Data Analysis and Decision Making – II
Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 02
Utility Analysis

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you. And this is the second class in DADM-2, which is Data Analysis and Decision Making-II. And the course has just started, we have completed one lecture, this is the second one. And as you know this is a 12-week course for 30 hours each week will basically have 5 lectures, each lecture being for half an hour. And each after each week, we will basically have one assignment each. So, as you know I am by the way I am Raghu Nandan Sengupta from the IME department, IIT Kanpur.

Now, as you are aware, we were discussing the Utility Analysis in a very simple way. And as you can understand said when we are trying to rank any decision, depending on what the overall utility is there would be other ways of ranking decisions also, but concerning utility is one of the main parameters based on which we are trying to rank decision. We need two things. Number-1 is what is the utility function that is U W depending on the wealth W; and number-2 is basically what is the corresponding probability of that function U W.

And based on and that probability function can either PMF or PDF, which is Probability Mass Function or Probability Density Function, and which should have all the properties for probability we have discussed that in DADM-1. So, based on that we find out the expected value, and we rank them from the expected value from the highest to the lowest and take the decision. Now, as I pointed out, obviously if the expected values are same, obviously you will try to rank them based on their variance. Variance is basically a some sort of loss or negative value. And we will rank them from the minimum to the maximum, and take the minimum one. This point will come up later on also as we discussed.

Another point to remember is that is what if we have to be basically take both the expected value and the variance to consideration, how would you do that? So, in that case the raking system would be if we have both the maximum the expected value and

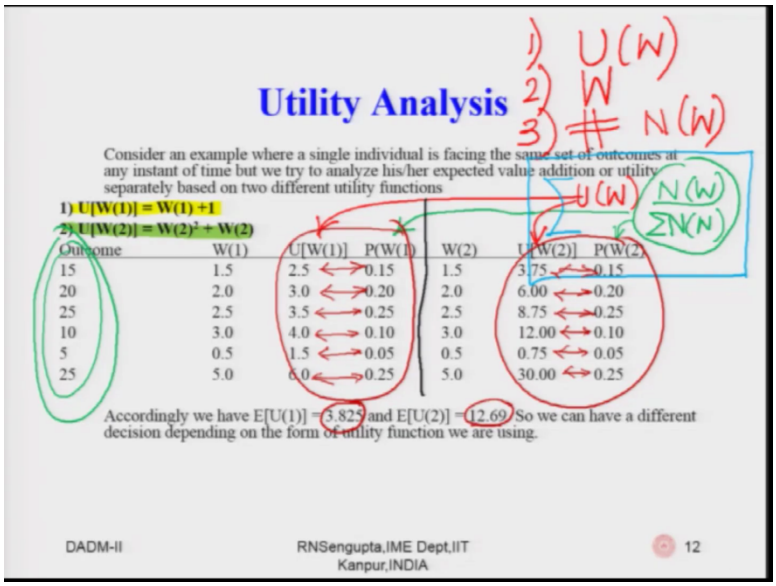
the variance also, we will find out the ratio of the mean to the variance, rank that ratio from the highest to the lowest. Take the one by the values with the highest, highest means the ratio of the mean to the variance. And we take that ratio, which is basically a sort of efficiency where you have putting weights in such a way that the raking is the numerator which basically the expected value, and the variance is in the denominator.

Now, what will happen? If we take the inverse ratio that means, we take the ratio of the variance to the expected value in that case. Variance to the expected value would be just the reverse, in this case we will be see rank name from the lowest to the highest and take it that accordingly and rank them, and take the values for which is it is the lowest value. Lowest value means, the ratio of the variance to the expected value.

Now, there are there are different measures of variance, I am not digressing, but I am just mentioning that variance can be as I mentioned in the finance concept, it can be beta. In another case say for example, it can be the standard deviation or it can be the as meant variance is already mentioned. In many of the distributions for finance, we considered conditional value at least, we consider expected regrade, we consider value at rest now.

So, all these things can be considered as a sort of loss or some sort of variance not exactly variance some sort of variance based on that we try to rank them. And then also we follow the same concept of trying to rank them from the lowest to the highest. If it is only the variance or try to basically rank them for the highest to the lowest, if we take the ratio of the expected value to the variance. And we take the ratio of the variance to the expected value; we will rank them from the lowest to the highest. So, depending on what the ratio we do it. So, without much a due, I will continue the discussion about utility.

(Refer Slide Time: 04:33)



So, let us consider an example. Consider an example where a single individual if facing the same set of outcomes or consider the an example where the individual is facing the same set of outcomes at any instant of time, but we try to analyze his or her expected value addition or utility separately based on two different utility functions. What are the utility functions?

