Data Analysis and Decision Making – II
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Lecture – 10
Safety first principle

Welcome back my dear friends and dear students. This is the DADM II lecture series under NPTEL MOOC. And as you know this total course duration is 30 hour which is for 12 weeks and each week we have 5 lectures each being for an half. So, total duration would be as you know 30 and spread over the total duration. I am Raghu Nandan Sengupta from IME Department IIT Kanpur.

Now, coming back to the discussion if you remember we are discussing the Safety first principle. And I did mention that how the normal distribution being true how we can solve it very easily. So, we will continue to discussing that and this is that 10th lecture which is the last lecture in the second week for DADM II and after that you will have the second assignment. So, even though for example, we have simplified our assumption by considering only normal distribution this point which, I am mentioned time again it.

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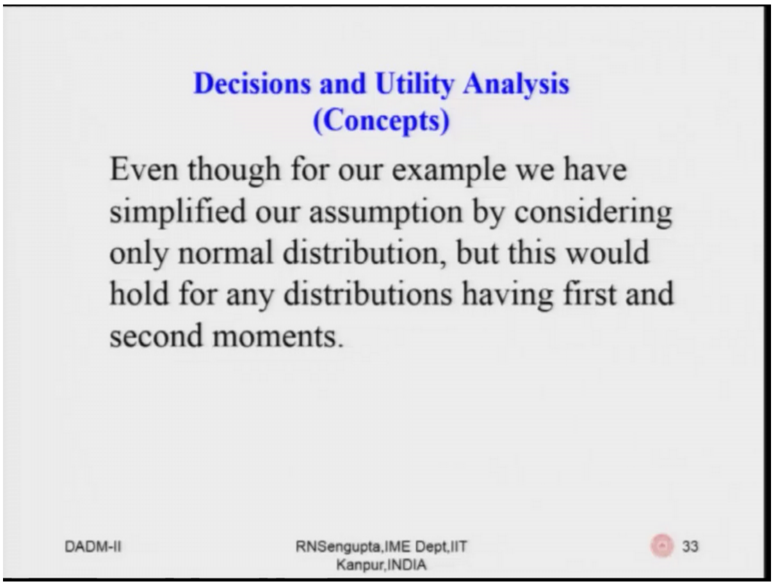

But this would hold true for any distribution having the first and the second moments. Because, the reason being first and second moments being you need to find out the expected value, you need to find out the standard deviation on the variance which is basically for A B C decisions R bar A, R bar B, R bar C would be the respective mean values for A B C decisions. And, similarly sigma suffix a sigma suffix b and sigma suffix c would be the corresponding standard deviation for a b c decisions.

I am using sigma in place of standard error, but that is without much of concern because if you get the concept you can solve it accordingly. Now I did mention about the Chebyshev inequality, Markov inequality very fleetingly I do remember in the case of a DADM I and to utilize that concept for the DADM II where you have the safety first principle.

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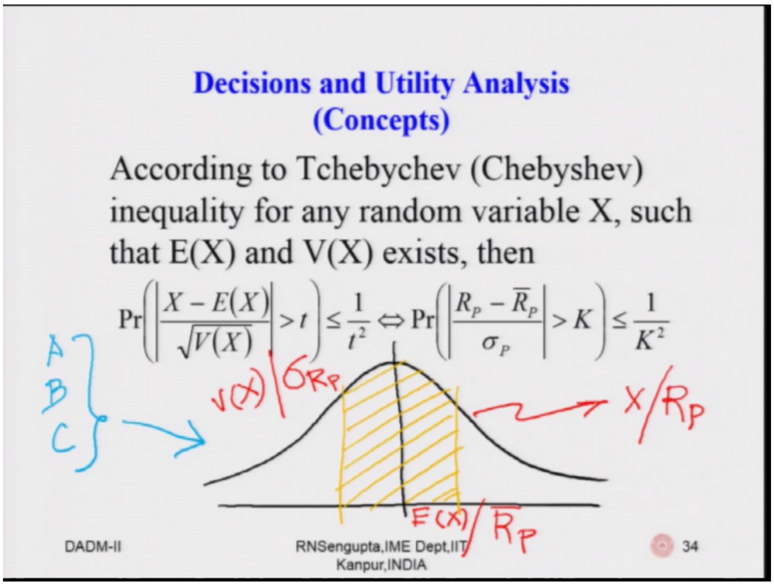

So, according to Chebyshev's inequality for any random variable X such that; first moment and second moment which is E of X and V of X exists then obviously, you will have this as true. Now let us look at from the diagrammatic point of view. So, that if I am able to draw the diagram it will be easy for me to explain to you and also you will get that concept decently clear. So, I will first draw the normal distribution, I will use the axis as neck so this is the normal.

