Data Analisys and Decision Making - II
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Lecture - 17
DEA

Welcome back my dear friends, dear students a very good morning, good afternoon, good evening to all of you. And, as you know this is the DADM II which is Data Analysis and Decision Making II course under NPTEL MOOC series. And, this total the course is for 12 weeks which is for 30 hours and each week we have 5 lectures is being for half an hour. So, we are in the 17th class, 17th class means we have already completed 3 weeks and in the 2nd class over the 4th week. And, as you know we were discussing about data envelopment analysis; data envelopment analysis being non-parametric technique where you basically use a very simple concept of linear optimization concept and the.

But based on the fact that the initial problem has been formulated as a ratio of input output to get an efficiency based on the fact that whether is an input oriented model or an output oriented model. And, then you basically try to solve it using optimization problem, but before that you convert it very simply into a linear programming problem very simple case.

And, if you remember I have been mentioning time and again that whether you take the efficiency or the inverse of the efficiency you have to either maximize or minimize depending on how you look at the problem. And, if it is an output oriented model or a input oriented model you basically formulate in such a way that you normalize either the numerator where the numerator it is basically the output or normalize the numerator whether numerator is the input; depending on how your trying to analyze.

So, if it is input being normalized; that means, you will fix input at a level of 1 and try to increase the output and if you basically give the output fix at the level of unit 1 normalize it then you are basically trying to reduce the input. Now, we were discussing the problem the last class the formulation of a problem when they were basically 3 DMUs K is equal to 1 2 3 capital K is equal to 3. They were basically 2 inputs and there was only 1 output and we are formulating it using the concept of output oriented problem. Now, we will basically consider the input oriented model.

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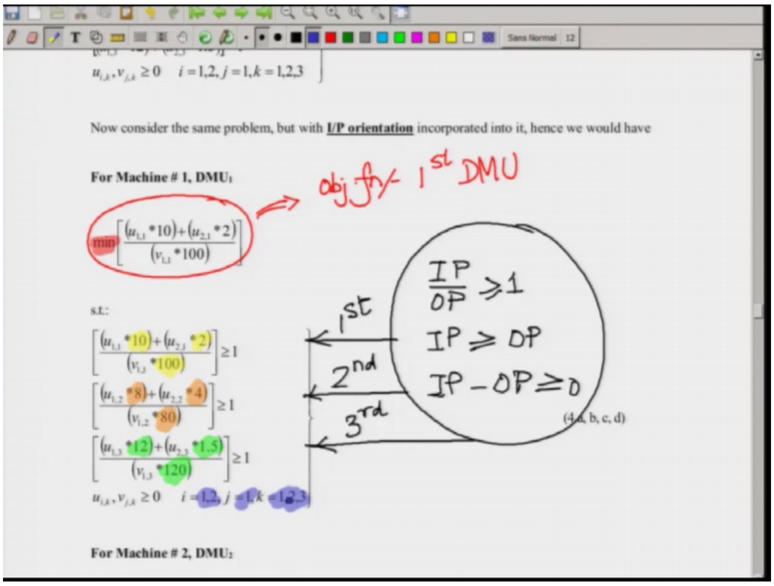

So, I will follow the same scheme of coloring so, it will be easy for you to understand. So, consider that you have an output oriented model in input oriented model. So, in input oriented model what you are trying to do is basically trying to minimize and earn a ratio. Why you are trying to minimize the ratio, because you are trying to take the inverse of a efficiency and taking the inverse of the efficiency, it would mean that you putting the output and the input in such a way that the input would be in the numerator and the output would be in the denominator.

And, try to basically normalize the output of the level of 1 and try to basically decrease the input as low as possible to minimize it. Because, minimizing would basically bring down the numerator as low as possible; that means, it will actually increase your efficiency for the DMU.

So, for as there are basically 3 DMUs that is DMU 1 2 3 which are 3 machines. So, will basically format the problems accordingly because, formulation would be more important and then solving it would be very simple optimization problem. So, in the 1st model this you have the objective function and this is minimization so, remember that. So, I will try to highlight the minimization even though I did not highlight it the maximization in the last class, but I will try to highlight the word minimization in each and every step. So, I will basically highlight it using the red color. So, I am trying to basically minimize and the minimization would be done for the case. So, this is the objective function for the 1st DMU for this 1st DMU so, this would vanish.