The first one I am just highlight it for first one which I have highlighted using yellow colour is a linear utility function, because you see UW1 1 in the bracket means, the first utility function, it has no numeric concept is basically the first utility function, so that value is W plus 1. So, again W 1 means the corresponding utility functions wealth.

And the second utility function let me use a different colour, it may light a one let three in one. So, the second utility function is again is a quadratic one that is W 2 in that the bracket the 2 is basically the nomenclature to define the wealth. Obviously, the wealth values W1 and W2 can be same, but is the nomenclature to define, so that is W square plus W, and the utility function is denoted by UW 2 just to differentiate that.

So, now concentrate on the table which is shown. So, the outcomes are given in the first column. So, again if you remember the outcomes are corresponding to the values, now the outcomes concept, and what we did in the example for the football match or for the machines a and b, those outcomes are an intrinsically different. In that case outcomes were given per say as the utility of the value, which was coming out from taking the

decision. But, here in this case the outcomes are the corresponding values of W based on which we will try to find out the utility.

So, even if the nomenclature of the column is same outcome, but they have different connotations. So, just I wanted to mention that, so be careful about that. So, the outcomes are given as 15, 20, 25, 10, 5, and 25. Again remember these values of 25 can basically are two different instances where the wealth is being invested, and you will have different out utilities based on that.

So, the wealth's are given the this, this outcomes which I will denote later on also. They can have a third meaning, so it is like this. In the first case outcome, it will be specified depending on how the problem is the outcomes are the corresponding values of utility. So, it can be I will just write it down, so outcomes if we mean we can mean UW also. So, obviously the final value. So, wealth is not given utility function is also not given only the functional value of utility is given, so that is the outcome so that is case-1.

In case-2 depending on the problem. The outcome can also mean W that is the wealth. Generally, we will denote it by wealth or amount of money, but it can be the denoted by outcome also depending on the problem. In the third instance, it would depend basically denote the number of such happenings as number of the experiment or numbers when you basically play a game, take a decision those values would basically be denoted by NW such that given NW for all the which are basically the outcomes, we can find out the corresponding relative frequency, we can find the corresponding probability that means, we are able to find out the probability mass function all the probability density function. So, outcomes would have different connotations accordingly.

So, coming back to this problem. So, this point 1, 2, 3 which I have noted down on this slide would come up again, and it will become very apparent as we as we solve the problem. So, the wealth W1 basically gives you the wealth which is being invested by the person corresponding to the fact that utility function one is valid. So, there 1.5, 2, 2.5, 3, 0.5, and 5.

And the corresponding values I will just check it, so there is we have to check this value just wait when we do the calculation, we check it. And the corresponding utilities would be given by this is fine, so you will basically have 1.5 plus 1, which is 2.5, 2 plus 1 is 3,

2.5 plus 1 is 3.5, 3 plus 1 is 4, 0.5 plus 1 is 1.5, 5 plus 1 is 1 is 6. So, this is right should we erase it.

So, the utility functions UW1 are given, which is the third column where I am hovering my point. Now, obviously there would be corresponding probability. So, when you find out the explicit value of the utility, you remember we multiplied by two terms one was utility, which is already here. And one was NW by summation of NW, which this value has to be found out corresponding to the fact that the fourth column PW1 would basically give the problem, because as the relative frequency.

So, what we need to do later on, I will just put another colour combination you have to basically sum it up. So, our main interest is I am using different colours to in order to make you understand. So, the red colour is basically the utility corresponding to the W value, which is given in second column. The second colour which is the green is basically relative frequency, which is given on the 4th column. And the summation we have to find out based on which we will basically rank them.

Now, from where does the relative frequency of the property comes. So, these outcomes which are given the outcomes which are given are utilize to find out the relative frequency. So, they are basically number of outcomes which are favorable. So, 15 divided by total sum or 20 divided by total sum or 25 divided by total sum all those values are given in the fourth column. So, those corresponding values are because the total sum is 100 is the ratios are 0.15, 0.20, 0.25, 0.10, 0.05, and 0.25. So, once you do this, so this is the first set of calculation. So, you will basically be multiplying utility with the probability. So, once you multiply the value comes out to be 3.825 is given.

Now, let us go to the second utility function. So, with this I will erase all the colours, and then proceeds. So, it will be so we use summation as rather than erasing I will just point it out again. So, it is not difficult for you, so that we use the colour red. So, these utilities would basically come from here. The wealth's again remember the wealth's have been purposefully kept at the same values that is W2 and W1 are same in order to make an ease of comparison.