Now, I have the expected value which I will mark as red; so, E of X. So, it can be R P also R bar P R bar P a or P b what I am using the expected value and there is the variance. Now this X has a distribution which is normally distributed and what I am considering is that a bandwidth depending on the variance. So, consider this one so this is the value. So, I am considering X minus E X divided by V X. So, I am basically normalizing the difference between the random variable and the expected value divided by the variance.

So, the variance being very high; obviously, it will be normalized accordingly. So, I want to take this whole area and that is greater than some value of t, t is the so called the constant which I will take. And the value of that would always be less than or equal to 1 by t square depending on the Chebyshey inequality. So, what I do is that in this case I replace X by R P E of X by R P bar. And there was a variance also I did not write V of X would be replaced by sigma actually should be sigma of R P.

So, and you consider value of K, K would be the some threshold value. So, that probability would be for this mod of this normalize distance being gate greater than K would always be less than or equal to 1 by K square. So, higher or lower the case where detect that what is the so, called normalized length based on which you can take this decision. So, if you basically have two or three different type of distributions; these are not normal I am using over two or three different type of distribution based on the fact. There is the mean values and the variances for the decisions are different.

So, see for example, we have A we have B we have C for all of them we will basically have such distributions where the expected values and the variances are different based on that when you find out the Chebyshey inequality you will have some value of K. And being less than or equal to 1 by K square would give you the bound such that you can find out that how you can rank the decisions based on the safety first principle.

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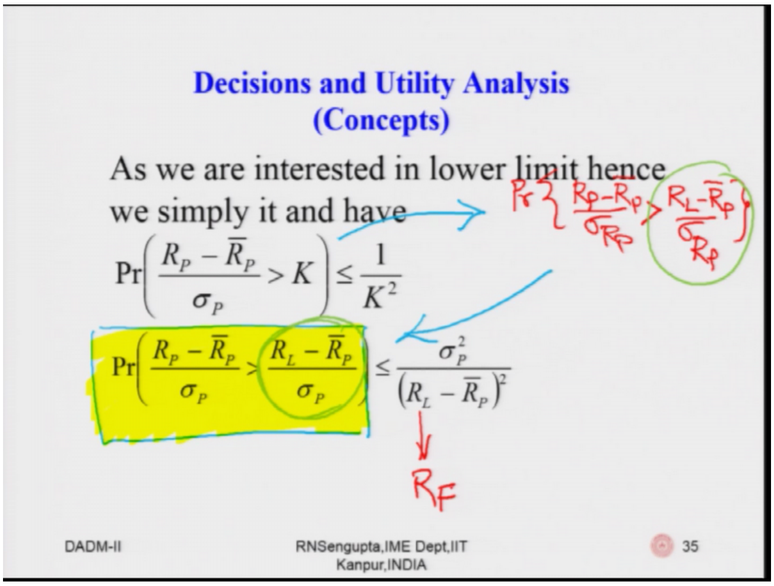

So, as we are interested in lower limit; that means, we are interested more on the negative values. If you remember it was basically minimization of probability of R P is less than R L. So, you simply we can write down so if it is we remove the mod. So, we find out the value. So, in place of in place of X it will be R P in place of E of X it will be R bar P and in place of variance of X it will be sigma of P or sigma for R P. So, this is greater than K such that it is less than 1 by K square.

Now, what I do is that again I now normalize them. So, normalizing this basically means now, coming back to the actual formula so here this actually this is coming from here and then I will write down what I am missing and then I jump here. So, if you remember we already have this formula; I should use a different color it will be easy for us to differentiate and highlight it first. So, this is the so, let me write down, what is that actual thing which we were looking for. So, we already have seen in the previous discussion.