So, this is for the 1st DMU and what do you have for the constraint. So now, if you remember for the input oriented model the constraint would basically formulated according. I will write the general the way how you formulate or write the constraints; it would basically input by output was greater than 1. So, in the output oriented model, it was actual efficiency which is output by input less than 1. Now, I should use a different color there I was using the black one my apologies please bare with me. So, it was basically the input divided by output is greater than 1, then input is greater than output, then input minus output is greater than 0. So, this was general one and your 3 DMU is so, there will be 3 equations which are equation 1, equation 2 and equation 3.

And, these are corresponding to the 1st DMU the 2nd DMU and the 3rd DMU. Now, if you consider the objective functions (Refer Time: 06:14) and if we consider the constraints. So, I will basically use the coloring scheme as same as I did for the output oriented model. So, for the 1st DMU the input and outputs would be denoted likewise. So, this is the combination of the input and output; the 10 and 2 are the corresponding units which are basically or the conversion rates which are being used for the input. And, that 100 is basically the conversion rates is being used for the output. Similarly, when I go to the 2nd DMU I use the coloring scheme as orange which I have already done. So, correspondingly 8 4 and 80 are the conversion rates of the units for the inputs which are 2 inputs and for 1 output..

Similarly, when I use a coloring scheme green which I had done it in the last class. So, 12 1.5 and 120 are the corresponding conversion in which are or multiplicative factors being used for the input inputs and the output. And the weights which will basically u i k and v j k where, i is basically the number of inputs ranging from 1 to capital I, j the number of outputs ranging from 1 to capital J or m and n which ever you denote we I think we have taken m and n. So, m would be the number of inputs so, i would basically range from 1 to capital M, j would basically range from 1 to capital N and k which is been basically the number of DMUs would basically range from 1 to capital K.

So, you had 2 inputs 1 output and 3 DMUs means we are in the same color scheme the dark blue. So, this is the number of inputs number of outputs and the DMUs. So, we have been able to at least give the initial formulation for the for the objective function being there for the 1st DMU and this being for the 1st optimization problem for the 1st DMU and this is an input oriented model. Now, let us go for the 2nd DMU which is 2nd machine again we use the same coloring scheme.

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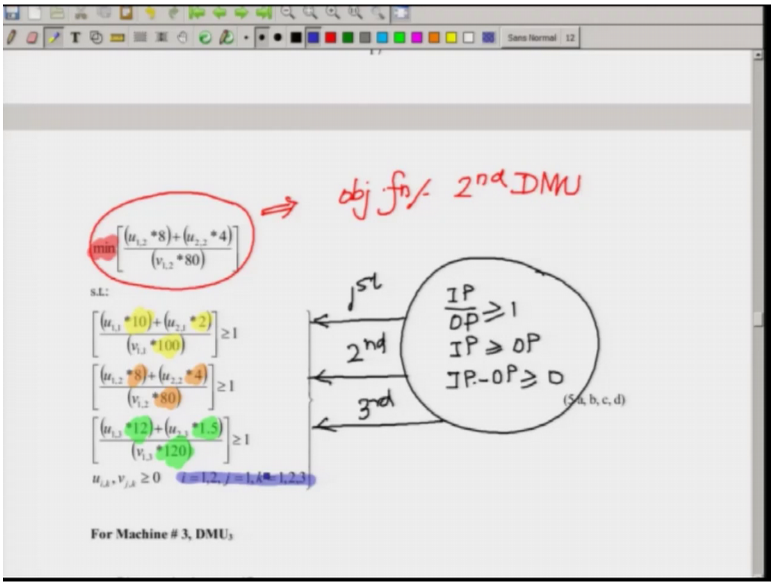

So, this is the objective function. So, we will mention it very explicitly; objective function for the 2nd DMU and this is minimization problem. So, I should highlight it using the red color which I have not done in the last class, but I am trying to make it much more explicit. Now, if you go it is basically the input by output being greater than 1 input and greater than output because, I am taking to the right hand side. So, input minus output is greater than 0. So, these 3 equations are coming for the 1st DMU for the 2nd DMU and for the 3rd DMU. So, this is for the 1st this is for the 2nd and this is for the 3rd.

