Data Analisys and Decision Making - II
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Lecture – 16
DAE

A warm welcome to all my dear friends, a very good morning, good afternoon, good evening to all of you and as you know, this is the DADM 2 which is the Data Analysis and Decision Making 2, lecture series under NPTEL MOOC and this total course is for 12 weeks which is 30 hours and each week we have five lectures, each being for half an hour. And after each week we have assignments. So, we have completed three weeks and we are going to start the fourth week which is the sixteenth lecture. And my name is Raghu Nandan Sengupta from the I M E department, IIT Kanpur in India.

So, if you if you remember we were discussing about data envelope analysis as a non as a parametric tool. But it is a nonparametric tool in the way you utilize that there is the parametric tool because there is a equations you have optimization problems. But when you are trying to utilize it, actually it would try to convert some of the subjective ideas. So, into some objective functions with their constraints and solve them accordingly. Like if you remember the problem, the ideas which I gave you of trying to compare hospitals, trying to compare schools, try to compare corporations, municipal corporation, try to compare government, states so and so forth. There would be many subjective and objective criteria's. We will try to combine both of them such that the efficiency would be utilized to compare them.

Later on, we considered three different types of way of trying to analyse them; one was output oriented model, input oriented model and one was the combination of output input. So, in the output oriented model, basically or in the input oriented model, the main focus was that when you are trying to find out the efficiency, it is basically the ratio of output to input. So obviously, we will try in that case you will try to maximize the efficiency. Subject to all the constraints, what are the constraints? I will just repeat it in few minutes. And in the other way, when you are trying to take, consider the input oriented model. Your main effort would be to minimize the ratio of the input to the output and subjected to some constraints. The type of constraint in both the cases would be the same.

Now, when you consider the output oriented model or the input oriented model, your are considering respectively the facts that in the output oriented model, you will keep the input fixed at a normalize it and level of one or one unit. And in the input oriented model, you will keep the output fixed at a level of one unit normalized. And based on that, hence you are trying to maximize in the output oriented model and minimize in the input oriented model; maximize the output function and in the second case, minimize the input function. And the subject to constraints would be are the corresponding ratios for the k number of DMU s, in the output oriented model it less than type; in the input oriented model, it would be greater than type. And you will try to basically use this fact for the initial optimization problem idea.

Now, as I had mentioned it I mean again I am mentioning, these are always non-linear problems. In order to convert the non-linear problems into linear part, you will basically as I just mentioned few minutes back, you will basically normalize the input to 1 for the output oriented model and normalize the output to 1 in the input oriented model such that the objective function respectively now changes to maximization of a linear function in the output oriented model and minimization of a linear function in the input oriented model. And the subject do not constraints now also becomes linear functions of less than type for the output oriented model and greater than type for the input oriented model.

So, this basically becomes a simple linear example. Now, considering that we had three machines, we will consider these three machines as the DMUs which is decision making units DMU 1, DMU 2, DMU 3.

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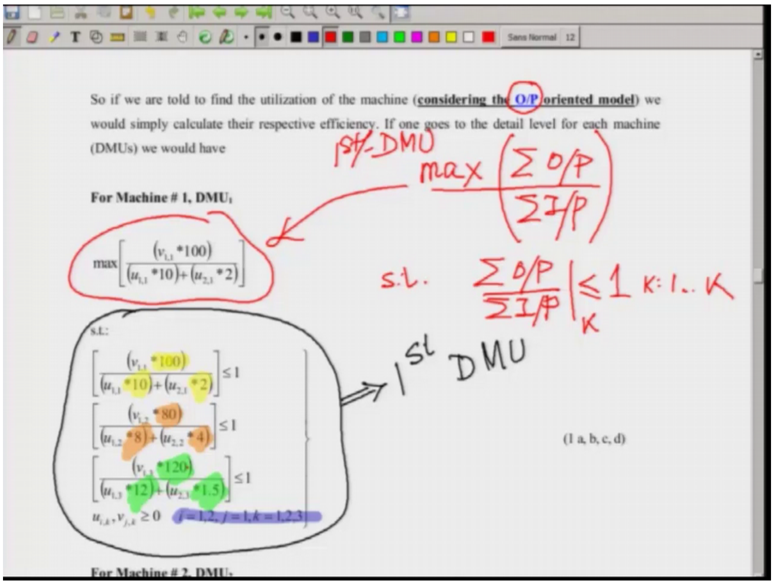