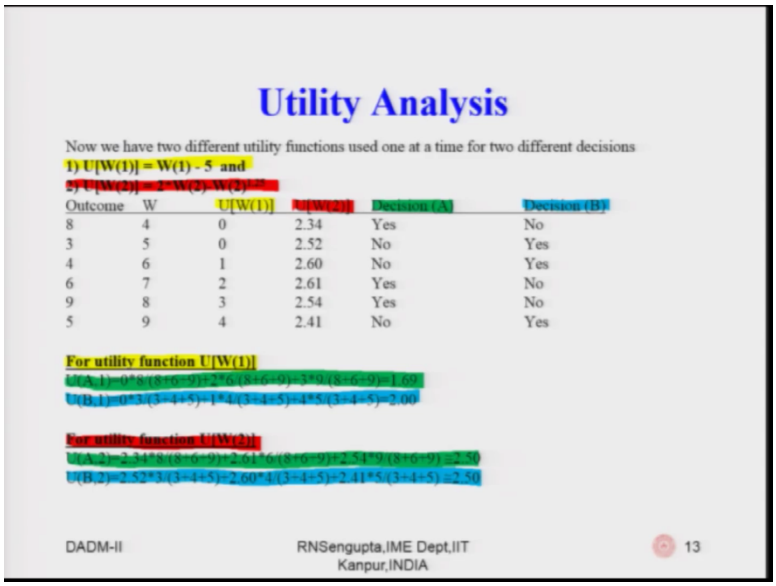
The green values which is the ratio or the relative frequency is given by this, and the summation obviously remain the same. And these outcomes again I purposefully again taking them and to its same. So, the ratios also that is the last column probabilities 0.15,

0.2, 0.25, 0.1, 0.05, 0.25 are the same. So, again these are calculated using this. So, I have just circle it twice in order to make you understand that they are being used in both the calculations of for UW 1 and UW2.

And finally, when we multiply I will be just note it down, so we multiply these values they are utilized and the value comes out to be twelve point twelve point 12.69. So, the utility expected utility based on utility function two, which is a quadratic one comes out to be about 30, I am just finding out the integer values. And the expected value of the utility based on the first utility function, which is the linear one comes out to be four. So, we can have a different decision depending on the utility function, which is there that means, a person can take different decisions as the utility function changes.

So, obviously you would may ask that we could have different wealth's also. So, the reason for taking the wealth I am again repeating it, it is basically for ease of comparison. And the outcomes are also same for reason ease of comparison. But, in reality the wealth's can be different, the outcomes can be different, here the outcomes again I am mentioning are the number of outcomes, which are favorable for any decision.

(Refer Slide Time: 14:57)



Now, we have two different utility functions used one at a time for two different decisions being made, and they are like this. Again I will highlight the first one was yellow, so I will use the same coloring scheme. So, the first utility UW1 is linear W1

minus 5. And the second utility function, I will highlight using red is a not a quadratic one, but it is non-linear because W to the power 1.25 and again the outcomes which are given on the numbers, which support a decision.

Again I have taken this numbers in such a way such that later on we can differentiate the decisions based on utility-1, utility-2, you will see it very soon. But, ease of comparison I take the wealth's are same which is the second column. The wealth values are 4, where I am just pointing out 4, 5, 6, 7, 8, 9 just for ease.

And the utilities are taken correspondingly. So, if you consider utility-1, so for wealth of four it will be 4 minus 5, which is minus 1. If you remember we have taken or considered that if it is negative we will consider at 0, so 4 minus 5 is minus 1 it will be taken as 0. 5 minus which is now we are going to the second row, so for the second column. So, 5 minus 5 is 0, it is 0. 6 minus 5 is 1, which is 1. 7 minus 5, it is 2, it is 2. 8 minus 5, it is 3, it is 3. 9 minus 5, it is 4, it is 4. So, the utilities based on utility function one are given in the third column.

Now, if you consider I will come to the decisions later for, let me first consider the utility values. So, if I considered utility function-2, which is highlighted by red. The values are given in the 4rth column, and how would you find that you will basically find out using the wealth, which would be if I consider the first row, it would be 2 into W2, which is 2 into 4 minus 4 to the power 1.25 value comes out to be 2.34.

Second value would be 2 into 5 minus 5 to the power 1.25 comes out to be 2.52. Third value comes out to be 2 into 6 minus 6 to the power 1.25 comes out to 2.6. The fourth one would be 2 into 7 minus 7 to the power 1.25 comes out to be 2.61. The second last one comes out be 2 into 8 minus 8 to the power 1.25 comes out to be 2.54. And the last value comes out to be 2 into 9 minus 9 to the power 1.25 comes out to be 2.41.

