Data Analysis and Decision Making - I
Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 47
Risk concepts

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you. And as you know this is the DADM, which is Data Analysis and decision making one course under NPTEL MOOC series. And we are as you know this is a 12 weeks course for 30 hours total number of lectures is 60 and that is each a lecture being for half an hour and each week we have 5 lectures of half an hour each and after each week you have assignments.

So, we are now in the 47th lecture and we have just started about considering the utility theories in the last 3 4 lectures. And by the way my name is Raghu Nandan Sengupta from the IME department IIT Kanpur.

So, if we remember we were discussing about utilities for decision making and whenever you are making a decision your main ideas to basically try to find out the so called expected utility that is a notional concept based on which discussion maker will proceed. And when you are making a decision we said that that in order to basically compare the decisions based on the concept of expected value, we will take a certainty equivalent.

Certainty equivalent means, that value in units of wealth, whatever the units of the wealth is, that corresponding utility based on that certainty value of wealth would give you the same value of utility or same sense of liking or. I will not use the word disking or liking such that, the gamble or the lottery or then uncertain in event which is there on the other hand would give you the same values such that you will be equally disposed between both of them. Which means the expected value based on C which is basically a shear event and the expected value for the case when you are trying to basically consider a gamble or a lottery those basically should exactly be equal and that is why we you will basically have the certainty value.

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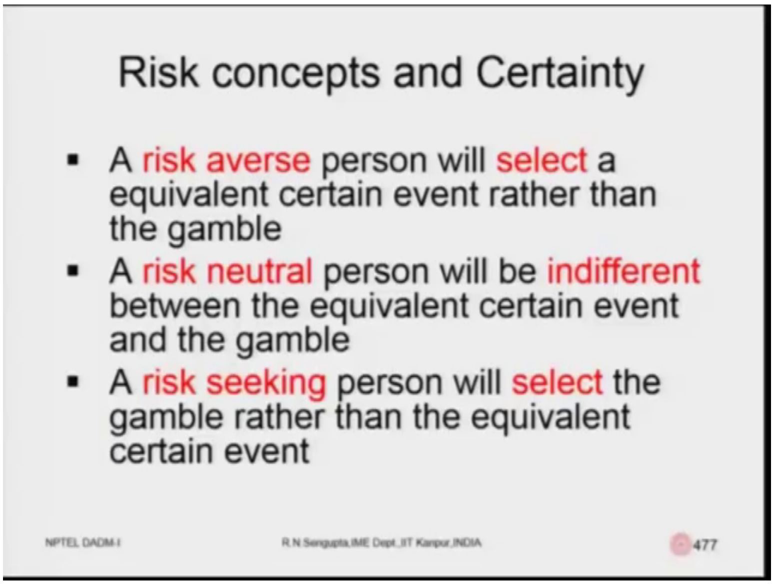

Now, when you are considering the certainty values, I will draw it and I will basically explain it in more details within few minutes. So, but first let me discuss. Consider you have a set of a gamble or a lottery or an uncertain decision on one side of the table and consider there is a fair gamble, in the sense there is a coin which you toss is an unbiased coin probability of head is half tail is half and based on that you get some outcome as W 1 and W 2. And obviously, based on W 1, W 2 you will basically now formulate your own utility, which would be corresponding to W 1 would be U W 1 and corresponding to U W 2 will be U W 2.

Now, if I would ask you what is the expected value, you will basically multiply U W 1 multiplied by half plus U W 2 multiplied by half and basically find the expected value of the gamble or the lottery. Now on the other side of the table there is a fixed value in the sense fixed value in the sense that value has a is based on the fact that the probability is 1 because, there is a sure event and that value is c So, if that value is c, obviously, with the accrued utility it for you would be U of C.

Now if I want find out the expected value of U of C, you will basically sale this very easy because it is the deterministic event so, you we will basically multiply the corresponding probabilities 1. So, your total utility will be U of C.

Now, in case if both of them give you the same net worth, in the sense same sense of liking or same sense of values, in that case I will be in equally disposed or equally have equal level of liking or disliking or indifference between both the gamble, which is one side of the table or the lottery which is on the one side of the table and the certainty event which is on the other side of the table.

