Data Analysis and Decision Making - I
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Lecture – 44
Decision sciences

Welcome back my dear friends and dear students, very good morning, good afternoon, good evening to all of you. And, as you know this is the DADM which is Data Analysis and Decision Making I course under the NPTEL MOOC series and this is a 12 week course, total number of lectures is 60 which is of 30 hours total duration. And, we are in as you can see on the slides we are in the 44th lecture which is the last, but one lecture in the 9th week and this each week as you know we have 5 lectures each being for half an hour. And, my name is Raghu Nandan Sengupta from IME department IIT Kanpur.

So, if you remember we were discussing about utility theories and concept of utility expected utility, how these things can be considered and we are given an example that utility theory for to find out the expected value there are basically 2 forms which needs to be understood. One is basically the functional form the utility and one is the corresponding probability. So, one is considering one is a simple random variable, it has a function g of x or x or g of w or w whichever way you define and you have the functional form the PDF of the distribution for the utilize is a preference set.

And we saw that for different utility functions given different decisions, your ranking would change because your expected values are changing. So, consider an example for the utility analysis of the decision sciences.

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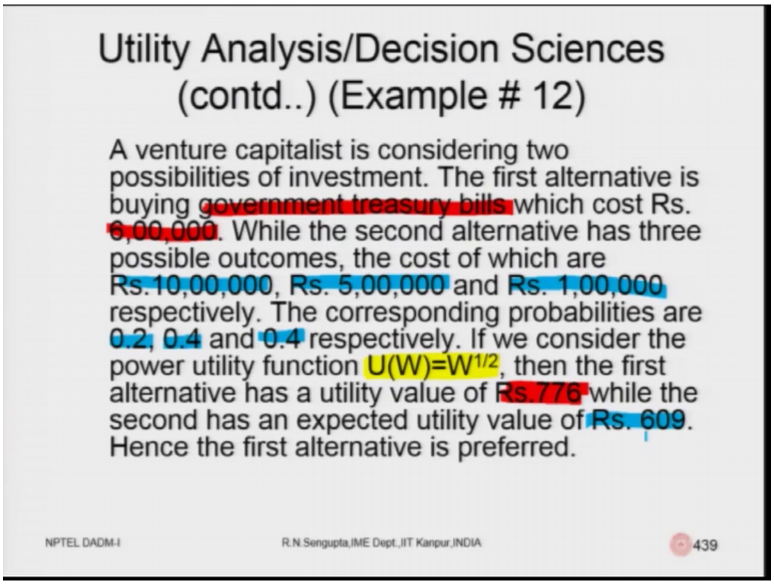

So, this is the situation is like this. A venture capitalist is considering two possibilities of investment. The first alternative is buying government treasury bills which cost rupees 6 lakhs, while the second alternative has three possible outcomes. The cost of which are accordingly 10 lakhs, 5 lakhs and 1 lakh respectively and the corresponding probabilities of these outcomes are 20 percent 40 percent 40 percent. So that means, adding them 20 40 40 makes one as it should be.

So, this is basically what when here when I was talking about the PDF. These are the or PMF, in this case this is a PMF they are discrete events I hence the corresponding probabilities are 0.2, 0.4, 0.4 and the corresponding x values or W values are 10 lakh 5 lakh and 1 lakh. But obviously, you have to find out the expected value based on the utility. So, what is the utility function based on which you are doing the going to do the calculations. So, this is what you will see, let me continue reading. If we consider the power utility function is there and I will discuss what is the power utility function later on which is given by UW is W to the power half then the alternatives would be and the ranking would be accordingly.

So, let me first make a blank slide and write down the values and then we can because this whole slide is full of information. So, I do not want to clutter it further. So, there are 2 slides done. So, we can solve the problems. So, this is the problem. The utility function is W to the power half. So, this is the utility function. I coloured its yellow.

Now, what are the two decisions? I will mark it with different colours so it easy. So, government treasury bills, this is the W value and for the other one 10 lakhs with 0.2, 5 lakhs with 0.4, 1 lakh with 0.4. So, these are the 2 alternatives. So, let me now so, remember government one is red in colour and the uncertainty event whether 3 outcomes probabilities are 0.2, 0.4, 0.4 as blue in colour. So, it will be easy for me to draw. One is red and one is blue. So, let we write the cases. So, this is case I which is for the government one.