So, the utilities for the second utility function utility values for the second utility functions are given in the column 4. So, I will just highlight them with using different columns colours. So, this is the utility for the first one or I should basically use the same color it would be easy my mistake sorry. So, so this is for the first one, this is for the second one.

Now, if you remember I mentioned that I would not be considering the decisions immediately. The reason was the (Refer Time: 18:31), it would become clear to you. Concentrate on the last two columns. So, it gives decision a there are some yes no yes no it continuous both for the last but one column in the last column.

Now, it means that if decision a is yes, it means that if you take that value based on the utility, you get a positive outcome. And if it is no which means that even if you are investing some money, the outcome is basically 0 or taking a decision outcome is 0. Obviously, in that case when you multiply the corresponding properties with the utility function, actually the out the total value of that multiplication values that is utility into the corresponding probabilities would be 0, it means that for that particular decision the corresponding probability mass function would be 0, because these are mass functions only they are not continuous distribution functions.

So, if you take decision A if you see the column second last one, it is yes, no, no, yes, yes, no which means for outcome 8, it is possible under A outcomes 3 and 4, it is not possible; outcome 6 and 9 is possible under A; and outcome 5 is not possible under A. Now, corresponding if you move to decision B, again you will see it is given as no, yes, yes, no, no, yes the corresponding logic would be for outcome 8, 6, and 9 the decision under B is not possible while outcome 3, 4 and 5 under decision B is possible. Now, within information we will try to find out the expected value, how we do that.

Let us come one by one. So, now they would technically be four combinations why four for utility-1, I will consider both decision A and B at one time, then again for utility function-2, I will consider decision A and B another time. So, they would be 2 into 2 4 decisions. So, let me will be come to this. Consider utility-1, so we utility 1 we are using yellow colour, so I will highlight it. And let me utilize the utility values let me use a different colour for utility a decision A and decision B. So, it can one can be green, so this is green. So, I will highlight this, I will come to a different colour for decision B.

Now, if you look, it was yes for 8, so obviously the corresponding utility for that particular decision would be what is the utility about 0, it was here I would not highlight it, I will just mark it is 0, multiplied by the corresponding value which you have. So, how many of the total outcomes are corresponding to 8 it will be eight which comes in the numerator divided by out of the total one of 8, 3, 4, 6, 9, 5 only 8, 6, 9 are favoring

decision A. So, obviously the corresponding probability will be 8, which favors that particular first outcome divide by the total number of outcomes which is 8 plus 6 plus 9.

Similarly, if I go to the second decision, which is favorable under decision A so, 3 and 4 are not possible, so we go to basically 6, so it will be 6 divided by 8 plus 6 plus 9. Similarly, if I go to the second last one, which is 9 9 is favorable, so obviously the corresponding probability would be 9 divided by 8 plus 6 plus 9. And obviously, for both these probabilities we will multiplied by the corresponding utility values, which would be 2 and 3, which you can find out from the value which is there in the third column. So, the value which comes out for the total experiment value is 1.69, which is here. I am just covering my pointer over 1.69.

Now, when I go to decision B, let we use a different colour this is only for utility-1. So, the corresponding utilities we know as is 0, 1 and 4, why it is 0 and 4, because there are 3 yes and 3 no's yes are for 3, 4 and 5. So, obviously, the corresponding values would be 3 divided by 3 plus for plus 5. Next would be 4 divided by 3 plus 4 plus 5. And the last one would be 5 divided by 3 plus 4 plus 5.

And what are the corresponding utilities, utilities are if you see the corresponding column value which is basically now the fourth one they would be sorry sorry, because we are considering utility one. So, we will not go into the utility 2 two-fourth column, we still are in the third column. The third column values corresponding the utility for it is yes, yes, yes for the last row are values 0, 1 and 4. So, when you multiply the corresponding probability in the utility sum them up, the value comes out to be 2, which I am hovering my pointer. So, if you rank them under this under utility function-1, decision B would be highly based on a higher level than A.

Now, let us change a scenario, we come into the utility function-2. So, the values I will just utilize the colours, so it is so this is the decision A, decision B I already colored it, so it is easy for me to explain. So, if I go to utility-2 corresponding to the decisions A and B. The corresponding values are if you see yes, yes, yes and no, no, no for the corresponding last column and the second last column.

