Data Analisys and Decision Making - II
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Lecture – 24
AHP

Welcome back my dear friends and students; a very good morning, good afternoon good evening to all of you. And this is the DADM which is Data Analysis and Decision Making II course under the NPTEL MOOC series. And as you know this total course duration is for 12 weeks and total number of lectures would be 60 that is 30 hours. And each week we have 5 lectures each being for half an hour and after each week we have assignments.

So, as you can see from the slide we are in the 24th lecture which is the last, but one lecture for the 5th week. And as you know that we were discussing about AHP which is Analytical Hierarchy Process. And my name is Raghu Nandan Sengupta from the IME department at IIT Kanpur. So, the problem was if you remember we were discussing that to buy a car and there are four different cars, three different criterias alternate is being the cars and we are making a decision.

And if you remember we are only considering the decision of one person and it can be replicated this concept can be replicated for different type of decision makers who are in the same family who want to buy that car that is; one car they want to buy collectively. Now, when the decision is being made you are giving scores I am just repeating it whatever we have done; we are giving scores the principal diagonal is one of the diagonal elements are not symmetric.

Because if you can give a score for the ith with to the jth one if you give a score of 2 to the ith one then the jth one would be half if you give a score of say for example, 9 to the ith one then the score to the jth one would be 1 by 9, signifying the liking and disliking propensity for those decisions. It means it as I have repeatedly mention that I will again repeat it means that if you are in a position to take a decision i which is favourable for you will be very happy.

Hence you will give a score of 9, but due to some circumstances you are being forced to take the decision j which you do not like. Hence you are you are love for the decision propensity for the decision would be 1 9. So, this is just a scaling concept which has been used. And then also I am I mentioned that you want to find out the lambda max; that means, the overall score you have for the ranking of either the criteria when you are comparing the criteria among themselves or the decision when you are trying to compare the decision among themselves.

And the value of a is a priority matrix would basically give you the priority vectors or the scores. And then basically you find out the average, find out the consistency ratio, consistency index. And you will compare that whether the value which you get does give your consistency scores such that you are consistent in your ranking process or there is no much no such ambiguity in your decision.

So, let us continue more in discussion about the problem. If you remember there were three alternative and they were the primary level one there were no tertiary level like CFT was not divided into for the groups and so on and so forth.

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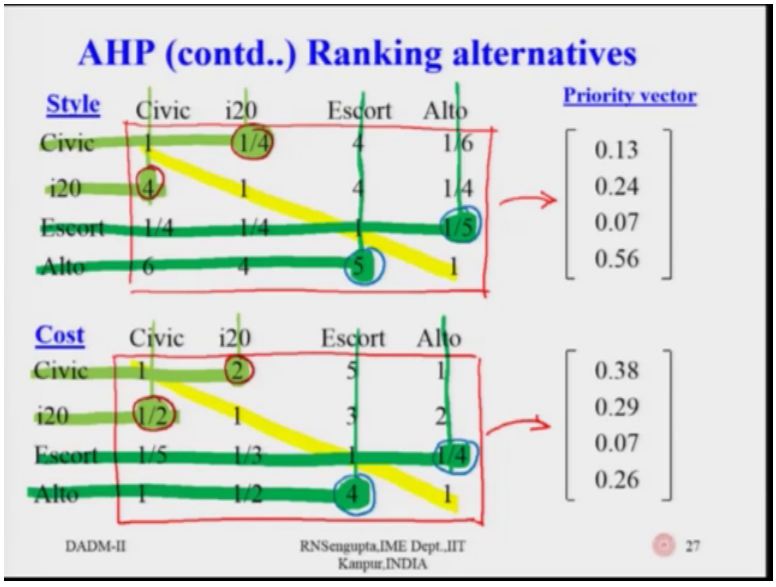

Now; obviously, the cost factor will come we will consider that in a very simplistic manner. So, in the AHP method in the problem when you are doing the ranking alternatives then once you do; now you have 3 alternatives style cost. So, I will only

discuss first the stand and the cost and the other things will come out automatically. So, consider the first matrix I will use the red colour to highlight.