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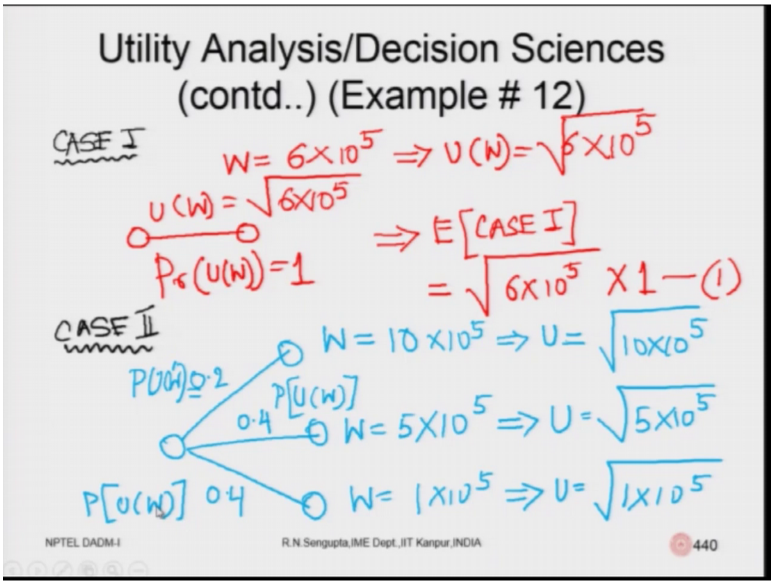

For the government one, you invest some value of W which is equal to if I let me I think its 6 lakhs right 6 lakhs. So, the corresponding value of UW would be 6 into 10 to the power 5 square root of that. So, the decision is W is the investment the probability for UW in this case is equal to 1 and the UW value is given as square root of 6 into 10 to the power 5. So, which you will imply expected value for let me write it as case I will be equal to square root of 6 into 10 to the power 5 into 1.

So, this is the first one. So, that value what it will come is let me remark it here. This is the first order utility is coming on to 776. You can do the calculation. So, find out the square root of 6 lakhs and that will come out to be 776. So, the first part is done now I want to do the second calculation. So, second calculation is 10 5 1, 0.2, 0.4, 0.4 blue in colour. Case II; so, what are the (Refer Time: 06:50).

So, let me use blue colour. So, the outcomes are 3, 1, 2, 3. What are the probabilities? 0.2, we double check; 0.2, 0.4, 0.4. So, the W values; obviously, I will write the UW values. So, W values is given as 10 lakhs, W value is given as 5 lakhs and W value is given as 1 lakh. So, which will imply the utility, I am only writing U so, the space is limited. So, it will be utility will be equal to power 5; it will imply utility is equal to 5 into 10 to the power 5; imply utility is equal to 1 into 10 to the power 5. Now, if I want to find out the probabilities, so, if everything is I will go to the next slide.

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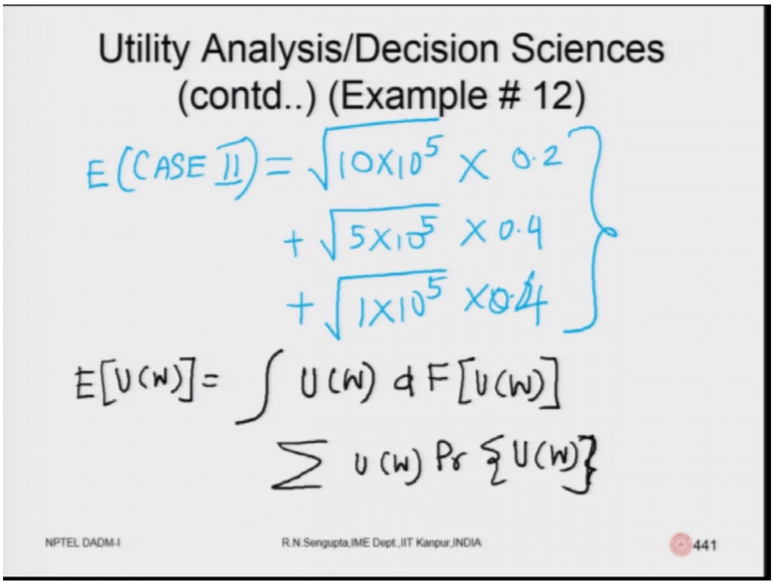

So, case so, expected value for case II is equal to I will show it and so, it will be 10 5 1 into it will be true. So, what are these? These are the probabilities corresponding to first case, this is the probability corresponding to the second case and this is the probability corresponding to the third case; 0.2 plus. So, again I will show for no mistake should be done. This value when you add up that will come out to be 609; let me mark it. So, this is the outcome for the second case. So, once you when you are comparing you will compare 776 with 609. So, the expected value for the first case is more. So obviously, you take this into the under solution.