Now, you say for example, if another person comes and the person basically has a liking for the lottery, which means the person is willing to take the risk. And the values are same, they are not changing. It is W 1 W 2 and the value C, but now you will be asking that why did not I mention about U W1 here because, the main fact remains that the utility base for the second person would change because, his or her perception of the lottery is now totally different between this lottery on the certainty events. That is why he is he or she is willing to take the risk.

Consider that the other third person comes and in that case the person is willing to take the certainty even not the lottery which means, the again let me again reiterate the values of w1, w2 and c are same probabilities; obviously, are half and half for the shear for the gamble which is the fair gamble. But the corresponding utilities for W 1 and W 2 for the third person is totally different what was there for the first person is also different what with respect to the second person. Hence he or she that the third person will be more inclined to take the certainty event because the person is basically not willing to take the risk.

So, in that if I talk about qualitative perspective of the risk perception of the three human beings of the 3 persons, I would say that the first person is risk indifferent because, he is neither willing to take the gamble or nor willing to take the certainty event.

The second person who is willing to take the risk is basically is a risk lover person because, he is more interested to take that value of the gamble such that, he or she thinks that in the long run the overall net worth of that gamble is higher than the certainty event or certainty value which is kept on the table. I am using the certainty value as a word I am it should basically be the value of C which I am talking about.

And in the case if the person is willing to take the value of c which is the deterministic event in that case, the person is not willing to take the risk hence is basically a risk hater. So, if I basically try to draw it, so let me read it what I said and then I will basically be able to draw it in an much more details.

So, risk averse person will select an equivalent certain event rather than the gamble because, he is trying to away run away from risk. A risk neutral person would be indifferent between the equivalent event and the gamble, while risk seeking person will select the gamble rather than the equivalent certainty event. So, let me draw it, so it will be easy for you to understand.

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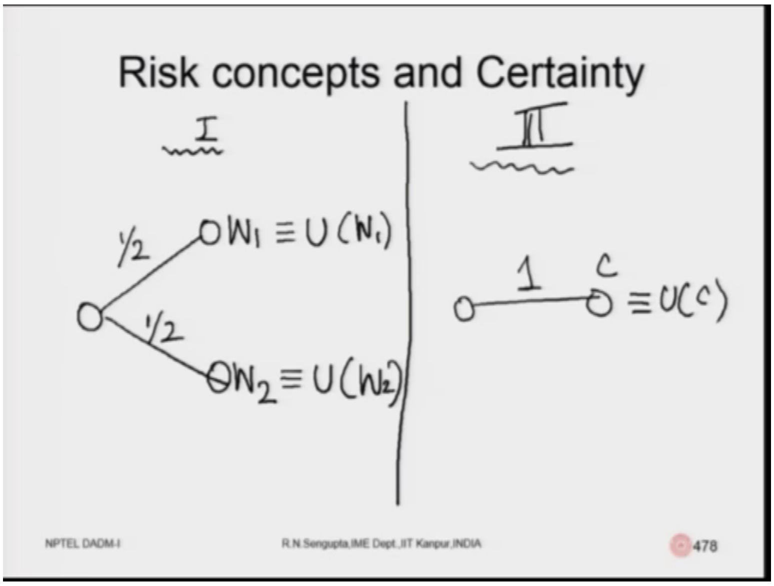

So, consider the let me draw the diagram first, so with the black color So, this is I and this is II, this is W 1, this is W 2, this is half, this is half, this is a value of so called let me consider it C.

So, the equivalent utility would be, I will denote as U of W 1 and the equivalent would be U of W 2, then equal this is probability 1 equivalent to be U of W C sorry, so this is the scenario. And then I will basically draw it in order to basically denote the risk aversion person, risk neutral person on the risk liking person or seeker person.