So, the equation which is written here when I am hovering my pointer is actually R P minus R P bar divided by sigma of R P, this was greater or equal to so in this case it is greater. Now you already had RL minus R P bar divided by sigma of R P. So now, what we do is that we are replacing this with K sorry. So, this is K I should remove it K; so, which means, that the K value is known to you given the fact that the standard deviation and the values of R N and R P bar are known to you.

So, R L technically would be and use the red color. So, actually this would be R F. So, you have the standard deviation of the decision, the mean value of the decision and R F being the risk interested to which is coming from outside. So, immediately you can find out the bound K, K square and solve the problems accordingly. So, lower and higher would basically give you how can rank them from the highest to the lowest or lowest to the highest depending on what your main criterion is.

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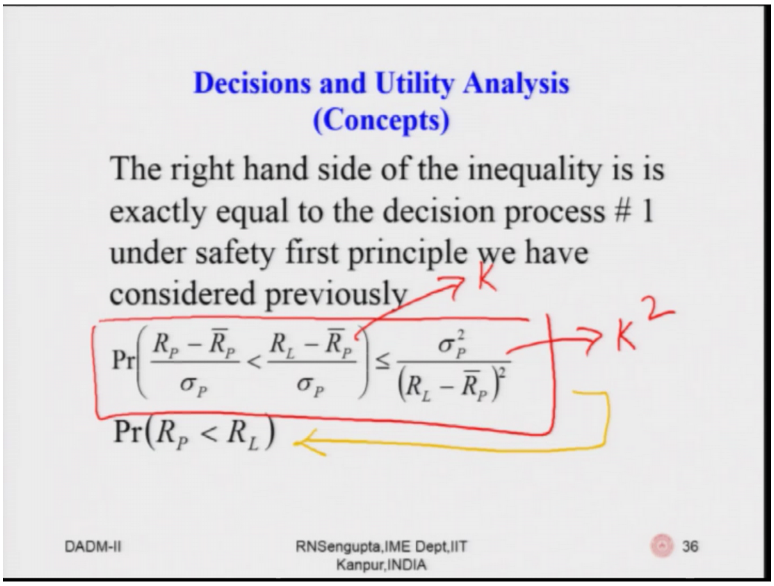

So, the right hand side of the inequality is in exactly equal to the decision process 1 which we are taken, 1 means minimization of the probability of R P less than R L under the safety first principle. So, we have considered initially so once you basically put this technically this is K, this is K square. So, actually the whole formula leads to the first safety first principle or bullet point which we have considered.

So, technically the Chebyshev inequality and the safety first principles are the same considering the normal distribution. And similarly you can modify that that rule of the safety first principal the first bullet point with respect to the Chebyshev accordingly. Provided, if you remember I has mentioned the first moment which is the expected value and the second moment which is the variance exists. For the second criterion if you remember now I am basically trying to change the problem.

I am not going to solve these problems from the optimization point of view you remember that, we will only come into this optimization concept later on in DADM III is basically to highlight that how they can be utilized I do not want to burden all of you with trying to solve the optimization problem. Because this is the total different concept we will consider in DADM III. For the second criterion we already had maximization of RL.

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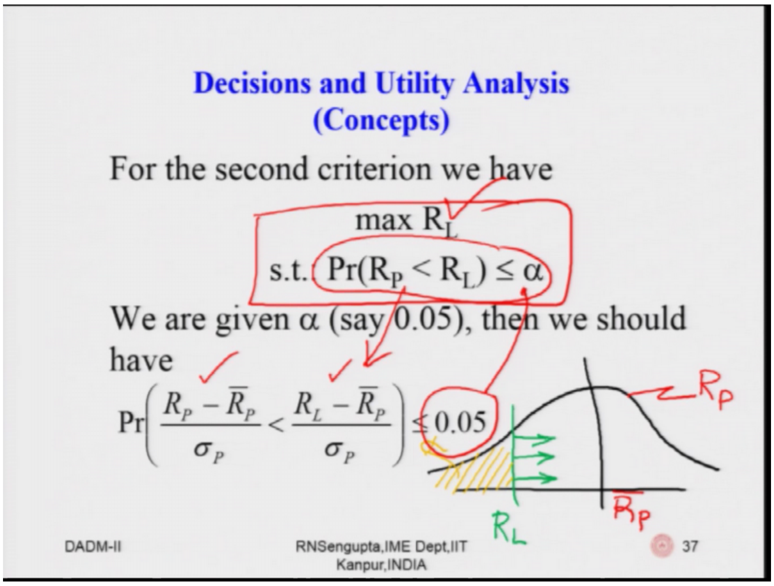

I should use the color red so it easy for us to read. So, you maximize R L and what you do is that you bring such then constrained. So, you are trying to push RL on to the right. If I am looking at the distribution from my side R L is under the left you are pushing it on to the right. But subject to the cases that the probability of R P less than R L is kept fixed at alpha. So, obviously it would been technically your actual optimization is now let us think it little bit philosophically or let us try to understand it.