So, you have basically given you the same way how I denoted for the 1st DMU. So, this the 1st 2nd 3rd then when I again come to the coloring scheme the conversion factor of 10 2 100 are the respective 1st two being for the inputs and the last one being for the output for the 1st DMU. Then 8 4 18 are the corresponding units conversion factors multiplicative factors for the 2nd DMU corresponding to the inputs which are for the 1st two which is 8 and 4 and the last 80 is for the outputs and then when I use the last coloring scheme.

So, the multiplicative factors are 12 1.5 and 120 corresponding to the 2 inputs and 1 output for the 3rd DMU. And, again the weights are given by u n v with the corresponding suffix to with respect to i k and j k, i is basically the nomenclature being used for the inputs j being the nomenclature being used for the outputs and k is for the DMUs and again here 2 inputs 1 outputs and 3 DMUs.

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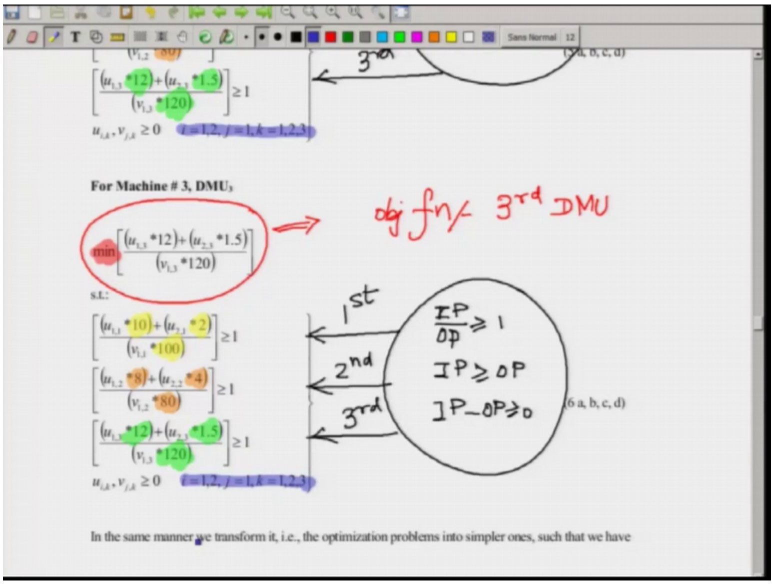

Now, let us go to the 3rd DMU formulation. So, this is the objective function for the 3rd DMU and again is red color highlighting the fact that is minimization problem. Now, actual formulation is input by output in greater than 1 input greater than output input minus output is greater than 0. So, they are being utilized for the 1st constraint which is for the 1st DMU. So, 2nd constraint which is for the 2nd DMU and the 3rd constraint which is for the 3rd DMU so, this is the 1st, this is the 2nd and this is the 3rd. Now, if I consider again the same coloring scheme sorry this I should use a highlighter.

So, the 10 2 and 100 are the corresponding 1st two being for the input conversion factor for the weights of whatever you try to denote for the 1st DMU and inputs and for the 100 being the 1st DMU is outputs. Correspondingly 8 and 4 are for the inputs for the 2nd DMU and 8 is basically the output related conversion factors for the 2nd DMU.

Similarly, 12 1.5 120 are the corresponding inputs conversion rates 1st two 12 1.5 and 120 being from the outputs; then again the weights which you want to find out are given by u and v with the corresponding suffix. So, and you know the color is basically dark blue. Now, this was an initial formulation; now I will basically utilize both the input oriented model 3 formulation for the case when we convert them into the actual linear optimization problem.