So, if you if you consider it, so,, so if you are told to find the utilization of the machine for the problem which you have just discussed the in the fifteenth class, a lecture, considering the output oriented model, we would simply calculate the respective efficiency. So, when is an output oriented model, this case you will basically maximize.

So, what you are doing? I will just write it here and then come back to the equation. So, you will be trying to basically maximize the summation of the outputs. I am not putting the summation to what limit to what limit because that is already explained. If you remember, there were the three subscripts i, j, k; i being for input, j being for output, k being for the DMUs. So, so once you consider this, this is the function because there you are multiplying the weights v 1 and this being done for the first machine, similarly for the second machine, third machine so on and so forth.

So, the number of hours was given input as 100, output was given as units of 10 and 2 and the weights were given as with u, variable and the suffix was u 1 and u 1 2 comes corresponding to the u 2 1 corresponding the fact that you are considering the inputs as 2 and the DMU is 1. Now, when you come to the actual formulation for the problem considering it is an output oriented model, what you actually had again for the first DMU you remember. This will change corresponding to how many DMUs you have. So, it was basically the subject to constraints were their summation is less than type.

So, this will be basically I will put just suffix k denoting the DM, DMU. So, this is not only implication for the first DMA it will be applicable for all of them. So, small k is equal to 1 to capital K. Now this if I consider, so for the first DMU, the output was 100 units. So, I will try to use different colours. So, it will be easy for all of you to understand and use the yellow one for the first one. So, it is 100 being the case for the output for the first one and the corresponding input things are 10 and 2. So, that would basically be the first equation. When I go to the first equation is first DMU, when I go to the second DMU, so it is less than I 1.

Now, when I come to the second one, so the weight corresponding to the output or so called the units was 80 and for the input for the second DMU was a 8 and 4. If you remember the labor cost and the conversion rates, all these things over there. Similarly, when I go to the let me check the colour. So, it is green for the third DMU, it would be 120 called the corresponding value in the output and the corresponding so called units not the weights. Units for the input would be 12 and 1.5. So, you would basically take care for k small k is equal to 1, 2, 3 and obviously, the weights which are the corresponding symbol for the output were v and the corresponding symbol for the input were u. So, the corresponding u and v s for all i, j and k would be greater than 0.

So, j is basically 1 because there is only one output; i is basically 2 because they are two input and k is equal to 1, 2, 3 because they are three DMUs. So, let me highlight that also with a different colour. So, i is equal to 1, 2; j is equal to 1; k is equal to 1, 2, 3. So, when and now if I go to the second DMU, the second DMU remember all these sets. So, let me highlight it using black. The sets of constraints these one which are specific for the first DMU would be same for the second, for the third, for the fourth so on and so forth till the capital k th DMU. Only change would be in the objective function, they would change. So, I will come to the formulation.

So, remember the colour scheme. Yellow one for the first DMU, the orange one in for the second DMU and the green one for the third DMU.

