

Data Analysis and Decision Making - II
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Lecture – 22
AHP

Welcome back my dear friends a very good morning, good afternoon and good evening to all of you. And as you know this is the DADM - II which is Data Analysis and Decision Making - II course another NPTEL, MOOC series. And we and obviously, I repeat it time and again, but in order to make you how things are as we progressed.

This is a 12 week course of 30 hours, each week we have half an hour lecture of 5 each in total number and the and after each week which is of 5 lectures we have an exam or assignment which is given. And after the end of the course we have a question paper or a final question to paper to be solved. And we have already started the 5th week and we are going to start today the 22nd lecture. And my name is Raghu Nandan Sengupta from the IME Department, IIT, Kanpur.

So, if you remember we were discussing about the decision which Ram and Shyam has and they have to take a decision where they want to go to IIM Ahmadabad or IIM Bangalore or IIM Calcutta and each has different level of importance they want to assign to academic rigor with respect to placement potential. And when they individually consider these 3 institutes under this criteria and they arrive at different weights for this alternatives; alternatives being A, B, C, IIMA, IIMB, IIMC.

Now, just at the end of the 21st lecture I did mention that what if their parents come, I have not included that I will come to that in explanation, what their parents come into the picture. So obviously, for the parents this both the sons would be of the same important, but I did mention that they have their relatives in Bangalore, they want both of the kids to go to Bangalore, but Bangalore cost is high while Calcutta cost is low so obviously, they have to make a judicious decision with respect to cost, with respect to safety of the place, with respect to what is what are the subjective criterias they want to analyse in order to arrive the decision. Similarly, both the sons also already arrived on the decision based on these two criterias which was mentioned, some may be subjective and some

may be objective. So, with this we brief background let me continue with the 22nd lecture.

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So Ram's and Shyam's collective hierarchy is as given

Decision	Select IIM			
Hierarchy # 1	Ram		Shyam	
	0.5 (p)		0.5 (q)	
Hierarchy # 2:	PP	AR	PP	AR
	$\frac{1}{3}$	$\frac{2}{3}$	$\frac{1}{4}$	$\frac{3}{4}$
	(p ₁)	(p ₂)	(q ₁)	(q ₂)

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So, collectively when I consider Ram's and Shyam's, I am not considered their parents. Now, it can be expanded may be the slide space is too small hence I have not been able to do that, but I will try to explain that. So, Ram's and Shyam's collective hierarchy is as follows. So, we basically it is the hierarchy and the decisions are and analyse accordingly.

So, say for example, for the parents if you remember, for parents when they analyse Ram and Shyam basically get the same score of 50 percent 50 percent as 0.5, 0.5 which is basically by of p and q. Now, for hierarchy 2 if you remember Ram had a score of one-third is to two-third for placement potential with respect to academic rigor, while Shyam had a score of one-fourth is to three-fourth for placement potential and academic rigor which is mentioned here. So, these were 1 is to 2 and 1 is to 4, that is twice and thrice. Now, let us mention them as points p the values which are given as p 1, p 2 for Ram, and q 1, q 2 for Shyam.

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Alternatives:	IIMA	IIMB	IIMC	RAM
	0.30	0.40	0.30	
	(p ₁₁)	(p ₁₂)	(p ₁₃)	
Alternatives:	IIMA	IIMB	IIMC	RAM
	0.30	0.40	0.30	
	(q ₁₁)	(q ₁₂)	(q ₁₃)	
Alternatives:	IIMA	IIMB	IIMC	SHYAM
	0.25	0.25	0.50	
	(q ₁₁)	(q ₁₂)	(q ₁₃)	
Alternatives:	IIMA	IIMB	IIMC	SHYAM
	0.35	0.35	0.30	
	(q ₁₁)	(q ₁₂)	(q ₁₃)	

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Now, let us go into the next level. If you remember Ram and sham, so this to coming back to the last slide 50 percent, 50 percent was the weights being given to Ram and Shyam by the parents and 1 is to 2 and 1 is to 3 who are basically the criterias weights which are being assigned by Ram and Shyam respectively. Now, let us come to the fact that for each of these criteria how do the decisions or the alternatives rank themselves or are ranked by both the decision makers separately which is Ram and Shyam.

