Data Analysis and Decision Making - I
Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 06
Conditional Probability

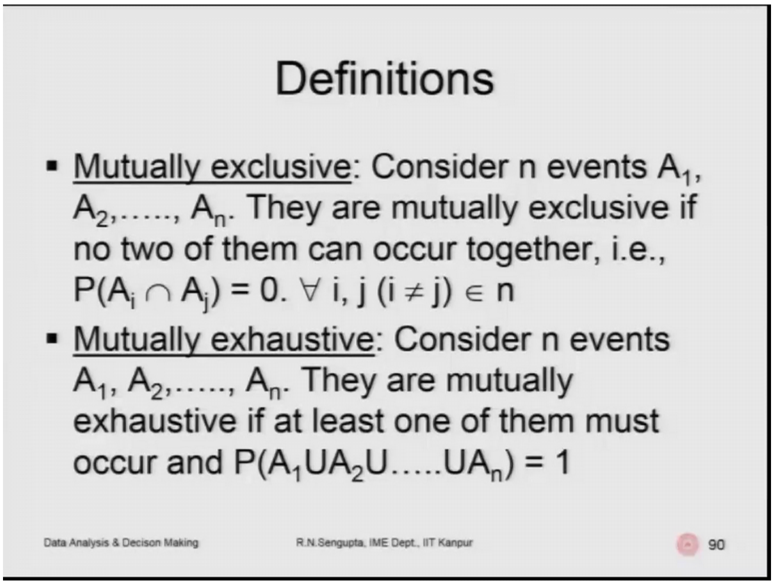
Welcome back my dear friends, a very good morning good afternoon good evening to all of you. And this is the DA, DM one lecture which is for 12 weeks which is 30 hours. Each week we have 5 lectures for half an hour each and I am starting the second week hence is the 6 lecture. So, if we remember you are discussing about the axomatic definitions of probability, then the concept of say for example, the general definitions of probability in the axomatic, you have basically into you can have technically infinite sets of sample points.

The probabilities of each sample points not been equal and for the other general form of definitions of a probability obviously, there as finite set of sample points. And the probabilities of each sample points had equal and example given for the axomatic one that you are basically trying to toss a coin till you get the first rate. So, they technically they can be infinite such points and in the other case from the normal definition, you can basically roll the die. And obviously, the number of sample points is finite which is 6 considering the 6 phase die and the probabilities are 1 by 6.

But generally the definitions based on which we will try to work for the probabilities were as we as you remember is that if A and B are sets of the universal set. And A is A subset or A proper subset of B then the probability of A would be less than equal to B because probabilities always increasing it is between 0 and 1 and cannot be negative it cannot be less than 0 or greater than 1. Then obviously, we also we have discussed and I have shown that is diagram that the corresponding probability of A union B would be probability of A only plus probability of B only minus the intersection which you have if we consider a very simple when diagram and it can be extended to 3, 4, 5, 6 so on and so forth.

So, I will just draw a diagram in order to basically highlight the fact if there are three or more such events which are there. Then the probability of a universal set is 1 probability of a null set is 0 based on that we were basically discussing and I give simple very simple examples. Now we will consider the concept of mutually exclusive and exhaustive event and they will be utilize later on for the different type of conceptual study.

(Refer Slide Time: 02:44)



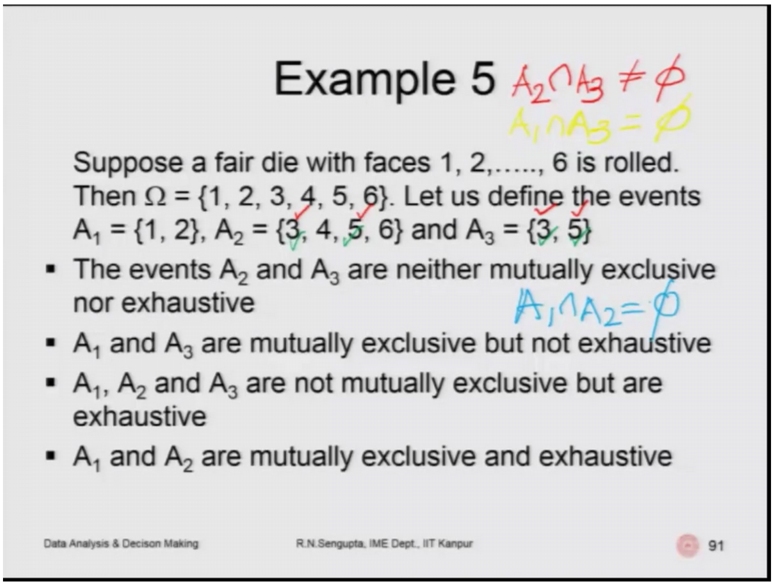
So, with definitions for mutually exclusive 1 is considered an events which are basically A1 to A n and A n is basically finite they are mutually exclusive.