So, this is style so this is the criteria which you are going to highlight. And what are the decisions? Either you can buy a Civic or i20 or an Escort or an Alto. And if you see the principal diagonal is 1 as it should be because when you compare Civic with the Civic the decision is 1 is to 1; that means you are equally dispose on both the sides. You are just considering ith decision and the jth decision on the ith alternative and the jth alternative and in this case i and j are the same which is the Civic.

Similarly for i20 i and j being same the second values 1 I am just hovering my electronic pen over that. When you compare Escort to Escort it is 1 when you compare Alto Alto it is 1; now let us go to the of the diagonal element. So, consider that I am let me use a different colour; I am considering i20 with respect to Civic; so this is the value 4 given here. And when I compare Civic with respect to i20 the value one fourth is there. So, what does it mean if I am taking the decision i20 then the propensity or the positive value which I accrue to myself for taking the decision of i20 with respect to Civic is 4 or 1 is to 4 times; 1 is to 4 time means is I am 4 times more inclined to take i20.

And in the case when I take Civic due to some reason may be; due to some cash consider Civic may be let us costly consider say for example, my friend is insisting that it should buy Civic. Or I am heard that the Civic some of the qualities of Civic is basically the mileage and all these things are better. So, obviously, it will have a so called influence in my decision when I consider the style 1; I am basically giving a point one fourth. Now remember this I am only considered from the point of view of style it will change depending on other alternate criteria.

So, I give a score of one fourth so this is definitely not symmetric as I mentioned when I consider Escort with respect to say for example, Alto and Alto with respect to Escort. So, Alto gets a score of 5 with respect to Escort while Escort gases score a one fifth with respect to alto; that means, I am more inclined towards Alto on the style front. So, again you have the matrix this is matrix a you know what is x x is the eigenvectors lambda is the.

So, the values of lambda has to be found out eigenvalues. So, a x is equal to lambda max x the formula you put that solve it in the initial matrix a which is the priority matrix is

this one. That leads us to the priority vector priority vectors being 0.13 0.24 0.07 0.56 and if you add them up they come out to be 1. If you remember we are following the principle of trying to normalize either through the rows or the columns and checking the sum should be one correspondingly.

And I had mentioned that whether you normalize along to along the row or the column the answer would be the same point 1. Point number 2 normalization basically mean that you are basic bringing the ratio or sum of the points which you are assigning to the alternatives or to the criteria should add up to 1. And here the normalization has been used where you basically divided by the sum of the total scores that is in a way you are trying to basically depict what type of utility function you are trying to use.

I had mentioned that in the last class which was in the 23rd class. And I have also mentioned that if you are trying to follow the utility function for a particular person you are going to use that same utility function for that person for different type of comparison of the criterias of the alternatives in between you are not going to change. Another point I did also mentioned that the utility functions or normalization concept which you are going to use for different person may be different.

But we will try to stick to the point where we are using the same normalization criteria for all the decision makers for a particular decision here in this case we basically buying a car based on different type of criterias which are there style cost and other things. Now, let us go to the cost factor; the principal diagonal is one which means when you compare Civic to Civic on the cost front i20 to i20 on the cost front Escort to Escort on the cost front all to all Alto to Alto on the cost front the score is 1; 1 is to 1 when I am considering the cost factor of i20 with respect to Civic.

So, let me use the same colour this is half Civic to i20 this is 2. Now you see in this style case you gave a score of 4 to i20 and one fourth to Civic. While in the cost factor you give a score of 2 to Civic and half that is inverse of that to i 20. So; obviously, on two different criterias your ranking the two decisions differently point 1. Point number 2 the differentiation scores would be different in both the criterias which is possible it need not be that you give a score of 4 here.