So, let us consider Ram if you remember had the for the points or the scores being assigned was basically p with the sum suffix 12, 13, 14, I will come to that 1 23 4 later on. So, Ahmadabad had 30 for Ram, 40 for Bangalore, and 30 for Calcutta. So, the scores are given as p 11, p 12 and p 13. Now, let me go into the alternatives with respect to. So, these are the alternatives are the A, B, C are with respect to the criterias which are placement and academic potential rigor.

Similarly, the weights are 0.3, 0.4 and 0.3, these are normalized. And they are denoted for Ram as p 21, p 22, p 23. So, the first set which is yellow and red highlighted are that for A, B, C, IIMA, IIMB, IIMC for these two criterias which is academic rigor and placement potential for Ram only, hence they are marked as with the main symbol as p with the corresponding suffix.

Let us now switch our attention to Shyam the other son. For Shyam they are now denoted by the symbol q with the corresponding suffix suitably q 11, q 12, q 13, then q

21, q 22, q 23 depending on which level you are. So, Ahmadabad has 0.25, Bangalore has 0.25, Calcutta has 0.5, again a normalized score and these are given as q 11, q 12, q 13. So, you will basically assign them points accordingly. When you come to the other set of alternative based on the same set of alternatives based on the other criteria. So, they are q 21, q 22, q 23 and for the decision alternatives the weights are 0.35, 0.35, 0.3 which is 35, 35 and 30.

So, if you consider this the first set is for Ram, the second set is for Shyam. So, they will basically consider that and that 50 to 50 or 0.5, 0.5 was the weights being given by the parents to Ram and Shyam respectively.

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AHP (contd..)

So

- IIMA: $p^*p_1^*p_{11} + p^*p_2^*p_{21} + q^*q_1^*q_{11} + q^*q_2^*q_{21}$
- IIMB: $p^*p_1^*p_{12} + p^*p_2^*p_{22} + q^*q_1^*q_{12} + q^*q_2^*q_{22}$
- IIMC: $p^*p_1^*p_{13} + p^*p_2^*p_{23} + q^*q_1^*q_{13} + q^*q_2^*q_{23}$

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So, hence when I consider Ram's point and Shyam point and then combine that for the alternatives A, B, C. Now, here let us pause one thing. It can be analysed in such a way that I can rank the decisions individually or I can basically collectively consider the overall score for each decisions, where all the scores of Ram of Shyam and parents would be considered in a collective manner. So, you have to just basically make the hierarchy in such a way and give and multiply the points accordingly. For, so far Ahmadabad the scores are like this and if you see here you are combining all the weights for Ram and Shyam.

So, if I consider the part for Ram, Ram was p, so obviously, this set of scores are with respect to Ram because they were the p with the corresponding suffix and these sets of

scores were for sham. So, this would be S, I will just use S for Shyam and R for Ram. Similarly, when I come to Bangalore again, I take the collective weights, only remember we make a note of the suffixes which are there. So, p and q are basically for Ram and Shyam respectively, but the suffixes would denote to you in which way the hierarchy has been denoted. So, I will use a different colour. Let me use the same colour. So, it will be easy for us. So, again this is for Ram I think I should be used a different colours. Excuse me, one minute please.

This is for Ram and let me use the light blue for Shyam. So, for Shyam is this one where the q suffix are there the q symbol is there, but the suffix are accordingly I will draw that, please wait. So, this is for Ram I will put a R, this is for Shyam and put an S. Then when I go to IIM Calcutta, so initially I finished IIM Ahmadabad, then I will be finish Bangalore, Bangalore and I am going to Calcutta. So, this is Ram and similarly this will be for Shyam for Calcutta. If I put a highlighter using red colour, so this is Ram, this is Shyam. So, in this way I will can be able to denote that.

So, in order to make it much more clear let me draw the diagram. So, I will use first the black colour and then mark it accordingly with the colours to give a better picture about Ram and Shyam. So, you have the arms this is Ram, this is Shyam, I am using the same colour please note that. And this will come for academic rigor then for placement potential everything.

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AHP (contd..)