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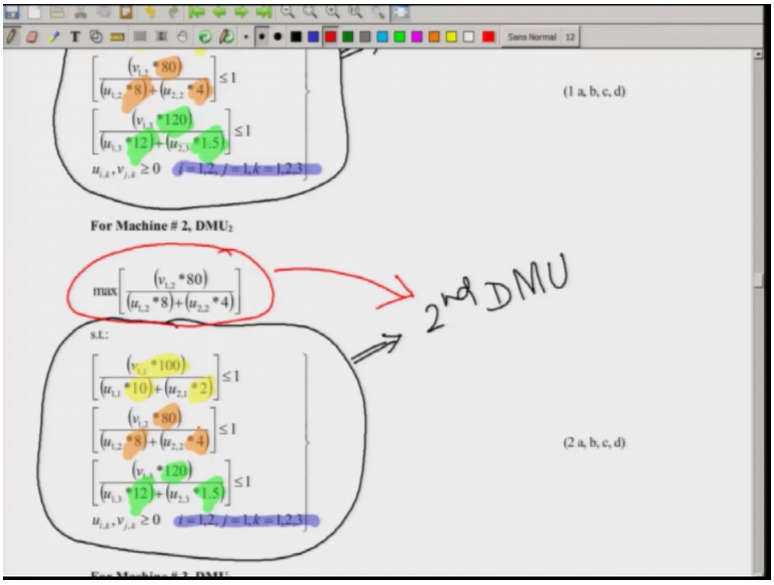

So, if I go to this first, I will basically highlight this as these constraints. So, they remain the same. So, let me highlight it. So, this is the constraints corresponding to the second DMU. So, they would be orange being the case for the first. So, let me highlight it, would be for the first DMU, so 100 units for the output; 10 and 2 for the input. So this matches. So, if you see here, I would not highlight. This it would become too cluttered.

So, the first set of constraint equation remains the same for the for the second DMU equations also. Similarly, 80 for the second DMUs output, 8 and 4 for the second DMUs input; these are the conversion factors not the weights. It remains same here. Again I will not highlight. It will become too cluttered. Then I use the green colour, 120 for the output for the third DMU, 12 and 1.1 being the input corresponding units or the so called conversion rates for the third DMU.

So, again we see 11.37 i is equal to 1, 2; j is equal to 1, k is equal to 1, 2, 3; that means, input 2 output 1 DMU 3 only change which is happening is. Now, let me change the colour to. So, this is the change which is the maximization problem corresponding to the second DMU. Technically, it is this only the second constraint. In the first case, it was the first constraint. So, if I go back, the objective function and if you see the first consequence, there are same because that has to be.

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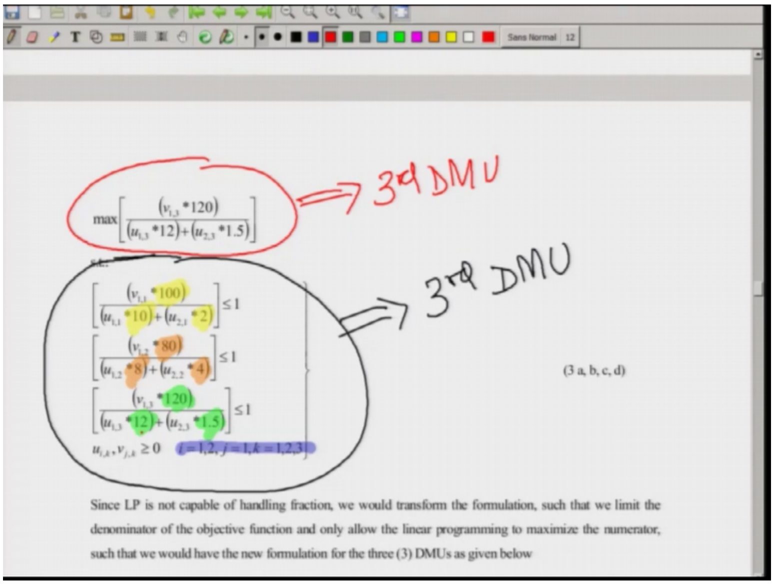

So, when I come to the third DMU, again the colour scheme I will use the same thing this is the constraint. This is the third DMU constraints. So, the colour I will again use is yellow for the first DMU; 10, 2 being for the inputs and 100 being for the output.