If no two of them can I occur together, so when we are talking the word mutually exclusive we will use the number of exclusively which is there. If you say that two of them on which we will exclusive it means that two of them cannot other occur to the together.

So, if they are mutually exclusive if and only if two of them can occur two of them can occur together. Which means that if I am taking the A i and A j i and j can be basically any one of the number between 1 and n. And probability of Ai intersection A A j is equal to 0 which is basically a null value and a null set. Corresponding to the fact that f A i occurs then the probability of A j not occurring is true and vice versa. How that is done I will come to that later on in for the discussion and the other definition is mutually exhaustive it means that if we have another ken n number of events again A 1 to A n and its infinite.

So, they are mutually exhaustive that means, you exhaust the overall search space. If at least one of them must occur such that the probability of A 1 probability of A 1 union A 2 dot dot till A n basically we will give you the universal set which is 1, I will come with examples.

(Refer Slide Time: 04:19)



Now, suppose a fair die with faces 1, 2, 3, 4 till 6 is rolled hence the overall sample space would be 1, 2, 3, 4, 5, 6. So obviously, it is a 6 face dies if you have a 8 face die or a 12 face die, so it will be done accordingly.

Now, they let us define the events accordingly to make our example very simple. So, consider A 1 is an event which consists of only faces 1 and 2 A2 is an event which consists the faces 3, 4, 5, 6; A 3 is an event which consists of faces 3 and 5 only. Now with this free we will basically give you their definitions are mutually exclusive and exhaustive. So, I am again repeating A 1 is an event 1 and 2 faces A 2 is an event which consists 3, 4, 5, 6 and A 3 is an event which consists of 3 and 5.

Now, the events A 2 to A 3 or A 2 and A 3 are neither mutually exclusive nor exhaustive. So, let us consider why it is being same so consider A 1 and A2. So, in A 1 you have 1 and 2 A 2 we have 3, 4, 5, 6, so if you consider the intersection of A 1 and A 2 what you will basically have is that the corresponding A 2 and A 3. So, if you are considering the intersection of A 2 and A 3 A 2 is 3, 4, 5, 6 and A 3 is 3 and 5. Then obviously, intersection would be you will have A 2 intersection A 3 what are the common elements you will have basically I will use red colour first you will have 3 you will have 5.

So; obviously, it is not a null set, so as and so obviously, probability would not be 0 as for the definitions of mutually exclusive. So, this is not mutually exclusive now let us consider the concept of exhaustiveness. So, let me change the colour now let us go to yellow. So, exhaustive means that union of all of them would should be the universal set. So, what is the universal set universal set as I mentioned is I am just not marking I am just showing it to you it is 1, 2, 3, 4, 5, 6.

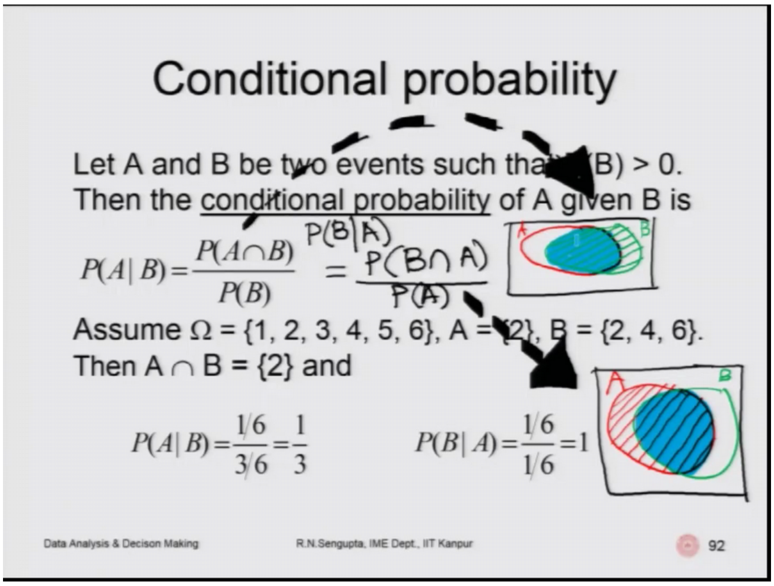
So, when I combine A 2 and A 3 it basically has 3, 4, 5, 6, but it does not have 1 and 2 so obviously, it is not mutually exhaustive also. Now considered A 1 and A 3 A 1 and A 2 are mutually exclusive. So, A 1 which is basically 1 and 2 and A 3 is 3, 4, 5, 6 if you combine them, there is no common element hence when we combine hope you can see the yellow colour.