So

- IIMA: $p^*p_1^*p_{11} + p^*p_2^*p_{21} + q^*q_1^*q_{11} + q^*q_2^*q_{21}$
- IIMB: $p^*p_1^*p_{12} + p^*p_2^*p_{22} + q^*q_1^*q_{12} + q^*q_2^*q_{22}$
- IIMC: $p^*p_1^*p_{13} + p^*p_2^*p_{23} + q^*q_1^*q_{13} + q^*q_2^*q_{23}$

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So, if I am considering, ok, let me consider in a better way. Here is academic rigor or p 1 is placement pleasure. I am considering this is a we will combine them accordingly and basically I have different type of trees and combine them. So, it is not the only single hierarchy they would be for Ram, they would be for Shyam, then combining from the point view of the parents also.

So, you will basically put weights accordingly. So, it is p 11, so this will p 22, now it will be p 12 sorry sorry p 12, p 13, then you will have p 21, this one will p 22, and this one will be p 23. So, if I go up the hierarchy, they would be multiplied accordingly. Similarly, they would be for again I am repeating they would be one Shyam and they would be combined for the parents. So, it basically combines all this course accordingly, I will come to that problem solving.

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AHP (contd..)

Wide range of applications exists:

- Selecting a car for purchasing
- Deciding upon a place to visit for vacation
- Deciding upon an MBA program after graduation

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So, what are the different wide applications for AHP? So, we will come to actual problem solving. So, wide range and applications exist say for example, you want to select a car for purchasing you want to buy. So, who are the decision makers? Can be the father, the mother, the two kids, may be the in laws.

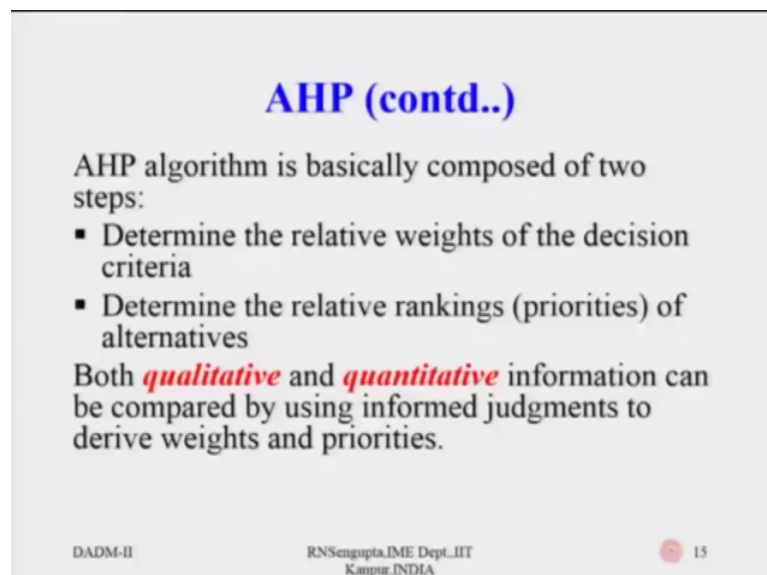
So, what are the criterias? May be the cost, may be the colour, may be the make, may be the what are the societal value it has, some may be very irrational I agree. But it may be some may be resale value, some may be maintenance costs, some may be boot space, some maybe the criterias, can be the safety principle, then what is the power in the EMI

you have to pay per month see for example, the family you wants to buy a car for which they have to take a loan. So, all these things can be considered.

You want to decide on a place to visit maybe the cost, may be the time to travel, maybe the safety of the place, maybe the hotel, may be say for example, how whether it is a mountain, whether it is a seaside, what they like. The parents may like say for example, mountain, the kids may like the seaside or father likes a very quiet place, while the mother likes a place where there are a lot of hustle bustle a lot of market, lot of happening things are taking place that she is quite happy. Or the kids may like a place where it has a lot of games to be played, the hotel has say for example, different type of games like maybe swimming pool is there for the kids, maybe snooker is there for the kids to play, maybe a football court or tennis court, table tennis court, badminton and all these things would be this. So, they will basically give points upon accordingly.