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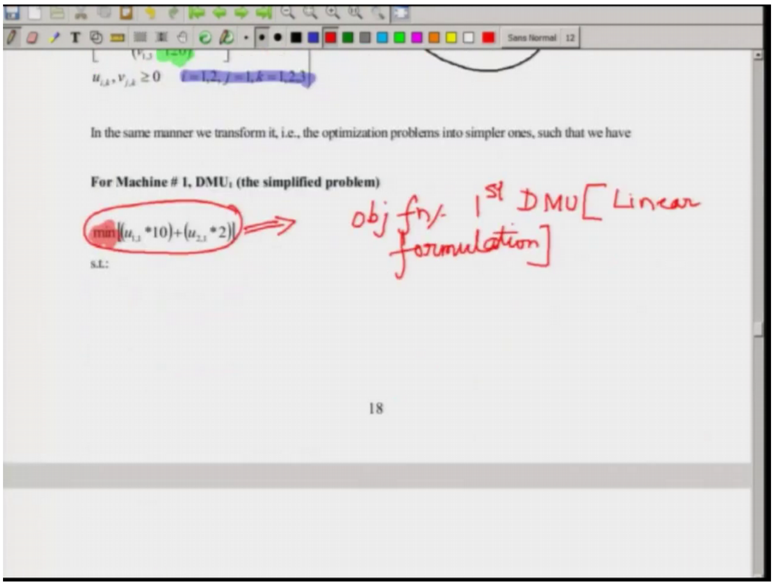

So, I will I will go one by one 1st DMU 2nd DMU 3rd DMU. So, if you remember I have repeated it again and again, but I will still repeat it in order to make things much clearer for you. For the DMU 1 we had basic assembly for DMU 2 and DMU 3 we have taken the case value your trying to minimize; minimize the overall factor which in the numerator you have the input and the denominator you have the output. Now, we are forcefully putting the outputs as normalized as 1 and try to minimize the input as low as possible for all the 3 DMUs. So now, based on this fundamental principle we will again give the newer version of the optimization problem which will now be linear; so, let us go to that.

So, if you remember the numerator had the leave the minimization word; in the numerator you had basically the base multiplied with the conversion factors for the input for the 1st DMU. So obviously, we will try to minimize that corresponding to the fact that the output which was in the denominator is being forcefully made as 1 and that will come later on. So, this will be the minimization 1. So, this is the objective function which I will denote, but in this case remember this is linear formulation; this is the linear formulation which is doing for the 1st DMU. So, because we have done away with the denominator.

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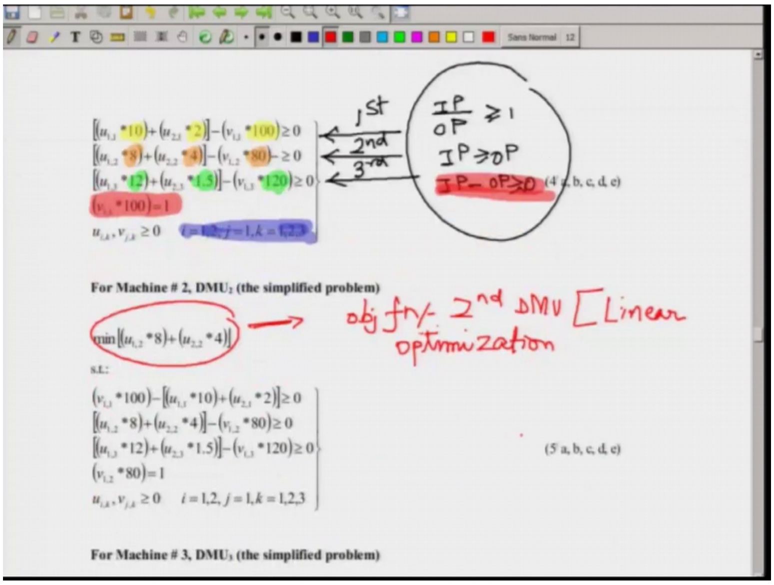

Now, corresponding to the fact that, we have been able to convert them the constraints. So, the constraints has already been given by the formulas; you remember I just write it down again this is 0. So, this part the last part actually it is now being utilized to convert the constraints. So, this is the 1st this is the 2nd this is the 3rd there would be one extra one; I will come to that. So, this is the 1st which was already there we are just converting to the linear part and actually we are utilizing the one here. Why I am highlighting it because, like in the minimization word this is also important that I am trying to utilize input minus output is greater than 0.

And, I am trying to utilize for all the 1st constraints where am just hovering my electronic pen. Now, if you remember so; obviously, let me come to the fact. So, the colour schemes are 10 2 100 for the combinations of input output of the 1st DMU. 8 4 80 is the combination of input ouputs for the 2nd DMU this is the conversion rates and, 12 1.5 120 is for the conversion rates for the 3rd DMU. Now, the denominator which has been forcefully put as 1 is now coming up as an added constraint corresponding to the fact that I am using the 1st DMU to minimize it so; obviously, this will come here. So, this part is the extra 1 which am getting corresponding to the DMU 1.