One A 1 union A 3 is basically a non-set hence the property of mutually exclusive happens, but if it is not exhaustive because if we combine A 1 and A 3 we are combining A 1 and A 1 and A3. So, the elements would be 1,2,3,5 so 4 and 6 would be left out, so it is not exhaustive. Now consider A 1 A 2 A 3 and take the property. So, A 1 A 2 A 3 if we combine mutually. So, let me change the color to green. So, A 1 A 2 A 3 and is not mutually exclusive because A 1 A 2 and A 3 the common elements is not mutually exclusive.

But they are exhaustive, so if you combine them the total set would be 1, 2, 3, 4, 5, 6 all the elements would be there. So, which is the universal sets so its mutually exhaustive, but we find the intersection 1 and 2 and A and A 3 4 5 6 does not have any common element. But the movement we bring A 3 into the picture again 3 and 5 are common. So, hence it is basically not mutually exclusive, but obviously, exhaustive and finally, if we combine A 1 and A 2. So, it is both mutually exclusive and exhaustive how let us see let us again change the colour to blue.

So, when we combine A A 1 and A 2 the overall universe set is form which is 1, 2, 3, 4, 5, 6. So, which basically mutually its exhaustive and exclusive because the intersection if you take of A 1 and A 2 it is a null set which is proved as per the definition.

(Refer Slide Time: 09:10)



Let us consider the con the concept of condition probability let A and B be 2 events such that the property of B is greater than 0. So obviously, that has to be done greater than 0 not greater than equal to 0 greater than 0 then the conditional probability of A given B is given by and let me denote it.

So, what we didn't actually mean is like this let me change the colors is better. So, this is an event A and then we consider an another event B. So, now what do I need we need to find out probability of A given B has occurred.

So, if given B has occurred this means this green one has occurred, now out of that what is the probability that given B has occurred A has occurred. So, that me mark it in blue colour that we use the highlighter it could be easier for me to use.

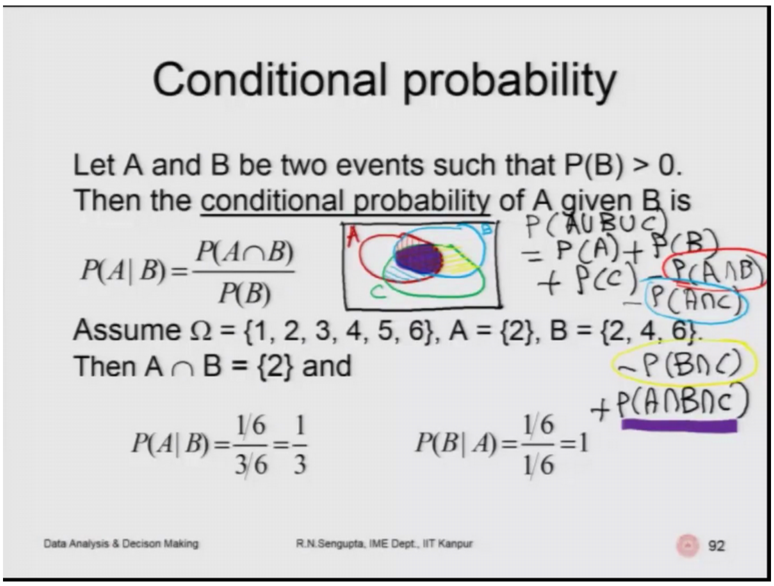
So, B has occurred, so A has also occurred is this one. So, when you trying to find the probability it will be the probability of this blue space divided by the green hash lines. Because it does not give you any corresponding fact that if B has occurred this portions which I am not marking, but I am just trying to highlight that with my cursor. This portions which are unmarked, which are white in colour has not occurred so obviously, they would not be coming to the probability sense. So, probability of A given B would be probability of A intersection B given divided by probability B.