So, the units I am going a little bit slow, but please bear with me. When I go to the corresponding colour for the second one is 88 and 4 for the second DMU which is 80 being for the output and 8 and 4 being for the input, the units or the conversion rates. Then, I use the green colour. This is what I was using in order to do away with all the confusions. So, the green and violet, so green would be for the third DMUs, input and output conversions or units which is 120 for the output and 12 and 1.5 for the input. And finally, i is equal to 1, j is equal to 1, 2 to j is equal to 1, k is equal to 1 to 3 corresponding to number of inputs, number of outputs and DMUs.

Now, if I come to the objective function, sorry because I am using a different colour scheme, we should stick to that. So, this is the objective function, this is corresponding to the third DMU. So, we see this one is exactly equal to the third constraint. So, now, we have form formulated the output oriented model. I am going to convert them. So, the conversions again would be very simple.

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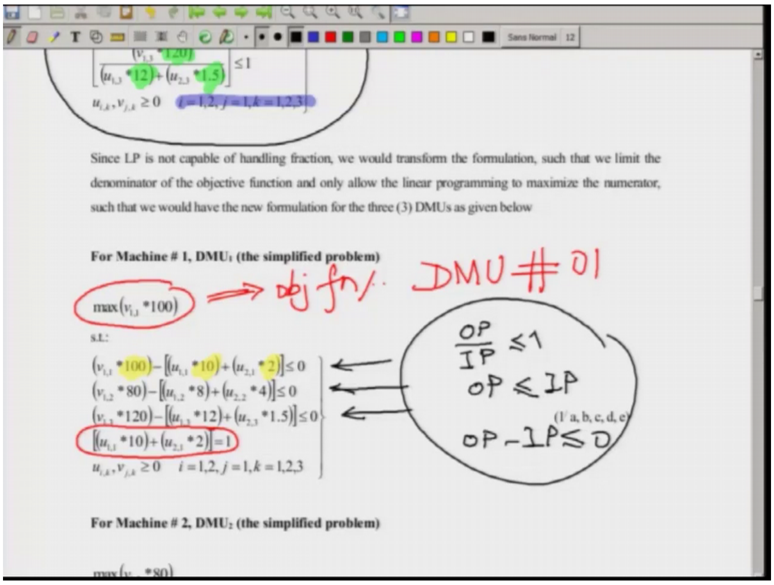

So, if you check the first equation, so I am going to basically do use the same colour scheme and follow it for the conversion in to a. These were till now we had considered the non-linear part. Now, we are going to consider the linear part conversions.

So, the objective function, so first I will highlight the objective function. So, I have removed the denominator which was respect to the input and I am and I am forcing that to be normalized to 1. Thus the object function now becomes a simple linear one, maximization, the bundle of inputs, outputs sorry. So, this is the objective function corresponding to DMU number 1. And where this did the this denominator vanish? It does not vanish. It has been converted into the constant. So, this is I will use a different colour now. This is red one. So, this one is the normalized input is equal to put it to 1 such that we have been able to convert the objective function into linear part. That is for the first DMU.

Now, if the question comes if you are doing that. So, correspondingly, the inputs for the first constraint, second constraint and third constraint which were there in the original formulation should also be converted into linear part. The question is yes, they are being considered. How? Let me go. So, the colour scheme will also follows the same; orange one, the yellow one for the first one. So, the first one was basically this 100 was the corresponding units or to the conversion factors for the output for the first, 10 and 2 or for the input corresponding to the first. So, they are less than 0 because if you if you consider the ratios, ratio was output by input is less than equal to 1.

So, the what we are doing is that, I will write something and I will again erase. Here other colours, I would not erase in order to make you understand. So, technically you had I write a very general one u at the output corresponding to the input less than to 1. So, what you do is output less than equal to input. So, output minus input is less than equal to 0. We are using this. So, you will be using it for the first constraint, the second constraint and the third constraint. So, we will repeat it for DMU 1, 2, 3. So, I have written for the DMU 1 only.