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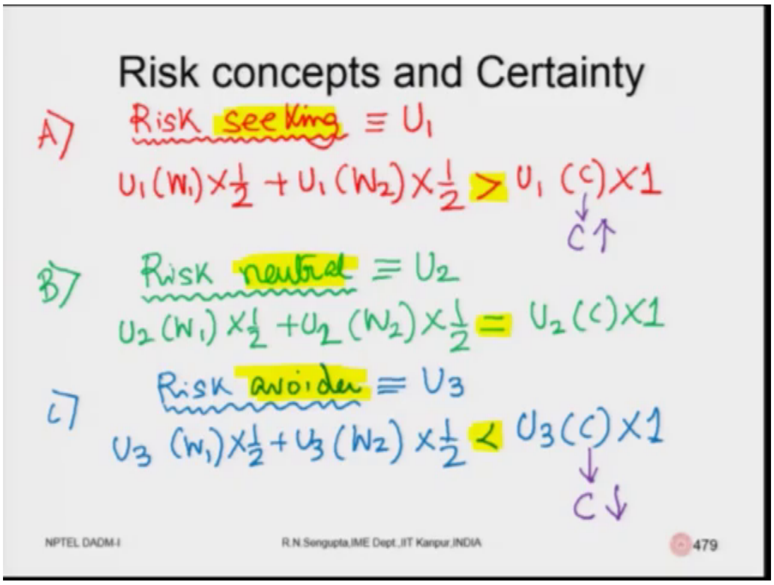

So, let I will use 3 different colors So, case A, so if it is risk seeking So, risk seeking person would be the, I will basically denote the utility first also So, it is basically U 1 function. So, in this case the utility 1 of the W 1 multiplied by half.

So, this would be true because, the sign would be greater than for the next person which is risk neutral, I will use the utility of U 2, so it will be U 2 of W 1 into half plus U 1 sorry it will be U 2 sorry my mistake. And the third one will be C, risk avoider, which will be a utility function U 3. I am using the utility function because, based on that that person is taking the decision. U 3 W 1 into half plus U 3 into W 2 into half is less than U 3 C into 1.

This value of C which I have written technically I should replace with W or because that value should C is a symbol used for the variable which is certainty value, but still I am using it in order to make you understand.

So, what is important is to understand this greater than sign, for a risks seeking person, the equal to sin for a risk neutral person or risk indifferent person and less than sign for a risk avoider, that is point 1. Point number 2 is when a risk seeker is taking that decision which is basically risky, here she is seeing at the level of value for a risky decisions which will yield an higher value even if the probability is there.

So, her or his concern is more to look at the positive side, so the output in the sense he or she thinks that is the there is a chance he or she will get a higher value based on which he or she can take the decision. Already it is neutral person obviously, would be indifferent. Now, if you ask me that if a risk seeking person is a taking willing to take the risk, so is there any certainty value which would basically make him indifferent; answer is yes.

So, I will use a color which is let me check violet, so violet should do yeah. So, in this case, the value I will use the same color for all the 3 case So, in this case the value of C based on which the risk taking person will become indifferent would be the case, where this value of C increases such that at a certain value this is new C value which I have written in violet would basically be higher than the C value which is written in that color. Such that the expected value on the right hand side increases hence, the equality sign would come.

While in the risk neutral person; obviously, the equality sign is there it would not be changing. For the case who is basically risk avoider so, in this case the C value will decrease that blue value, blue means what the value of C which is blue would basically decrease to a certain value such that, it would basically be equivalent to the case such that their equivalent sign for the equation.

Now, this would be true along with the fact that the functional form of U 1, U 2, U 3 are different, so in case say for example, U 1 is square root of W and U 2 is something else, so it does not mean that we are trying to basically equate the functional form of U 1, U 2, U 3, they remain as it is based on that I will taking the decisions. So, I am only considering 1 person, his or her outlook of risk based on which the decision would be taken.

Now, consider I will basically built up the background of the experiment. So, this is the use certainty of to find out utility for decision maker and consider that I do not know your utility function and I want to basically have some idea what you utility function is. So, this is how we will do. We will basically have and consider this being done very simply theoretically. So, you will understand what I am I am trying to say. So, you will have a basically a graph paper as shown here.