Now, if somebody is asking this question. So, let me write it for ease of understanding somebody wants probability of B even A. So, this would be so that means, A has occurred first then B will occur. So, how would you denoted. So, this diagram I am just make it little bit clear this one was for this. Now let me this one now I am going to draw it for this one.

Again the same Para scheme this is A the green line would be for B. So, A has occurred so that the market that means, A has occurred now B. So, this portion if I consider the common area and again you will the same color blue, so this is the overall area. So, this blue would basically signify probability of B intersection A provided A has occurred and this blue area would basically give you the probability of A intersection B. P intersection A intersection B and B intersection A are the same thing probability of A intersection B provided P has occurred.

Now, let us consider the I will basically delete the diagrams and explain one concept, if you remember that probability of A union B. So, those diagrams are just consider here for the time being, so hope you have noted get down I will just erase this.

(Refer Slide Time: 15:05)



I will this is a little bit earlier part, but we have already discussed the show it would not be de difficult is A, I should use colours we should follow this norm sorry.

This is A this is B and this is C good colour combination is coming out to define. Now the hashed areas I will use accordingly let me write the formula first. So, if I want to find out probability of A union, B union, C. So, it will be probability of A pro probability of B now as probability of C. So, now, I have to basically, so this would be A B C.

So, I have to basically subtract intersection of A B intersection of B C intersection of A C and add the part which is common to all of them. So obviously, it will be minus intersection C minus A B intersection C plus probability A intersection B. So, it basically get this area, so if I denote in the in the diagrams and colour sense let me use the color.

So, P this one A, A intersection B would be this part. Then if I use the blue colour A intersection C which is then I use yellow color. A intersect B intersection C and the common area and highlight it with say for example, Ajantha let me check no this would not with their highlighter Medanta.

So, this portion would be this area now let us so this was part of the discussion which you already had. So, I erase it with your due permission and I will come to this example again.

Now, assume an example where the universal set is 1 to 6 again you are role rolling un unbiased die. And the events are a is only the element of the phase 2 and B is they when where only the even number comes 2 4 6. So obviously, A intersection B would be 2. Now in this case when you are considering the probability that B has occurred first, so 2 4 6 has occurred. So, their 3 even 3 three is sample points and out of that if I want to find out what is the probability that A has occurred it will be one out of those 3 because it B is 2 4 6 a has occurred which is 2.

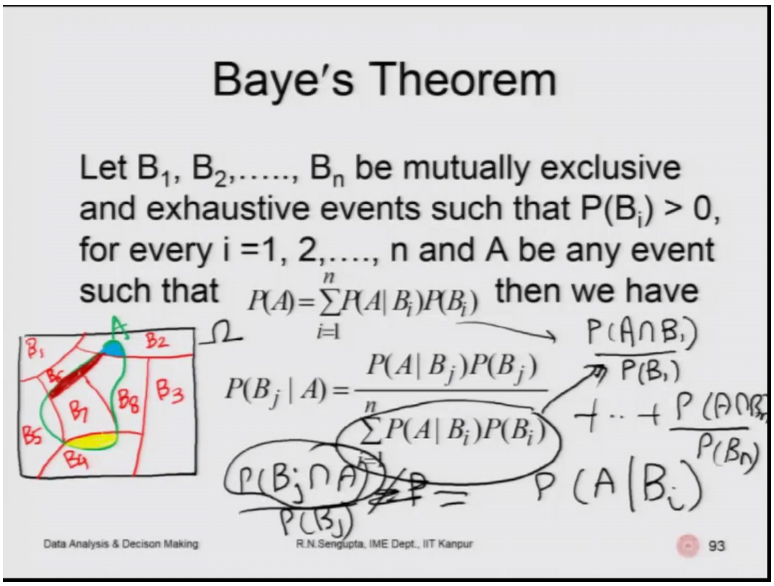
So, the probability would be technically would be 1 by 3. So, how do you find out technically is the probability of B is 3 by 6 for the overall universal set. So, 3 out of the 6 cases 3 means 2 4 6 out of the cases of 6 which you 1, 2, 3, 4, 5, 6. So, that will come into the denominator and in the numerator we will have probability of A. So, A is what one out of 6, so it would be 1 by 6 by 3 by 6 is equal to one third. Now consider other way around when you want to find out the probability of A has already occurred what is the probably B.