You want to decide about MBA program. Just the example which I considered like Ram and Shyam placement potential academic rigor, parents there also say is their which they want to also put on their kids all these things can be considered as actually very interesting examples for AHP.

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AHP (contd..)

AHP algorithm is basically composed of two steps:

- Determine the relative weights of the decision criteria
- Determine the relative rankings (priorities) of alternatives

Both *qualitative* and *quantitative* information can be compared by using informed judgments to derive weights and priorities.

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So, the AHP algorithm is basically composed of two steps point one you determine the relative weights of the decision criteria. So, if you remember I did mention Ram and Shyam giving weights of 1 is to 3 and 1 is to 4, those were based on the fact what is the

score you want to place for placement potential with respect to academia rigor, Ram and Shyam gave those points.

So, when you decide on the weights this is the relative weights are decided by the decision maker based on the decision criteria which he or she has that was the best one step. And then you determine the relative ranking or the priorities of the alternate is based on the cumulative scores a person has assigned to the different type of alternatives. So, you will come to that. AHP considered considers both qualitative as well as quantitative information and they can be compared by using informed judgment to derive weights accordingly, such that you are able to combine the weights to arrive and the final answer.

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AHP (contd..)

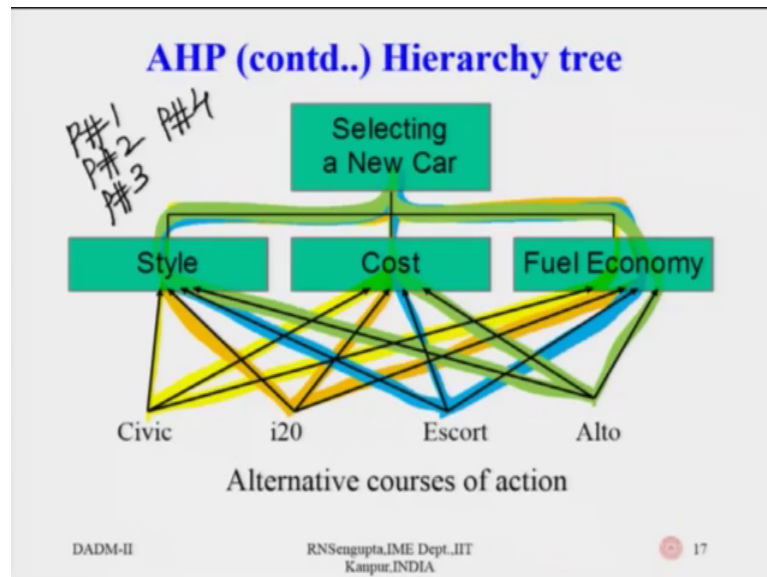
- Objective: Selecting a car
- Criteria: Style, Cost, Fuel-economy
- Alternatives: Civic , i20 , Escort, Alto

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So, let us consider a very simple example. I am not going to make it very complicated. You can make it complicated with different layers of hierarchy, different layers of criteria, different layers of decision making. So, objective is to select a car, you want to buy a car. What are the criteria you want to consider? The style of the car and the cost of the car and the fuel economy. I am not going to consider efficiency, EMI, per month maintain and cost resale, value safety, all this I am not going to consider they can be brought into the picture in order to make it more much more realistic. Let us consider a very simple problem.

What are the alternatives? Alternatives of Civic, the curve, i20, Escort and Alto so obviously, cost factor will also come into the picture later on, apart from this subjective criterias. So, this is the hierarchy.

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The hierarchies are you have style which is only style, ok. By the way another thing I am this style I am considering it is of a single level of criteria or hierarchy. It can be say for example, stein can be broken down into more sub level of criteria tertiary one we are not going to consider that.

The next criteriam, sorry criteria with respect to style on the same level with cost, again cost is only you know one set of criteria it is not broken down. It maybe the cost would have been the buying cost, the selling cost, the EMIs, the fuel cost, all these things maybe the insurance cost, all these things we are not going to consider in a very simplistic manner for our problem.

The third criteria is fuel economy. So, again fuel economy I am not going to consider that whether at high speed or low speed all these things. So, there is only one hierarchy the hierarchy consider, style cost and in fuel economy. So, what are the decision alternatives which you have? It is the Civic car, i20, Escort and Alto. So, alternative course of actions based on the fact you want to arrive to buy the best car for only one person, there can be many persons who are going to take the decision I am not going to

consider all the different persons and combine then I am only going to take one person's criteria.