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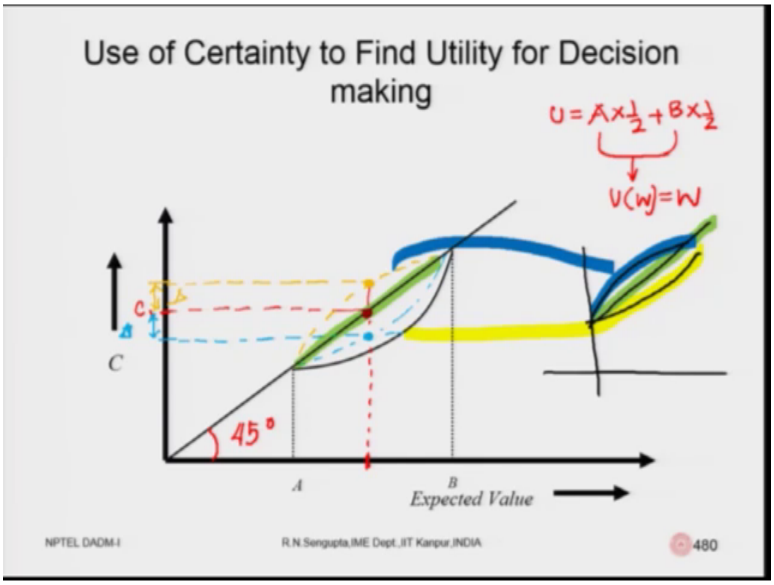

Along the X axis I will marks values of W. So, it will be 10, 20, 30, 40 whatever it is 100, 200, 300, I am not talking about the units, I am only talking about the values. And consider thus the I take 2 arbitrary values A and B and I also consider a fear gamble such that, the probability of getting an chance where the output is A and probability or the chance for getting the output B is half and half.

So, it is a fair gamble. Now also consider which is also interesting to know, consider the utility function to be linear in the sense U W is equal to W. So, if I take a decision and the output is A, my total value for that utility a is U A, which is A again. And the utility when I take a the decision which gives me an out of output of B is UB which is B again.

So, now obviously, in case just to I am not digressing, I am giving another example in case if the utility was square root, so for the decision A, the total utility would have been square root of A, for decision B, the utility would have been square root of B. Now consider that, if the utility is basically the same thing linear one, let me draw a 45 degrees line as shown. So, let us consider the example or the hypothetical methodology how you can find out the utility of a person. So, it can be anything, it can be you, it can be me whoever it is, I want to find our something to do with the utility function.

Now, let us build up the background for that. So, take the slide which is given here and mark along the X axis with the values of W. So, they can be W1, W2, W3 or 10, 20, 30, 40 whatever it is, units do not matter. And take arbitrarily any 2 values A and B and also consider the outcomes are base for a fair gamble for which the probabilities are half and half.

So, obviously, the probability of A coming out is if I put A is half and probability of B coming out is basically half. To add it on, may let us make us another on the other assumption that, the utility function based on which we want to basically draw this graph is linear; which means U W is equal to W. In that case if U W is equal to W, then the utility of a is U A which is equal to A again and utility of B which is U B is B again. Hence, the total expected value for that gamble for which the outcomes are A and B is equal to A into half plus B into half.

So, let me write it. So, the utility is equal to A into half plus B into half because, A and B have a utility which is going to equal to U W is equal to W. Now let us draw a 45 degrees line. So, if you draw a 45 degrees line, so the value of A plus B by half would be here and let me extend it vertically So, this is the point where it touches the utility. And then let me go on to the left hand side horizontally.

So, this is say for example, some value of some value which is along the y axis and consider those values which you draw along the y axis are the certainty values. So, consider this certainty value is C 1. Now with this let us do the experiment, third order experiment as such. You keep a value of C 1 on the table on the left hand side on one side and you keep a gamble fair gamble with values of A and B on the right hand side, where the fair gamble probability as you know are half and half.

So, a person enters, consider you enter. And you are asked that there is one certainty value, which probability one is C 1 that is money. Money some sort it what is the units let us not discuss, it can be anything Dollars, Euros, Rupees, Yens and the on the right hand side, you see there are other two values of A and B and the probabilities are half and half; that means, if you toss and get an head, you get A, if you toss and get a tail you get B.