When you are talking about R P which is basically we are we already have the returns and we want to find out the weights for the alternatives. Now, if the weight change R P changes. So, when you basically look at this optimization problem maximization with respect to some constraint. Maximizing R L is basically pushing on to the right, but provided also the probability of R L R P is less than R L is equal to less than alpha; so obviously, if technically if the distribution let me draw it no this is.

So, let me write this is R P bar this is R P this is R L. So, what you are trying to do is that you going to push RL, but subject to the cons fact that this whole area remains as alpha. So, if you are pushing RL; obviously, it would also mean that the f if only R L moves then the area alpha would start increasing. So, in order to restrict that the whole distribution also starts moving on to the right; so, the alpha value remains as it is if I consider the over R L value are to be alpha it can be less than alpha also.

So, we are giving alpha given alpha see for example, 0.05 then we should basically have if you convert this whole set of constraint consider in the normal distribution. So, what we have is R P minus R P bar divide by sigma of R P which is the left hand side of the equality inside the bracket and on the right hand side it will R L minus R P bar divided by sigma of R P which is the second term on to the right hand side of the inequality sign inside the bracket. And, this alpha value is given from depending on the reliability or the overall risk attitude the person has based on it he or she can basically invest.

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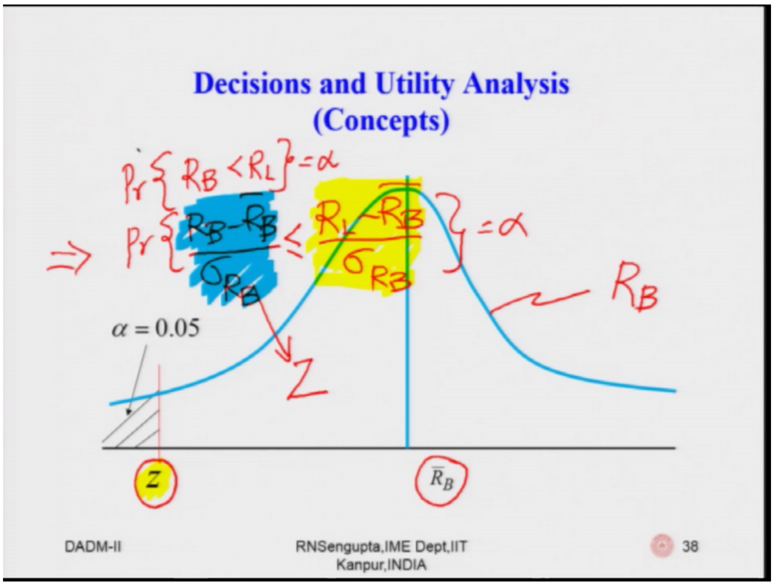

So, we have drawn it considering the distribution is true what is it is been showing shown here for decision B. This is basically RB bar; obviously, you will have the R B as the distribution this is R B bar then you have a standard deviation. So, technically this Z is being formulated based on the fact that you coveted that into standard normal; that means, you have started with this I am repeating it many times plus please bare with me.

So, this value of Z is equal to this and the value for this which is basically to the distribution of Z we have we have been able to basically draw the distribution of capital of Z. So, technically this is the Z capital Z. So, this distribution as been it should be drawn. So, I have basically drawn both of them I am basically marked them values accordingly. So, technically this should be some R L which has been converted into small z this is RB which has being converted into capital Z the standard normal distribution. And again I am repeating this can be done for or the distribution also.