So, I will erase it here. So, I will come back to all these things later on. So, again going back to the highlighting part, orange was coloured being used for the second DMU, is 80 for the output units or the rates; 8 and 4 being for the input. So, again we have converted it into a simple linear one. We go to the third DMU; green colour, 120, 12 and 1.5 being for the output and the respective two inputs for the third DMUs.

And this part which I should highlight using the yellow colour in order to make you understand that is corresponding to the first DMU. So, let me highlight it. So, for second and third DMUs, I will highlight using different colours. You will notice that. So, this is coming from the so called objective function for this first DMU. So, this takes care. So, now, of the conversion factor for the non-linear equation to a linear part and finally, i is equal to 1, 2; j is equal to 1; k is equal to 1 to 3.

So, we will follow the same concept of trying to formulate for the DMU 2 and DMU 3. So, that was. So, remember the colour scheme. Again, I am saying objective function for DMU red and it will be yellow, orange, green for the first, second, third and the k th plus 1 the extra constraint which is being formulated will be highlighted corresponding to the colour which I am using for the DMU 1 or 2 or 3. So, now, we will do for the second DMU converting that non-linear set of equations to linear part. So, let me consider the colour scheme.

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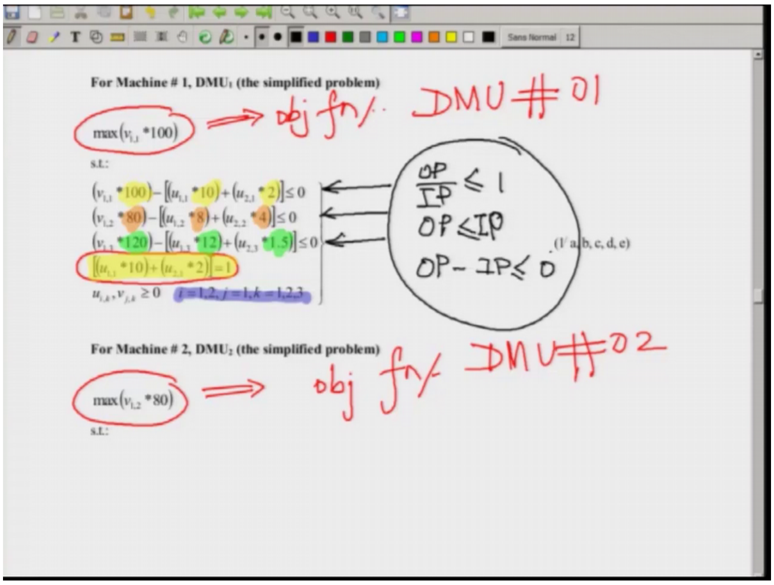

So, this is the objective function when converted into a linear part because we are removing forcefully removing in the sense forcefully putting the input as normalizes 1 and trying to only take the objective function as the bundle of outputs only. So, this would be the objective function for DMU number 2. So, what are the constraints? So, the equations are like this.

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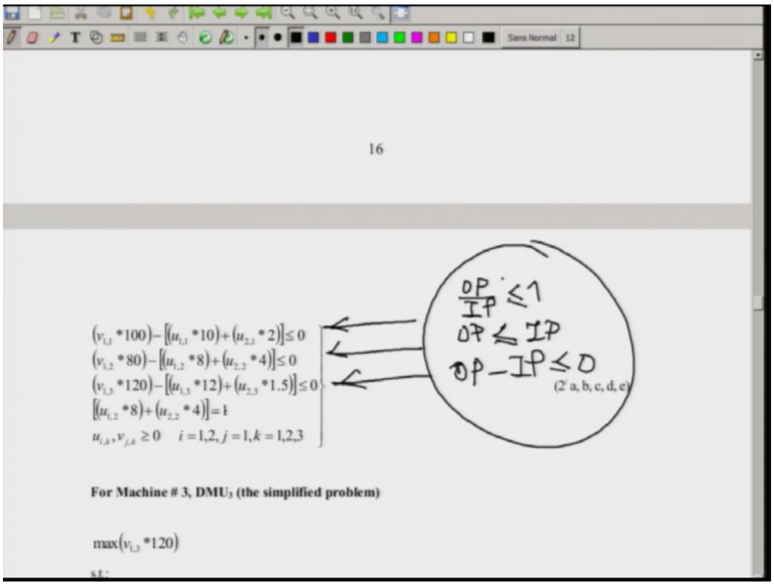