Now, I ask you that which one would you take. So, you look at these 2; case 1 and case 2. Case 1 is basically the certainty value C1, case 2 is basically the fair gamble A B with probabilities half and half and say for example, you say that you will take the value which is basically the gamble which now, technically means that if the value of C 1, you would one been little bit higher like C 1 from some plus delta at some point of time you would have been indifferent between the gamble which is A into half plus B into half this expected value and the certainty value which is now C 1 plus some delta value.

Now, consider the other notion. In case if you did not take the gamble, but you took the certainty value; which means the certainty value should basically decrease that, C 1 should decrease by some value of delta C 1 minus delta such that, the expected value of a into half plus B into half would exactly match the expected value which you gain getting from the certainty value which now is C minus delta.

Now, if that is the case the set of people who are inclined to take the gamble and the set of people who are not inclined to take the gamble, their utilities of the set of decisions would be either above this straight line which is 45 degrees line or below the 45 degrees line. So, let us consider, how it is true and that is true you can basically think that you understand. If my value of certainty value I want to be low, so it will be C1 minus delta. So, if I basically go along this x axis, let me take a so it is some C minus delta, I consider this is some delta minus. So, it will mean at a certain value, I would basically be now be equal inclined between the gamble and the certainty value, which is C minus delta.

So, if all the values are now lower hand side, I will basically have a graph which would be on the side below the straight line. And hence, it will basically have a risk aversion property if you remember, I had drawn it graph as this decreasing, constant, increasing. So, this human being basically would have a utility function, which will be like this.

In case the person has a utility let me check the color light green, where the value of C 1 and the gamble they are giving me the equivalent which equal. So, the 45 degrees line would be applicable. So, the human being would basically have a utility function which is W. And for the case if it is above, this is delta and this value is going, which is somewhere here, this would be the curve in that case, the values would be the graph here so, it will be the leakage rate.

So, depending on your characteristics, you can basically ask questions and change the value of C one below go done or go up such that you are able to plot it. Now you are asking the question that whether it is possible to change A A and B? Yes, it is possible. You change the gamble, the values of A and B and you can basically plot the different values of C 1 or C 2. So, C 1 is basically expected value A into A plus B by half; it could be say for example, A 1 A and B now becomes A 1 and B 1.

So, it will be A 1 plus B 1 by half that will be the equivalent of certainty value, which is now C 2 and then you can do the calculations accordingly. Now, in the other questions you may be also interested to know whether it is a possible to change the probabilities, but in this case it is a fair gamble. So, we will consider the probability as half and half only. So, this is the background.

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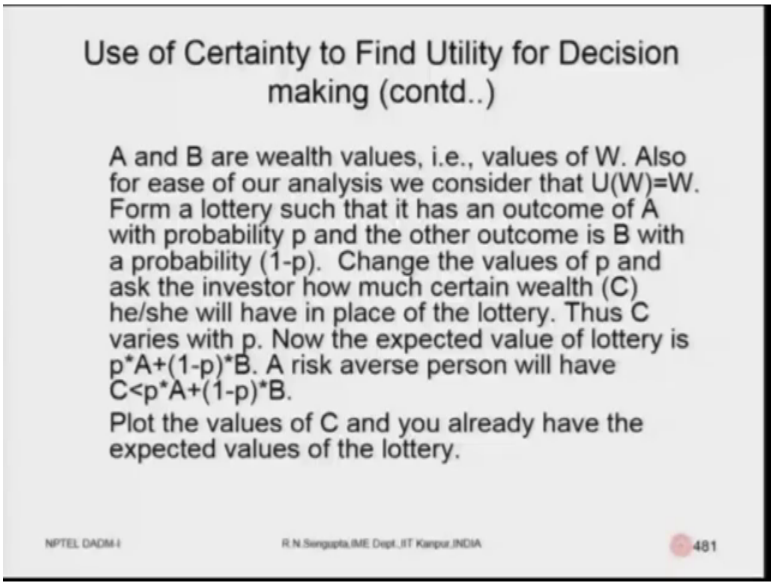

So, A and B are the wealth values, that is the values of W. Also for ease of our analysis we consider that UW is equal to W. You form a lottery such that, it has an outcome of A with probability p and that the other outcome is V with probability one minus p. Change the values of p and ask the investor how much certain wealth he or she will have to have in place of the lottery such that, she or she is equivalent the c varies with p. So, we are technically not considering p as to be changing that p does not become p 1, p 2, p 3, p 4 and hence the corresponding probabilities 1 minus p becomes 1 minus p, 11 minus 2 and all these things we consider p and p and 1 minus p as half and half only.