So, one of the decisions can be say for example, Civic for style you consider to buy a new car, then Civic for cost and then they can be Civic for fuel economy. Say for example, then you consider i20 it you can be style, then go can be cost, and then go can we fuel economy and then go. So, these are being cumulatively considered next for Escort can be style, then you decide the selecting car and the cost can we fuel economy.

So, these the colours which are giving have a specific significance. If it is yellow it is a cumulative score which you are going to combine multiply in sum them up for the Civic. Similarly, for the orange one it is a multiplicative factor of all the scores sum them up for i20, for the blue one which I have just done now is for the Escort, multiply the factors and some of the scores. And for Alto, I am using the green colour, when I multiply the scores being assigned for style, separately cost, separately fuel economy, separately combine them and add them up.

So, based on the overall score I will rank them from the highest to the lowest. This is our main idea and obviously, this can be done for person 1 father, person 2 mother, person 3 the in laws, person 4 the kids and so on and so forth it can go. Like as you saw it was Ram, Shyam, the parents.

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AHP (contd.)
Ranking Scale for Criteria & Alternatives

Intensity of importance	Definition	Explanation
1	Equal importance	Two factors contribute equally to the objective
3	Somewhat more important	Experience and judgement slightly favour one over the other.
5	Much more important	Experience and judgement strongly favour one over the other.
7	Very much more important	Experience and judgement very strongly favour one over the other. Its importance is demonstrated in practice.
9	Absolutely more important.	The evidence favouring one over the other is of the highest possible validity.
2,4,6,8	Intermediate values	When compromise is needed

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Now, how do I; this scores which have been giving repeatedly I did not mention anything about the scores, now I am going to come to that. So, look at this table as proposed by Saaty.

So, the intensity of the importance of the decisions are given on the leftmost column. So, there are odd numbers it can be even also which is in the last cell. The intensity of importance are given as 1, 3, 5, 7, 9 and obviously, of 2, 4, 6, 8 I will come to that explanation. And the definitions are given. If you give a score of one is to one it means there are equal importance and is mentioned in mentions which is this point which I would want to highlight I will just mark however, my electronic pen there. There are two factors which contribute equally to the objective hence you are giving a score of 1 is to 1.

If it is score of 3 which is somewhat more importance which you are giving because the experience and judgment slightly favour one factor with respect to the other. So obviously, we will give it a score of higher of 3 and what is the score you will give to the lower one, I am going to come to that within 2 minutes.

Considered score is 5 for the higher one that means, my your placing a level of importance which states by definition is much more important, hence there the experience and judgments strongly favoured one criteria over the other or one alternative for the other. The next one which is the 4th row is of a score of 7, 7 means very much more important which means the experience and judgment very strongly favours one over the other. Hence, it is its importance is demonstrated in practice, hence one would give a score of 7 other would give it a score which is definitely much much lower. What is that low score I am going to come to that which I just mentioned.

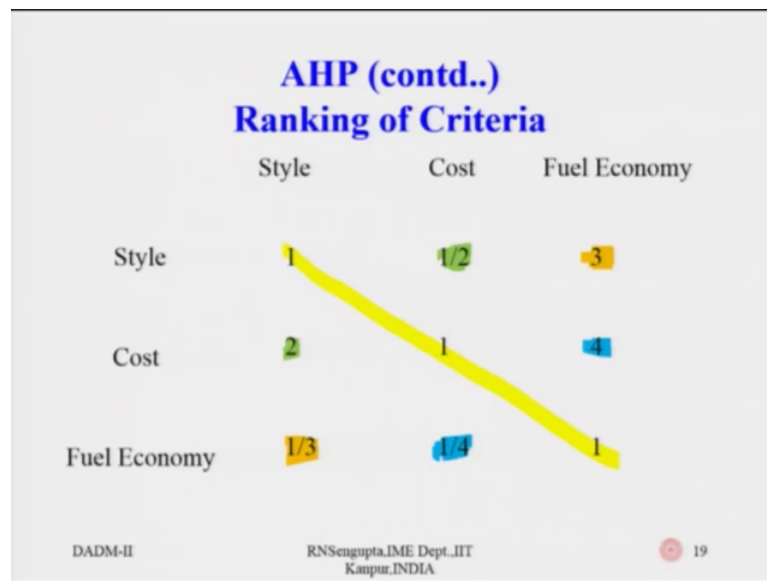
So, if the intensity of importance is 9 it basically means absolutely more important the one which you want to take with respect to the other. And as per the explanation it means the evidence favouring one over the other is of highest possible validities hence you will give a higher score of 9 and other you will give another score which is definitely much much less than 9.

