, I kept repeating it time and again the weights remain as it is. And it should also be remembered that you have normalized, so there were some of the weight should be 1. So, once you multiply the normalized x with the weights, you will get the total weighted sum of the alternatives and the criteria based on which you will proceed for the next step.

So, with this I will close this 1st lecture for the 6th week which is the 26th lecture and I will continue trying to solve the problem with all the steps which you will consider a very simple algorithm, and try to also analyze that what if the ranking positive and negative, concordance and discordance are not of equal importance, then how you will do? We will consider that later. First let us consider they are of equal importance that positive weights and negative weights would be same, weights I am using in the very simplistic sense not from the methodological point of view. So, with this I will end this class.

And thank you very much for your attention have a nice day.