Now, the expected value of the lotteries now given into p into A, which we consider as half into A plus one minus p, which is again half into B. A risk averse person will basically be will be more inclined towards the gamble and a risk seeking person will be more inclined towards the, risk averse person will be more inclined towards the certainty value.

And there is seeking person will be more inclined for the gamble, we plot the values of Cs C as they change depending on the value changing values of A and B properties we keep at half and half and you already have the expected values of the lottery based on that you draw it and basically find out whether the concave function, convex function and basically mention some property above the utility of that human being, whose utility you are interested to plot.

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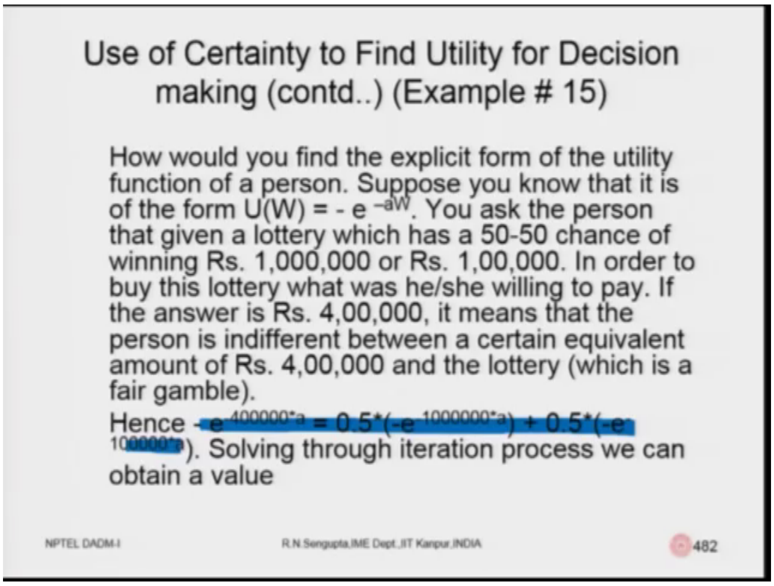

So, how would you find the explicit form the utility function of a person? Suppose you know that is the it is of the form of the of the of the exponential utility function, which is equal to minus e to the power minus A W. You ask the person that given a lottery which has a 50 50 chance of winning. The values are basically 10 lakhs over 1 million on the other values basically 1 lakh. So, you win 50 with 50 percent or half half a probability 11 lakhs and half probability you win 5 lakhs.

In order to buy this lottery what would he or she would be willing to pay? And answer say for say for example, is 4 lakhs, so he or she is willing to play 4 lakhs considering that the probabilities are half and half of wining 10 lakhs and 1 lakhs. It means that the person is indifferent between the certainty value amount of 4 lakhs and the lottery which is a fair gamble. Now because, in that case, the person will be calculating the fair gamble based on the expected value.

So, what would be the expected value? The expected value of the person would be the utility based on 10 lakhs, which would be now minus e to the power minus A into 10 lakhs into half the probability plus minus e into to the power minus A into 1 lakh into half that value would be equivalent to the certainty value. Certainty value now becomes W becomes now C so, it will be minus a to the power minus a into c which is basically 4 lakhs multiplied by 1. So, equate them.

So, if you equate them, what are the only values unknown it is only (Refer Time: 28:07) is unknown would be A you will basically do the iteration and find out the values A accordingly. So, hence you put these values, solving it you get the values away and proceed accordingly. So, you can have different type of utilities and calculate do the calculations accordingly. With this I will close this 47th lecture and continue the discussion further on in the 48th, 49th and 50 related to utility and then come back to the multivariate analysis.

Thank you very much and have a nice day.