And one fourth here to balance the score which is 4 and one fourth; that means, you are not forcing the level playing field for the decisions it can be any random depending on

what you think on a standalone basis the scores you should give between i20 and Civic on the cost front. Similarly you have consider what the score you should give to i20 and Civic on the style front. Similarly when I consider let me take the same colour if I consider Escort to Alto it is one fourth when I consider Alto to Escort which is 4.

So, initially the scores were so the initially the scores were 5 to 1 5 here it is 4 to 1 4. And in this case in the earlier case when I had done i20 to Civic it was 4 to 1 4 and here it is half to 2 on two different criterias style and cost. And if I consider again I do the same thing normalization across the row or the column checking the sum should be one I am using the same concept on normalization.

Because this is the same person who is doing it for comparing all the four cars or style front then I am comparing the four cars on the cost front. So, here normalization would be the values of the priority matrix respective values on priority matrix. So, this is the priority matrix for the cost you divide each cell by the sum of the scores and you get the priority vector. So, here there are 0.38, 0.29, 0.07, 0.26 and if you see the rankings are changing.

In the first case the highest score was 0.56 for Alto here the highest score is basically for Civic on the cost front when you are trying to compare. This 0.07 is just a coincidence that the values of for Escort at the same. So, the priority vectors would give you the information that how you will proceed to find out the consistency ratio and consistence index and then try to compare whether there is a rationality in the decision making process. The exactly what whatever we have done when we are considering style cost and the other factor.

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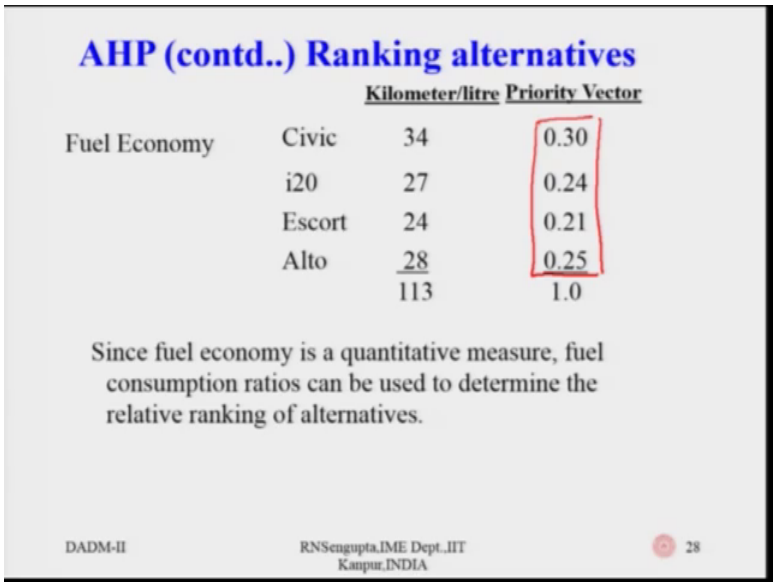

Now, I come to the fuel economy case here some values are given I am just considering very arbitrary values. So, I am considering fuel economy for Civic is 34 in the kilometre per litre which is in cost in rupees i20 is 27, Escort is 24, Alto is 28 these are a little bit higher values I know that, but just I am trying to give a comparison. And the priority vectors when I ever I basically come when I again I do the same comparison find out the fuel economic concept for all the four cars which are the alternatives again find out the priority matrix.

Then find out the priority vector the priority vector basically is as I found out for the other two. Here in the third case is 0.3 0.24 0.21 0.25 against Civic is higher. So, it is not that Civic will come higher in all the senses, but we can find out. So, now remember one thing in the first two cases when you did the style and the safety they were based on just subjective value you comparing 1 is to 1. That means, ith to the jth one considering others are not there.

But in fuel economy the picture is little bit different fuel economy is a quantitative measure fuel consumption ratios can be used to determine the relative ranking on the alternatives that what we have done. And, this ratio which you can find is 34 by 113, 27 by 113, 24 by 113 28 by 113 this ratios would give you the priority matrix normalize them either check up the sum of the rows is 1 or the sum of the column is 1 you follow the same thing. Another thing I am repeating it whichever normalization you use whether

row or column follow that throughout the calculation for all the decision on all the criterias do not change it in between.