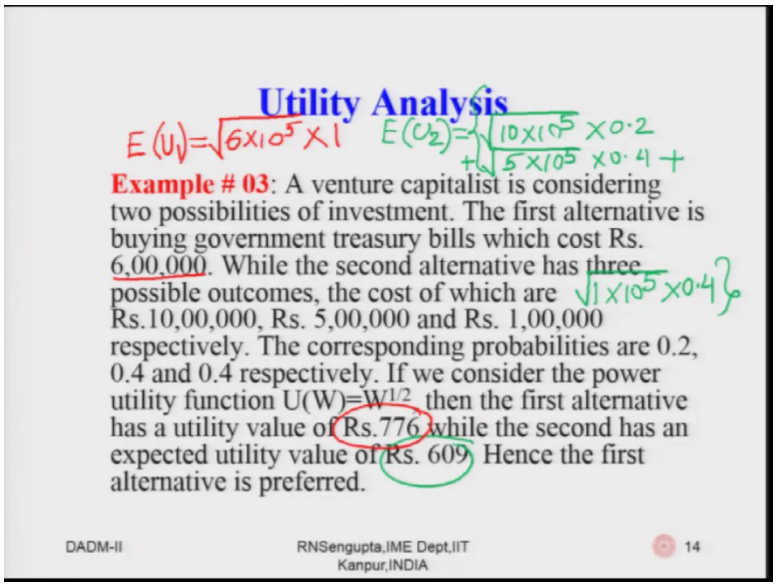
Then you multiply the values, the values would be 2.34 would be for the fourth column if you see this value multiplied by the value, so it is basically for decision A it is yes. So, it will be 2.34 into 8 divided by 8 plus 6 plus 9, then the next one would be 2.61 here into

6 divided by 8 plus 6 plus 9. And the last one would basically be 2.54 into 9 divide by 8 plus 6 plus 9. The value comes out to be if you multiply the probability and the utility values and sum them up comes out to be 2.5 approx.

Let us go to decision B. So, again the utilities you will find out from the fourth column. So, these values are 2.52, because it is yes. The next values 2.6, which is yes. And the last value, which is yes which is 2.541. And the corresponding probabilities are so if you consider only the yes, yes, yes the probabilities are let me repeat it for the all three values consecutively. They would be 3 divided by 3 plus 4 plus 5, then 4 divided by 3 plus 4 plus 5, and 5 divided 3 plus 4 plus 5. The total value if you find out, by multiplying the probability into the utility comes out to be about 2.5.

So, this case under utility-2, you are indecisive between decision A and decision B. I took a little bit longer time in trying to explain, but it was the main class of the matter was that depending on the utility, depending on the decisions, depending on the probability, depending on the outcome you can have different ranking systems, but an and these values of probability, utilities, outcomes would differ depending on what the example is being stated as.

(Refer Slide Time: 26:41)



So, let us consider as very simple example. A venture capitalist is considering two possibilities of investment. The first alternative is buying government treasury bills which cost rupees 6 lakh. While the second alternative has three possible outcomes the

cost of which are 10 lakh, 5 lakh, and 1 lakh corresponding probabilities are 20 percent, 40 percent, 40 percent, so the sum of the probability is 1 and it is a property mass function.

Now, let us consider power utility function, which is W to the power half. And then let us find out what are the decision based on the government securities and the decisions which has three outcomes correspondingly. So, the corresponding values are like this.

So, if you consider the first one so obviously the utility would be so if you consider let me, so the first one is 6, and 6 lakh I would not use the 0 6 lakhs. And the value and the probability is one, because there is only one decision. So, the utility let me use as U1 suffix one in order to make you understand. So, it will be because the wealth is six lakh, and the utility is power function.

So, it would be 6 into let we use the zeroes in order make you understand, and let us make no confusion. So, it will be square root of that multiplied by one this is the probability. So, the expected value let me write the expected value of U1 is this, why the expected value of U2 I am using a different colour so what are their three outcomes.

So, it will be a root of 10 into 10 to the power 5, because this is 10 lakhs into what is the probabilities probability is 0.2 plus 5 into 10 to the power 5 into 0.4 plus let me write the last value here this or rather than cluttering it. The last value would be square root of I will try to overcome this in the next class. So, it will be 1 into 10 to the power 5 into 0.4 find out this expected value find out this expected value.

So, this green one comes out to be 609 while the government one comes out to be 776. So, based on that you will obviously would take the government one, because utility is much more. So, this is simple example we make it a little bit more complicated easy proceed, but we will try to utilize the concepts of utility and trying to basically go into the later part of the multi criteria decision making, and the multi attribute utility theory.

Thank you very much for your attention, and have a nice day.

Thank you.