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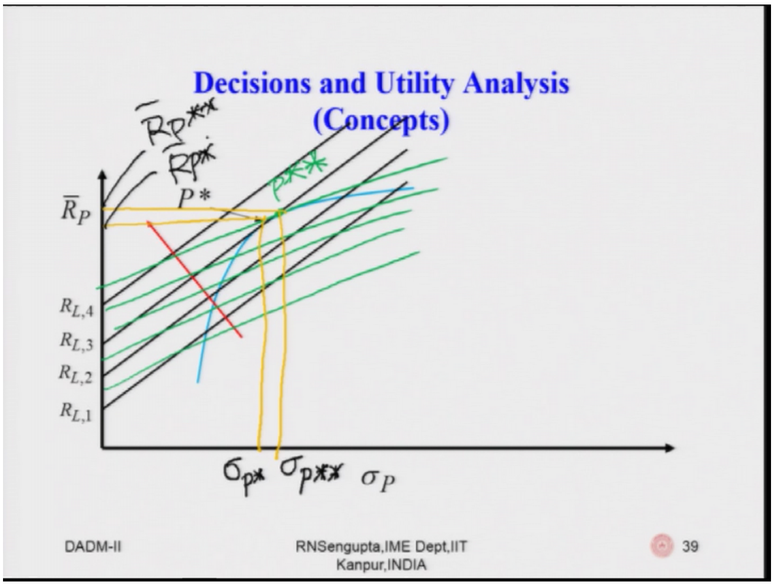

Now, coming back to the utility concept based on the mean variance concept for portfolios or any decisions. What I would not go into the details, but I will basically highlighted. So, if you see the mean variance diagram. So, along the y axis you are measuring R bar P it will change depending on the weights which you are investing in different alternatives or on the X axis your sigma P. And what you are doing is that you have drawn this blue line which is the efficient fact here.

So, our main task is basically to find out that P star value P star is basically also called optimum portfolio value where the weights which you are trying to invest in this different type of alternatives for the decision which is the portfolio is such that I am getting the maximum return based on the fact my risk is also increasing. But the proportional value of the increase of the risk is such that the ratio of the risk that it return or return to risk is the best for my case.

So, risk to return would be I trying to take the minimum value for return to risk I will take the maximum value such that the weights are the best combinations. So, what you do is that this R L value which is RR P RR sorry RF which is the risk interested you will be basically keep changing it. So, as you keep changing it will basically have a different contours based on which we can find out what is the best value of portfolio or the alternatives such that you get the R P star value or P star value.

So, you keep moving the parallel lines which are R L 1 R L 2. This suffix 1 2 3 4 are the different values. So, keep changing them. So, see for example, if you consider R L the values which are there in side this whole set of values or the points which are there inside the graph are basically different combinations of risk return profile for the decisions such that you had different type the same alternatives, but you are investing in different proportions.

Now, those are not the base because you have not been able to reach the best optimum value because; obviously, you will always try to go as highest possible for R P and as low as possible for sigma P. So, keep changing RL. So, go to R L 1 R L 2 R L 3 and the point where it is basically tangent to the graph that P star value would give us the best combinations such that my overall risk return profile is meet is met. Now, you may be thinking that why I have drawn this graph in this way they could be have been other tilted also. So, let me draw it let me use the green color.

So, say for example, RA l on the graph was this depending on the on the risk return profile. So, I could have drawn parallel lines like this. So, it would the R P star would have been some value. So, for the first case I will use the return and risk were this. So, I will consider as say for example, double star and another case is basically we have remove this and come back to the same color combinations which is easier for me to draw. So, the first value is basically sigma P 1 star. And this is sigma P double star double star and one star are just to differentiate.

Similarly, here you will have basically R P double star R value and this is R bar P single star. So, they would basically be to different such a portfolio or this or the two different sets of decisions. Based on the fact that are the alternatives of the same, but you are investing different of modes you can change it accordingly depending on or the problem is reformulate.