Even though I mention that the ranking process would give you the same thing whether you are doing the normalization along the row or the column some of the rows or the columns in 1, but they should be a standard procedure that it will try to utilize for each and every decision. The normalization concept of utility would remain fix and also the way you are trying to normalize whether rows or column should be fixed and you continue with that. So, using this quantity values you find out the priority values. So, you have now priority vectors for all the three criterias for all the alternatives the alternatives have been ranked.

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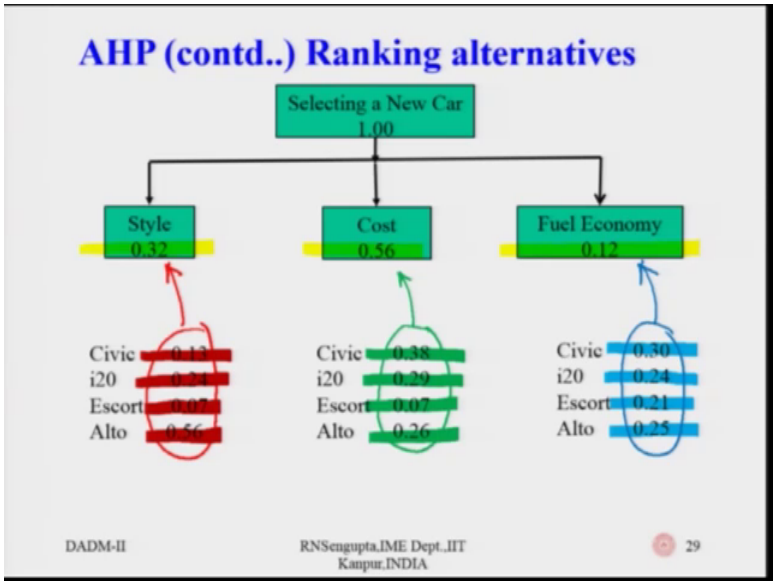

Now, you come here so when I basically if you remember when you are basically doing the style cost and fuel economy the comparison. The scores which are shown in the green box 0.32 0.56 0.12 they all add up to 1. Again the same normalization concept has been used. So, these are the scores which you have assigned to them self when they compared against each other without considering the alternatives in picture. But then when you come back to considering the alternatives with respect to each and every criteria alternatives means; Civic, i20, Escort, and Alto with respect to style.

This is the priority vector for the case of style. If I do the comparison of the priority vector for the case of cost for Civic i20 Escort and Alto this is the priority vector which

is assigned to cost. If I do the comparison of the alternatives based on the criteria fuel economy the priority vectors are as given. So, these are the exactly whatever we have done you have used the same concept priority I am again repeating priority matrix using the normalization concept, using the some concept, that is one either rows or columns in trying out trying to find out the priority vectors those average values all this things you have you will repeat it time and again.

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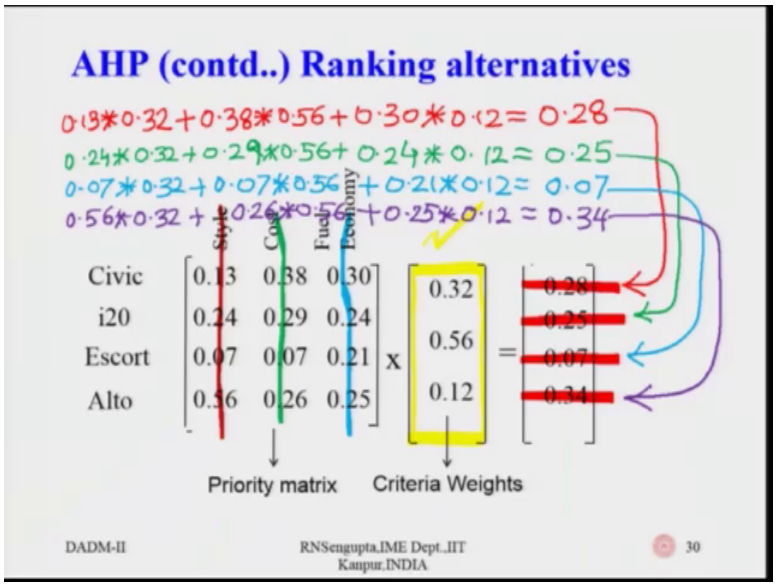