So, A has occurred is 2, so what is B? B is 2 4 6 so obviously, B has already occurred. So, the corresponding probabilities would be 1 by 6 divided by 1 up a 6 would be 1. Because probability of A is 1 by 6 and the corresponding probability that B A intersection B would be 1 by 6 1 by 6 by 1 by 6 is 1. Now consider a different example which is not given here considered the third event C is 1 only right.

And in that case try to if you try to find out the probability that given B has occurred what is the probability of C will occur. So, B occurred is 2 4 6 C is 1 so obviously, the probability of C occurring if B has already occurs is 0 hence the probability would be 0 condition probability was 0, technically the probability of C occurring 1 is 1 by 6. But the condition probability in the C will occur provided because has already occurred would be 0.

And you can go from condition probabilities from stage to stage. So, consider this I will I will come to the examples and in explanation terms in quality it sense later on. So, we will consider now the Baye's theorem which is basically the usage of conditional distribution and how Baye's theorem can be utilized that. Baye's theorem. Basically is if you remember the conditional distribution, which I just discuss about 1 minute back, is that given one situation what is the condition the other events would occur. So, now, we will consider the concept of Baye's theorem. So, now, in the Baye's theorem I will draw the diagram which is not there. So, that will make much things much easier for you.

(Refer Slide Time: 21:51)



So, let us consider first let me basically draw the background of the story based on which will discuss let B 1 to B n be mutually exclusive and exhaustive set of event. So, now, this is an important word set up words mutually exclusive and exhaustive. So, now, I will pass and draw the diagram and then make it very evident to all of you that what does the events B 1 to B n are there pictorially such that they are mutually exclusive and exhaustive.

So, this is the universal set now B 1 to B n I mutually exclusive and exhaustive. So, let me draw them let me use the red color, so the remember one thing when the moment you use the word exhaustive it means the B 1 to B n basically comprises if you take the union of them comprises the universal set which is shown.

So, this is, so you have B 1 B 2 B 3 B 4 B 5 B 6 B 7 B 8. So, these are all mutually exclusive and exhaustive now check here the concept of mutually exclusive and exhaustive. If you combine all of them B 1 union B 2 till B 8 it is a universal set for which the probability is 1. And if you find out the intersection of B 1 B 2 B 1 B 3 or B 2 B 8 any taken two at a combination it is always a null set because they know common elements, now consider any event a has occurred.

So, let me draw it in the green colour the consider this is A. Now how is A formed so now, if you consider A being form I will use a different colour consider is blue. So, AI would be formed by the intersection of mutually exclusive event. Obviously, they are not exhaustive which is A intersection B 2. So, this is the first part then it will be A intersection B 6. Now let me use a different colour as per as possible this would not be right.

So, when use a different color say for example, so this is intersection of A and B 6. If this I would use as a intersection let me used a book yellow colour this is A intersection B 4. So, we go in this way now if you considered A, A is being formed by the intersection of these. So, what I am technically writing is this one P A intersection B 1 divided by P B 1. So, B 1 has occurred and then intersection A and B.

So, the obviously, in this case is a null set consider we come to B 2. So, this event has occurred whole B 2 and then we want to find out that the intersection of A and B 2, so the which is this area. So, this area divided by the whole area will give me the first one I continue this still the last part Bn. So, here this is 8 here it is again null set obviously, all combinations would be done. And then if you form made the the corresponding Baye’s theorem would come out like this, but what the actual Baye’s theorem is. So, this is the diagrammatic representation I am now going to explain.

Probability of given a has A occurred what is the probability of any one of them B 1 to B 8 8 in this case or B 1 to B n is this is occurring. Now consider this, the new denominator is this which you already know which is basically P A. But now look at the numerator; numerator is technically P B j intersection A given P. So, this would be, so this is the numerator. So, this is given by if you consider now technically this is true.

So, if this is true I replace this numerator with the multiplication of probability of A provided B j has occurred or P i has occurred multiplied by Pj and then get the corresponding formula. So, which can be extended to 2, states 3, states so on and so forth. So, with this I will end the 6 lecture and continue a little bit more discussion number 10 with very good examples about Baye’s theorem and go into more depth about different distributions and cover it accordingly have a nice day and.

Thank you very much for your attention.