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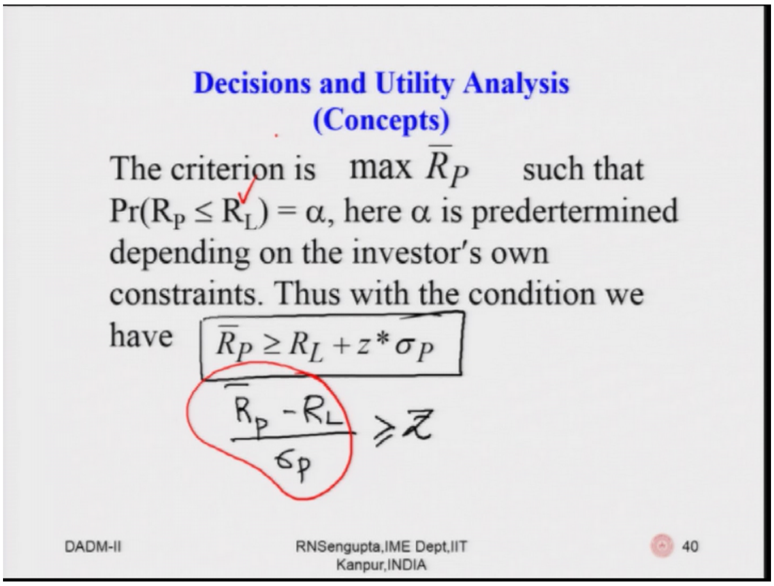

So, the criteria to maximas R P such that now your are again beginning constraint which is based on the fact that you had the first safety first principle is probability R P is less than R L is equal to alpha the alpha value is fixed pre depend determined or given by the investor. Here alpha is the predetermined depending on the investors own cons constraints thus with the con condition if you put it you will basically have the graph as this.

Now, this is simple concept how do you do it let me basically go reverse step. So, basically it is R P bar minus R L divided by sigma P is less than or equal greater than small z. So, this is the case. So, what we have considered is basically oh this should not be R bar. So if you consider the value here this is basically small z on the right hand side of the equation which is from here.

So, you have been able to covert that actual value of R L minus R P bar divided by sigma P into a case where it is basically the small z you put a equation for that have a linear line. And thus, the linear line it have been talking about. So, you can basically covert it depending on other problem has been formulated. Now, consider different way of try to solve the problem.

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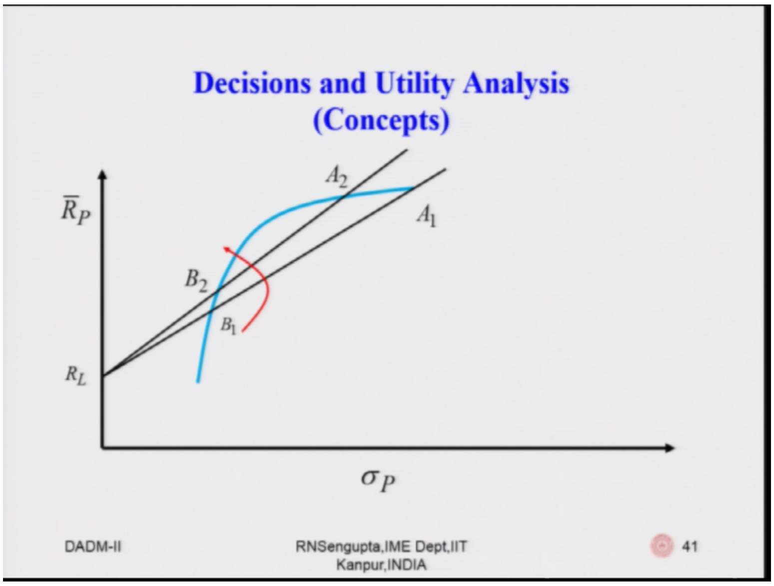

Here I will consider the RF or R L is fixed. So, I want to find out at what value of R P or P star I will get the best combinations. So, what I do is that I have the efficient front here which is the blue line. Now what I do is that I turned the straight line which is R L and A 1, A 1 is arbitrarily one point take in order to basically denote the straight line I will turn it count of clock wise keep moving it. So, basically it cuts or basically touches the blue line at B1 and A1. Then you turn it count of clock wise it touches or cuts it the blue line at B2 A2 continue doing it.

So, at one point of time A1 we are. So, this A values and B values are merge which would basically the tangent point and that is the P star value based on which you are going to take a decision that what is the optimum alternatives in the in the investment sense Which we should invest in order to get the best possible returns based on that you are trying to maximize the return or minimize the risk or some combinations so that. And again remember we are considering in a very sense the normal distribution.