Now, we come we are coming at the last stage of the problem. So, now, we want to combine them; combine them in the sense that all the alternatives corresponding to all the criterias will combine to find out the best ranking. So, what we have is this. So, if you remember I will just flip to between two slides so please bear with me. So, let us consider the criteria weight so I will use the highlighter yellow. So, I will just highlight this vector this is the side lines.

So, what are the values 0.32, 0.56, 0.12 so let us go back. So, these are the values 0.32 0.56 0.12. So, this is the priority vectors when the criterias are only compared without considering the alternatives so this is done we understand that. So, I put a tick mark this is taken care of. Now let us come to the comparison of the four alternatives with respect to each and every criteria. So, I will use three different colours. So, first it be so for the style 1 so let me consider the style 1 as this. So, what are these values for Civic 0.13

0.124 0.07 0.56 so let us go back. So, these are the values so they are 0.13 0.24 0.07 0.56 only for style. So, it is matching the style for all the four alternatives are as it is.

Now let us go to cost for cost it is 0.38 0.29 0.07 0.26 for the four alternatives. Let us consider 0.38 0.29 0.07 0.26 here it is cost done for all the four alternative. Then let us go to the fuel economy 0.32 0.24 0.21 0.25 0.3 wait ok; 0.30 sorry 0.30 0.24 0.21 0.25 0.30 0.24 0.21 0.25 this is the priority vector for all the four alternatives corresponding to the criteria fuel economy so this is done.

So, you multiply the priority matrix with the criteria weights; that means, you are trying to give the priorities and they are being waited based on the fact that what is the weightages or the priority values which you are giving to the alternatives to the criterias them self. Once you multiply so this is just remember the ranks and a and the size of the matrix should be important.

So, this is the 3 cross 3 and this is 3 cross 1. So, obviously, the end result would be 3 cross 1 as it should be. So, if it is increases or decreases it would not matter. Because you say for example, number of alternatives increases here so the corresponding values can be taken care of; because if this is in case of, but the style in the cost remains 3 so this will also remain 3.

If this in case of style cost following other increase the length of this vector will also increase. And you can find out the total weights which I will now highlight using the red colour 0.28 0.25 0.07 0.34; that means, you are multiplying the first row the first column which is 0.28 0.24 this second row with the second column will give you 0.25. Third row with the column will give you 0.07 then the fourth row multiplied by the column will give you 0.034.

So, I will just write one value here so if I consider the multiplication of so the values would be 0.13 into 0.32 plus 0.38 into 0.56 plus 0.30 into 0.12 that value should come out to 0.28 which is here. So, now consider 0.24 multiplied by 0.32 plus 0.29 multiplied by 0.56 plus 0.24 multiplied by 0.12 comes gives us the value of 0.25 the value is here. Then if I find out let me write it down I know that you have understood, but please bear with me. So, the second value 0.07 multiplied by 0.32 plus 0.07 multiplied by 0.56 plus 0.21 multiplied by 0.12 that value comes out to be 0.07 this value is on the third section. So, this is just the coincidence that it is 0.07 here also 0.07 here also 0.07 here also.

So, the last one 0.56 multiplied by 0.32 plus 0.26 multiplied by 0.56 plus 0.25 multiplied by 0.12 equal to 0.34 this value is the fourth value in the (Refer Time: 24:33) so using this you find out the overall score. Now the question would also arise that; what if style had a tertiary levels of discussion hierarchy, cost at two such levels of hierarchy? So, like style can be colour can be an important fact, seat adjustment can be important factor, having say for example, sun roof can be an important factor, dash board layout can be an important factor so all these things can be considered.