So, these first 1, 2, 3 which I will highlight are basically coming from here. Output divided by input is less than 1 because in output oriented model. So, output is less than equal to input, output minus input is less than equal to 0. So, this part is being done for the first, the second and the third, the fourth one which is being added k th plus 1 would be corresponding to the fact that we are forcefully normalizing the input as one for the second DMU.

So, this I will repeat in the third DMU. So, let me remove it. You can keep it because I am not writing, let me check one minute. I can I can, I am going back to the first DMU. Please write let me write it down when because it will be easier for you to recapitulate when you do it. So, it will be like this. Output , wait, so these three are here, not the last one. So, I will use this is the same scheme here. So, it is output by input .

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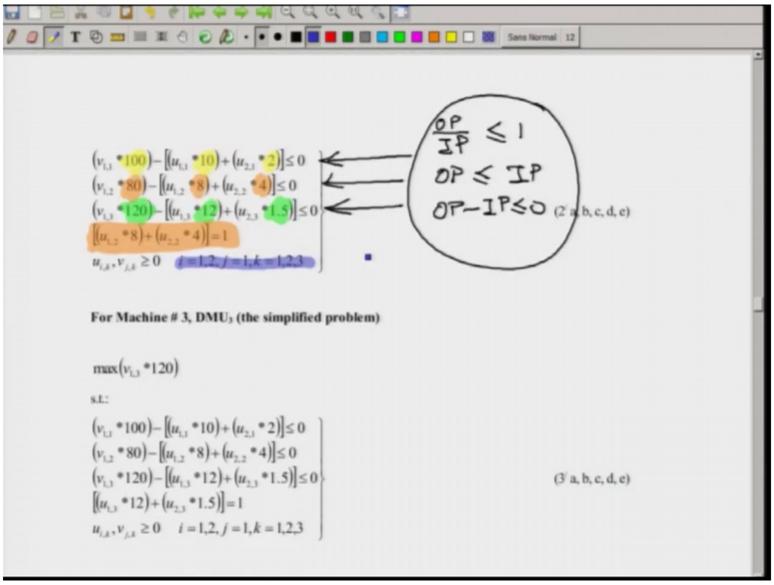

So, they are same and now let me come to the highlighting part. The yellow one is for the first DMU and 2 for the input and 100 for the output, so the conversions of the units. Similarly, when I go to the second DMU, I use colours highlighting 88 and 4 which is for the output. And then, the next 2, 8 and 4 for the input, for the second DMU; when I use the third DMU colour scheme is 120, 12 and 1.5 corresponding to the output first 120 and 12 and 1.5 for the input conversions or the units for the for the third DMU.

And the last one which is there, I will highlight it and this highlighted colour would be orange because this corresponding to the second DMU. So, this is the input bundle which is being normalized to 1. So, that input bundle which is being normalized to 1 is being brought back into the constraints for the objective function. And similarly, i is equal to 1;

j is 1, 2; j is equal to 1; k is equal to 1, 2, 3 corresponding to the number of weights and which are there.

Then, I go to the third DMU normalized part.