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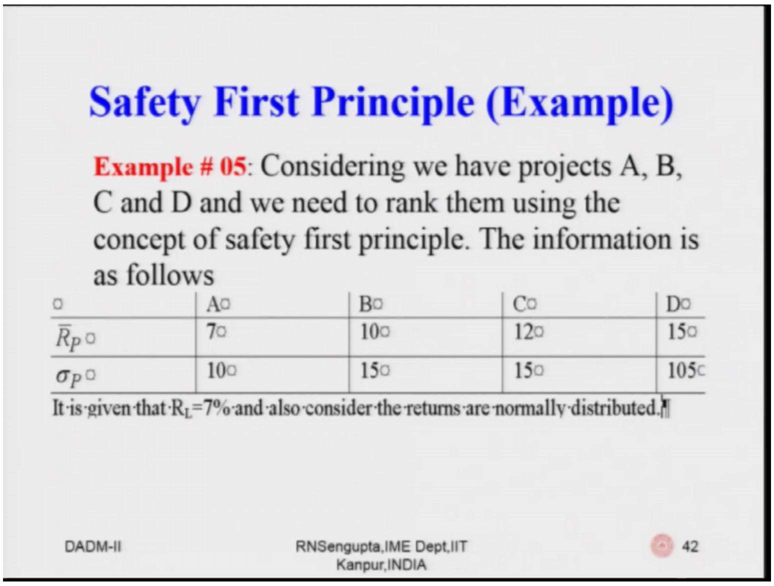

Now consider we have projects A B C D I am considering four different decisions and we need to rank them. So, I am not interested for the timing to find out what are the investment which is happening per alternative for the projects ABCD or decisions A B C D. They would become coming out an later stage first I want to rank them provided I have made an investment. So, consider we have projects ABCD and we need to rank them using the concept of safety first principal the informations are given.

So, along the first row you have basically the R P bar which is the return on the portfolio the decision for A B C D are given as 7, 10, 12 and 15. And the corresponding second row is basically standard deviation happening for the returns for these decisions which is A B C D their decisions and the standard deviations are respectively 10, 15, 15, 105. And you also remember this R L value RF value which is fixed before hand by the investor depending on what are the external inserter information which he or she has the values given as seven percent and also considered the returns are normal distribution.

So, what we need to find do is basically again covert them to a standard normal which have been doing at time and again in the in this class by drawing the diagrams. Find out the probabilities and based on the safety first principle find out the probabilities on the left hand side less than RR L RL is given as 7 percent. So, you can find them and rank them accordingly such that your problem would be solved.

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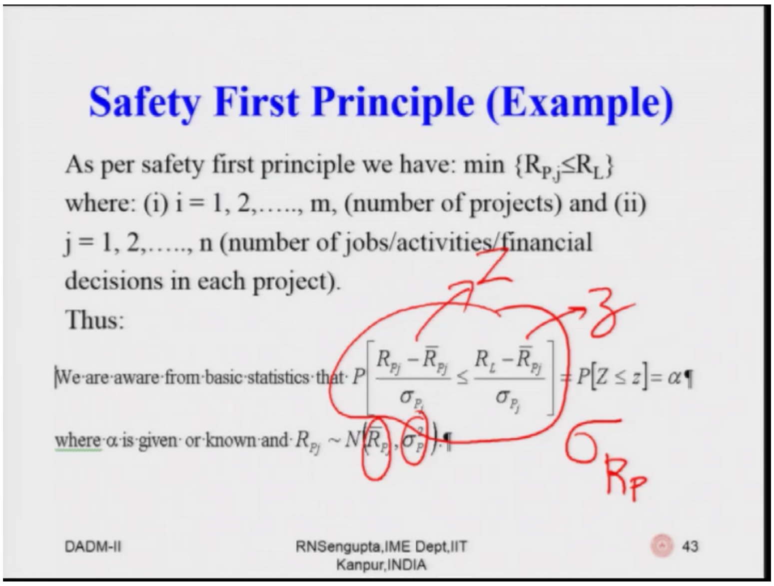

Now, as per the safety first principle; so, your actual decision is minimize the property of R P is less than R L where, we have basically 1 2 3 4 till m is the number of projects. And under each projects we have the investment we made in jobs activities or financial decisions or alternatives which are given by j is equal to 1 2 3 4 till n. Thus, we have from the basic concept of statistics this tool which we have already done repeatedly.