Cost can be what is the EMI which is available, what is the resaleable value, what is the work shop charges. If you visit the work shop what is the average cost which you have cost can be whether the insurance is high or low. May not may not depend on the person or the number accidents of the person has committed or how bad or good the car has been driven. But consider the due to some reason the insurance companies changing higher premium for one variety of car and a lower premium for other variety of car per month for the same type of person.

So, those cost can be considered fuel economy can be say for example, whether in the person drives the car along the highway and consider for when you consider Civic, i20, Escort and Alto the fuel economy is for the highways are totally different with respect to when you are driving in the city or in the in the sub urban areas. So, all these things can be considered in the tertiary level and you can also have the same type of priority matrix same type of priority values find them and basically proceed from the lowest level to the highest level.

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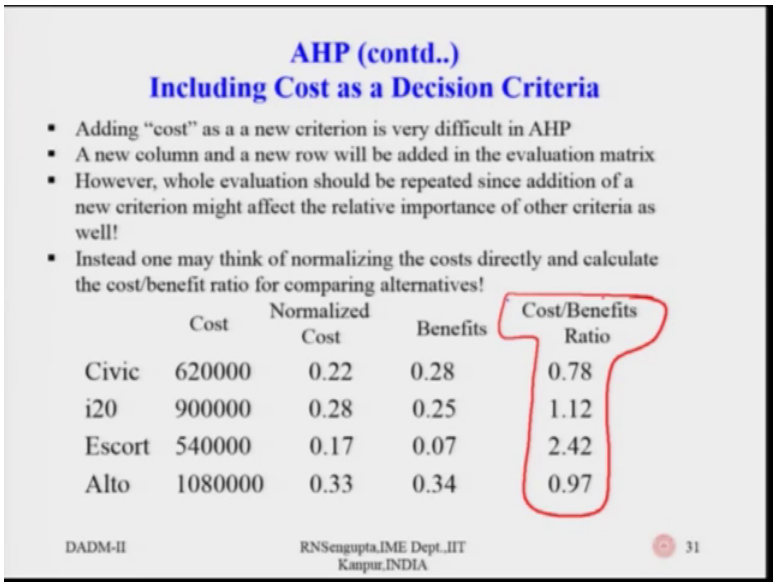

Now, let us add a cost. So, adding cost is a new criteria is very difficult in AHP. So, it has to be analysed accordingly. You have to either add this is your adding remember that a new set of information is coming. So, those were fuel economy in the sense per litre what is the overall distance the cars could travel on an average, but now you are bringing the cost factor. So, the cost factors are again arbitrary values I have considered.

Adding cost is a new criteria is very difficult in AHP a new column and a new row will be added in the evaluation matrix. However, the whole evaluation fluid should be repeated again. Since the addition of a new criteria might make the relative importance between the criterias totally may totally change. So, say for example, you had till now this three criterias of style fuel economy and the other one and the cost. But consider that you bring safety say for example, or resale value whatever it is.

So, in that case the relative ranking between the first three when we were there when we totally change when you considered the addition of that. Now, you have to basically consider the four one. So, instead one may think of normalizing the cost directly and calculating the cost benefit ratio for comparing the alternatives which will be much easier. So, consider the Civic, i20, Escort, Alto cost are given, you normalize them you find out the normalized cost. The same concept sum of all the values will come in the denominator and the actual value will come in the numerator.

So, the benefits which you get are also given are in the ratio values and you find out the cost benefit analysis and basically add the priority ratios corresponding to that. And once you have that the cost benefited ratios would be given this with this I will end suddenly; 30 minutes lectures for the 24th is over. So, we will continue discussing of AHP and other concepts in the 25th lecture; with this I will end today's 24th lecture and have a nice day.

Thank you very much.