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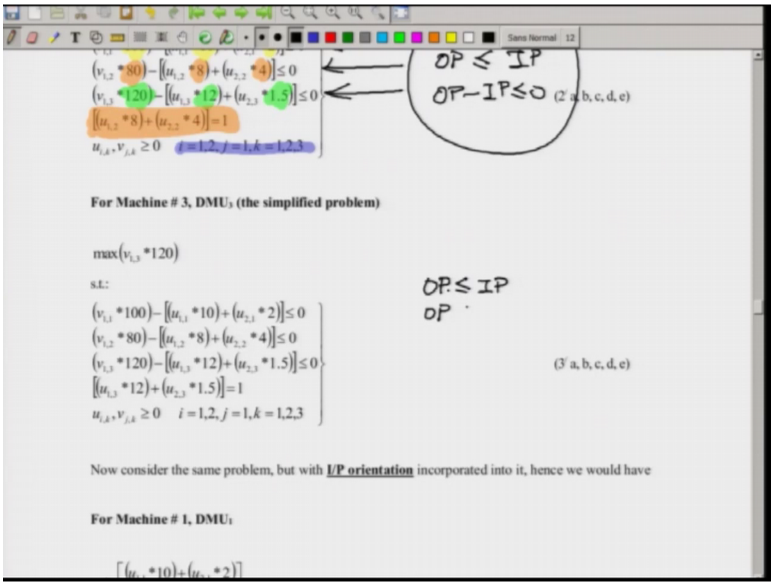

I will go through this slowly and then give the formulations in the they would be in the assignment. So, it will be much easier for you to understand if you go slowly and understand. So, again I use the same scheme. Output less than equal to the input, output sorry, I missed one step.

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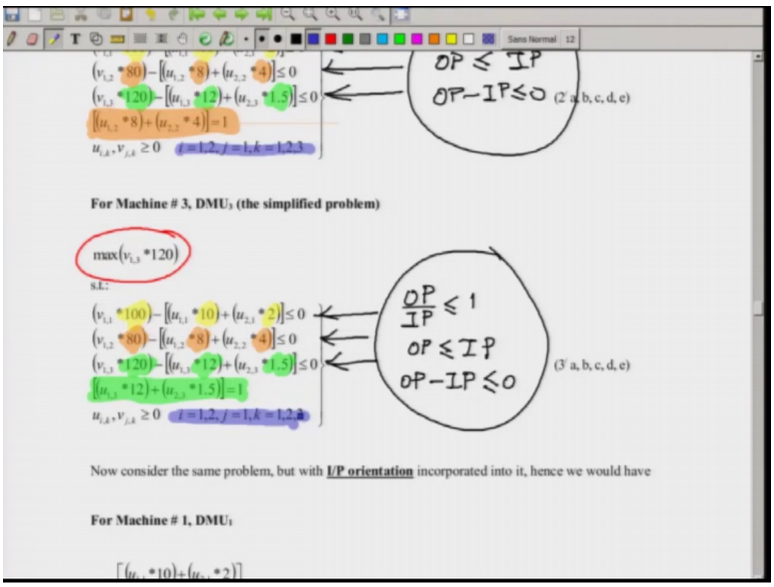

Output less than input and then, output minus input is less than equal to 0.

So, this sets of equations are being done in the first, second and the third. The last one will again we will see is a reputation coming from the objective function for the third DMU. So, let us use the same colouring scheme. So, this would be, so if I am using the same colouring scheme, I will highlight it using it red. So, this is what I am doing. This is the objective function normalized case for the second, third DMU and the corresponding constraints using this concept of output and input less than equal to 1 would be true for the case for first DMU, it would be true for the second DMU, it would be true for the third DMU and the and this the k th plus 1 which is the fourth constant which is coming is corresponding to the fact that normalize inputs have been put in 1 for the third DMU.

So obviously, this colour would become green and as usual the i, j, k would be violet. So, if you understand that, you will you will basically get a hang of this. So, what we have done is that, we have basically utilized the output oriented model considering the fact that we are trying to maximize the object function constraints are less than type. So, forcefully put or normalize the input to point to 1 and bring that into a constraint and then convert the objective function into linear part and the constraint is also a linear part considering this equation.

So, with then we will go to the input ordinated model and, but we will discuss that in the next class. So, with this I will I will end this lecture and continue further discussion in the seventeenth lecture about the DAE part considering the input oriented model and also considering the radial model. With this, I will end discuss and have a nice day.

Thank you very much.