So, this is capital Z this is small z which is given that value is alpha can be found out from the standard normal table. Rank them from the lowest to the highest depending on whether you are looking at the left hand side or the right hand side. Now why I am saying in the right hand side that depending on many different criterion is, if you considering the negative distribution of the losses negative distribution being the losses or if you consider to the positive distribution which have the gains see your overall rule remains the same. But with the sign change accordingly.

So, it is a maximization of problem because minimization or vice versa and the properties also rather than on the right hand side you will try to basically look on the on the other than the left hand side you will try to look at the right hand side. And you remember that the R P j for the each investments has have a normal distribution with an average value given by R bar I am not mentioning the j and sigma P being the sigma square being thus on the variance of that decisions.

Now, remember this standard deviations again I am mentioning actually the R sigma R of P these are taking the standard deviation on the return of the distribution, returns are drawn from based on that you find out the mean value based on that you find out the standard deviation.

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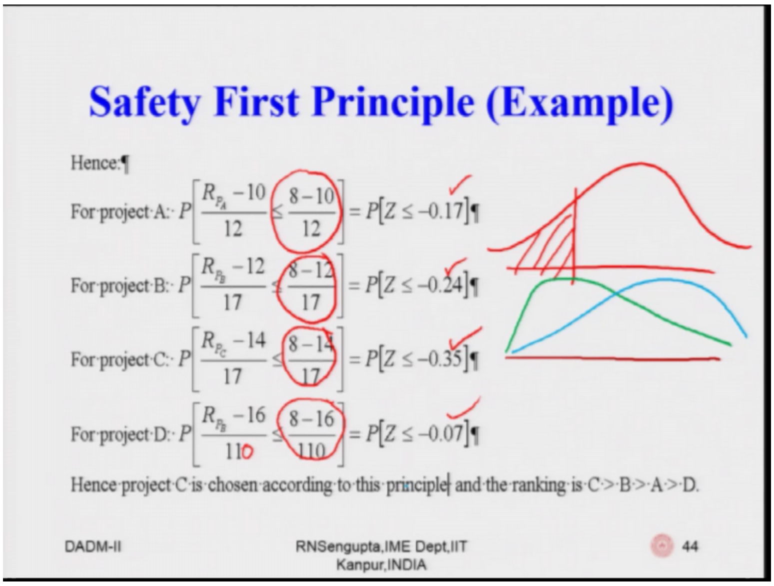

So, once you put plug-in those values. So, you have the value put on the project given as corresponding. So, I have taken different sets of values rather than R L being 7 I have taken 8 the returns for project A B C D or the decisions A B C D are given by 10 12 14 and 16. And the standard deviations corresponding to A B C D are given by 12 17 17 and 11 one minute it should be 110 sorry my mistake my mistake it should be 110.

So, based on that the small z values for the first case project is given by 8 minus 10 by 12. For the second one is 8 minus 12 by 17, for the third case C is given by 8 minus 14 by 17 and the fourth case is given by 8 minus 17 by 110. So, once you find out small z the probabilities can be found out. The small z values are given as I am not repeat I am not saying the minus value because they are on the negative side which is my 0.17, 0.24 0.35 and 0.07.

So, you want to minimize them so; obviously, hence project C is chosen according the principle the science because you want to basically minimize the probabilities. So, minus 3.4535 is as far as possible under the left. So, obliviously the alpha value would be as low. So, if I draw the graph this is the alpha value I want to basically make it as low as possible. So, it will be on the left and based on that you can find out that how will rank them.

So, the ranking system means C is better than B is better than A is better than D. Now I did not mention fleetingly the concepts of positive and negative distribution. They want a basically based on the fact that whether your skewed distribution on to the left or to the right. I will just draw it in order to make you understand. So, for a positive distribution these are the axises considered I will used different colors skewed on to the left my mistake.

Skewed on to the right, you will have that distributions caused in the positive and negative I will come to that later on in DADM III remember that because in DADM II. It is more related to the trying to understand the concept of utility theory and how we can utilize it for non parametric distribution. With this I will end the 10th lecture which is the end of the second week.

And hopefully by 11th lecture which is the first lecture for the third week I will able to wrap up utility. And then start in the 12th lecture which is the second class in the third week of about the actual concept based on which we will be covering in this DADM II lecture series under NPTEL MOOC.

Thank you very much for your attention and have a nice day, bye.