And if we give the weights of 2, 4, 6, 8 which are the even numbers they are intermediate values and when compromise needs to be made then you can basically give. So, the reason why 1, 3, 5, 7 is that you want to basically bucket or make the buckets of the of the scores in such a way that there is no ambiguity, but if there is ambiguity we can bring

the points of 2, 4, 6, 8 also in the scoring along with 1, 3, 5, 7, 9 in order to make it much more smoothen smoothen enough.

But obviously, the scores of 1 is to some value, 3 is to some value, 5 is to some value, 7 is to some value and 9 is to some value would give if you are able to with that it will give a much more rational and a start differential ranking system.

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Now, what does it mean? So, say for example, what you will do in the case of this ranking on the cars you will consider the 3 criterias based on which you are trying to rank. I am not bringing the car into the picture for the time being remember that please. So, say for example, it is like this. If the cars are there cars would be for a decision maker the same way as Ram's and Shyam parents would have analysed Ram and Shyam separately. So, Ram and Shyam are basically the the alternatives which they have in front of them.

Now, you consider Ram is going to consider placement potential and academic rigor, in the same way and same way you are going to consider the in the what I should say the ranking or the comparison or the alternatives in this way. So, what are the alternatives in the first criteria, for the first level? I am not going to the second level because there was no second level we would not consider that, there are style, cost and fuel economy.

So, when you come, so please pay special attention to the first value which is 1 1, the second cross 2 cross 2 value which is again 1 1 and 3 comma 3 which is the third value which such that if you consider those values they fall along the principle angle. So, what does it mean? And it is very logical. When I compare style to style I give a score of 1 that means, style with respect to style they are same of the same importance. When I rank cost to cost again, I give the same score of 1 is to 1. When I compare fuel economy to fuel economy, I give a score of 1 is to 1 since the score is 1.

Now, let us go to the off the diagonal element. So, let us consider, I will highlight each values accordingly. Let us consider two with respect half. So, what it means that when I consider cost to say for example, when I am going to consider the concept of cost to style and when I am going to consider style to cost, I give a score of a ranking of cost is twice important with respect to style, hence I give a score of 2 and when I am consider a style with respect to cost it will be just the reverse, in the sense I will give a half a score for style while I had given a two score for the concept of cost. When I am considering cost and style cost is giving is on a higher pedestal of a score of 2 while style has a score of half.

Now, when I consider say for example, cost, I am only consider for the timing the cost with respect to say for example, fuel economy. So, for me cost to fuel economy I give a score of 4 to cost and 1 for to score of to a fuel economy, hence when I consider that the concept of cost is coming out to much more important with respect to fuel economy then it has been with respect to style. And when I consider say for example, the concept of fuel economy to style I give a score of one-third to 3 that means, fuel economy is one-third ranking with respect to style and style has a value of 3. Now, let us pause.

The values which are written here are being given by a person, but obviously, they can be some irrationality in the decisions of trying to rank because if you I am ranking A with respect to B, A is higher and when I am trying to rank B with respect to C, where B is higher. Many of the cases it turns out that the ranking of A C is, A with respect to C is such that C comes out to be on a higher ranking for A which is logically not true. So, the table which is see in front of us or the slide is the ranking which has been made by the person and will basically try to combine all the alternatives in such a way that any inconsistency in the ranking would come out automatically as we do the problem.

So, with this I will end the second class for 5th week and continue discussion of AHP later on in the third and fourth class for the 5th week.

Have a nice day and thank you very much.