And obviously, they would be even if you formulate for DMU 2 DMU 3 they would be such problem formulation coming at the constraints; I highlight it with same red color and as usual i is equal to 2 i is equal to 1 and 2. So, their 2 j is equal to 2 1 k is equal to 1 2 3 and the weights are u and v. Now, let us come to the 2nd DMU; I will use the same coloring scheme again. So, this is the objective function for 2nd DMU, this is the linear optimization. And, use the same word what I would let me see what I will do; it was linear optimization of formulation. So, I should remove this word I should remove this; what I should use formulation.

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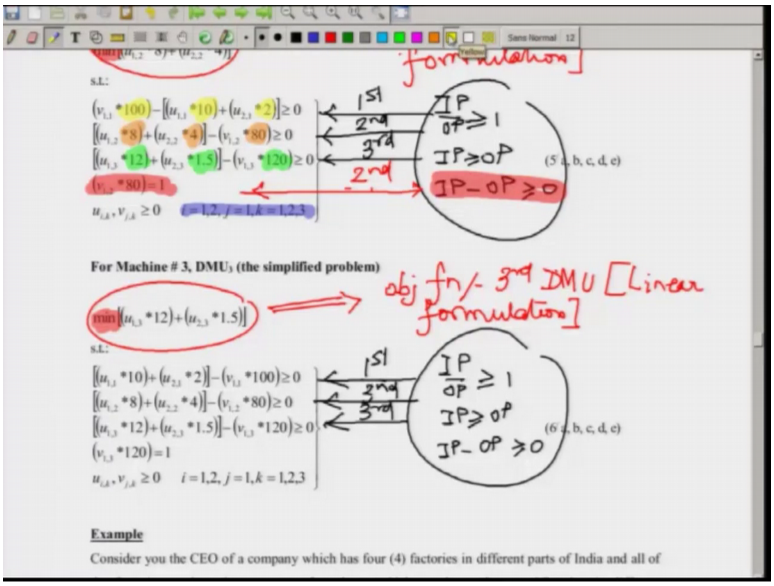

And here the constraints for the 1st P it was input output greater than 1 input greater then output input minus output is greater than 0. So, I have converted that accordingly. So, these are being utilized for the 1st the 2nd and the 3rd and the last constraint which has marked by red color this one is coming from the fact. So, if you see this formulation this formulation they are coming from the fact, that I am considering the objective function corresponding to the 2nd DMU the input to normalized to a factor of 1.

So, this is the 1 and again the corresponding color schemes means you know for the 1st DMU orange for the 2nd DMU green for the 3rd DMU. So, as I am doing it watch the color scheme whether it is exactly the same and i is equal to 2 j is equal to 1 k is equal to

3 and then let me come to the 3rd DMU. So, this is another thing which I should have done it.

So, it will be make things clear for us I should have highlighted word minimization here. So, I should highlight the word minimization here also. So, this becomes objective function 3rd DMU which is linear formulation to the linear formulation which has been done. And, if I consider again same scheme input my output greater than 1 input greater than output input minus output is greater than 0.

So, these 3 have been done using the corresponding concepts. So, this is the 1st this is for the sorry sorry this is for the 1st, this is for the 2nd this is for the 3rd. And, I should also mention because I did not mention let me check how I can do it; I should have mention in the DMU 1 2 3. So, technically this one I need the color red. So, this was basically coming on from here which is also corresponding to the 3rd. So, this is the in the objective function the denominator is been shifted here because we are normalizing it; similarly this should have been there. The 3rd no my mistake my mistake just wait on wait this would be for the 1st, this is the 1st DMU similarly this would be for the 2nd.

So, 1 2 3 are all the concern existing both in a non-linear part and the linear part; only the added on constraint which is coming up is corresponding to the DMU where we are trying to optimize. Similarly, the color scheme are yellow for the 1st then orange for the 2nd, 1st 2nd means the DMU green being for the 3rd. And, this color of red is basically the added constraint which is coming corresponding at the fact that is coming for the 3rd DMU.

And obviously, u and v are the weights and the coloring schemes remains the same which is i is equal to 1 2, j is equal to 1 and k is equal to 1 2 3. I will just discuss one problem and then going to the objective formulation for this input output oriented model.

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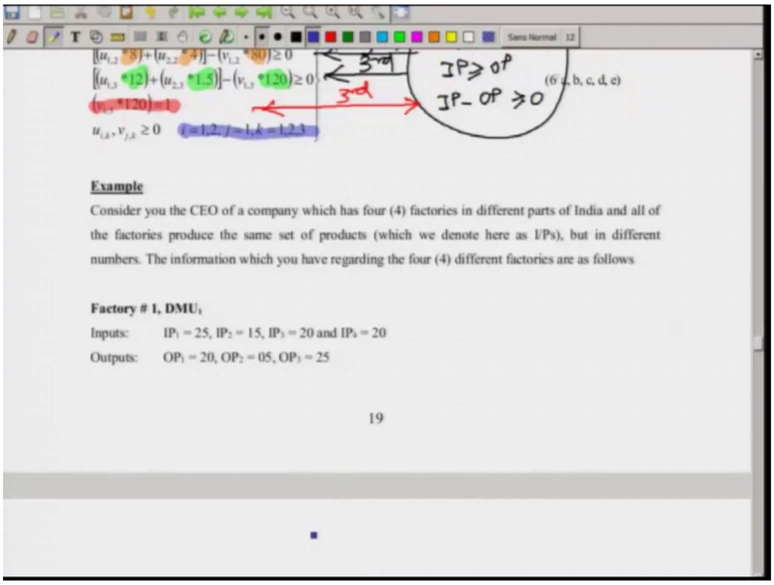

So, consider the problem is considered that you are a CEO of a company which is four factories in different parts in India and all the factories produce the same set of products which we denote here by the inputs, but in different numbers. So, they are given as follows. So, you have for factory 1 which is DMU 1 you are basically 4 inputs 3 outputs. So, i would change from 1 2 3 4 j will change from 1 2 3 and; obviously, k will change from 1 2 3 4 because there are 4 DMUs.

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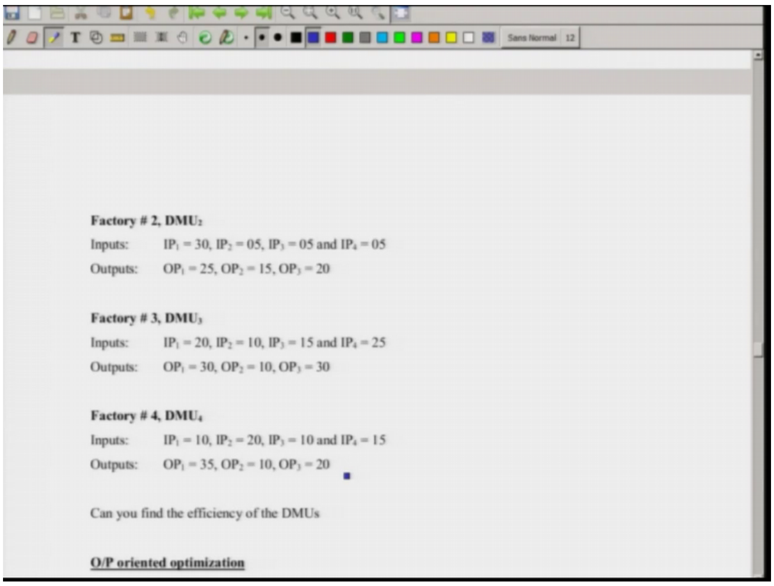

Similarly, for 2nd factory inputs are given I am not reading the values. So, inputs are given I mean again 4 inputs and 3 outputs and this is the 2nd DMU. Similarly, when you come to the DMU 3 inputs are 4 in number outputs are 3 in number and for the 4th DMU it is an inputs are 4 in number and outputs are 4 in number. So, I will just formulate it very fast; once you understood I will do just spend 2 minutes or 3 minutes of the formulation for this problem in the next class. And, then immediately go into the other topics related to DAE in simple format and then come to the solution later on when we do the simple optimization problem.

With this I will end this this 2nd class in the 4th week which is the 17th class and continue discussing more about DAEs in the later part.

Have a nice day and thank you very much.