Now, again I will repeat few important things. The probability and utility may be dependent which is right. So, the utility is W to the power half it may change depending on the problem formulation and the corresponding properties would also change. So, when I was talking about expected value, let me change the colour of the highlighter. So, when you are trying to basically find out the expected value, I will use expected value of this one.

So, remember it will be an integration of UW, this is the random variable multiplied by this distribution function of UW or in the other case if it is summation it will be UW multiplied by probability of this. So, this would give me the expected value. Now, remember one thing; even though it is not at all to be discuss in this problem and in this course this is a note. So, let me erase the first part of the problem which I am sure you understood.

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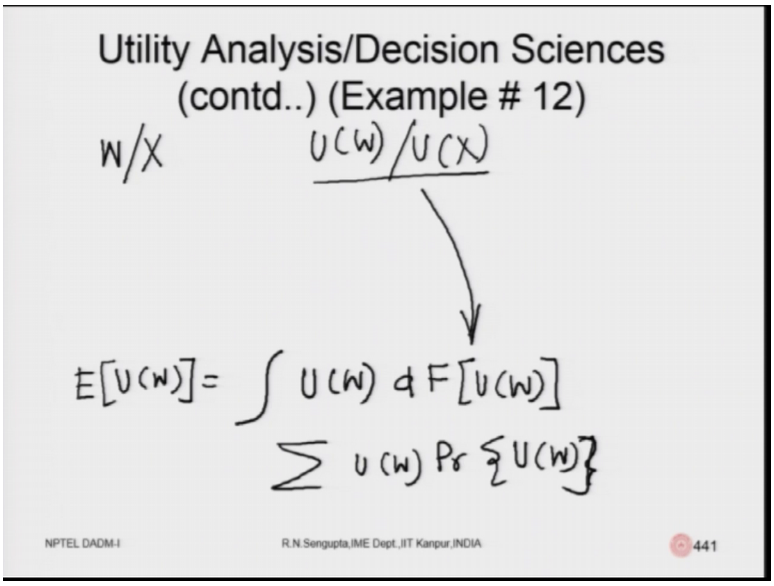

So, in case if the distribution of X or W is given, you can use Jacobean transformation to find out the utility function PDF or the utilization of X and this one which you find out after the transformation are the values which you are going to use here. So, W and X, the probabilities being the random variables probability is being given, you can find out the corresponding PDF or PMF or UW and or UX.

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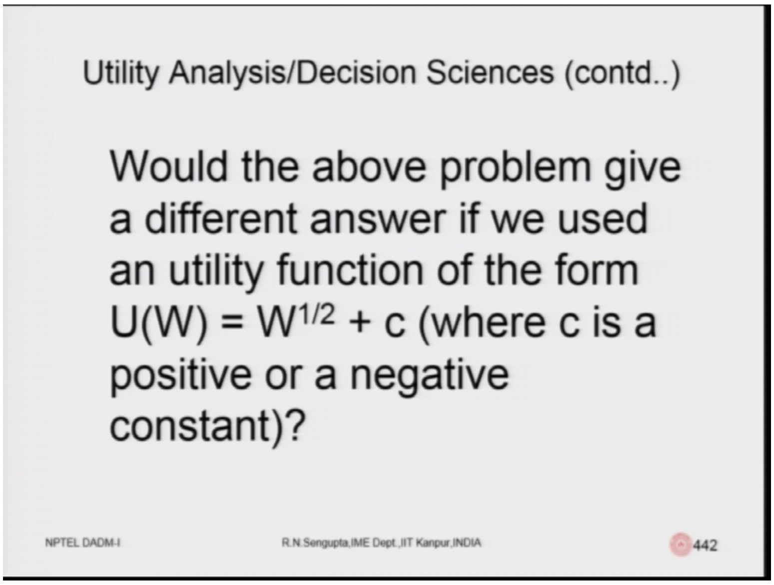

So, the question would rise that what would happen to the other problem given. Would it give a different answer if we have used an utility function which consists of W to the power half plus a constant value? So, where c is positive or a negative value. So, if is a positive value; obviously, the ranking would not be much different provided; obviously. W would not be negative, W to the power half we have already considered, it would never be considered negative. In the practical sense, so any positive ranking which you have already done that would not change if you keep adding c in the same quantum to the function or from the utility.

But, in case if utilities positive and you basically add a negative of these values then you have to basically check depending on whether utility function is quadratic or non on or linear. So, that would always hold to corresponding to the utility function form which you have. So, positive value would basically not have an effect.

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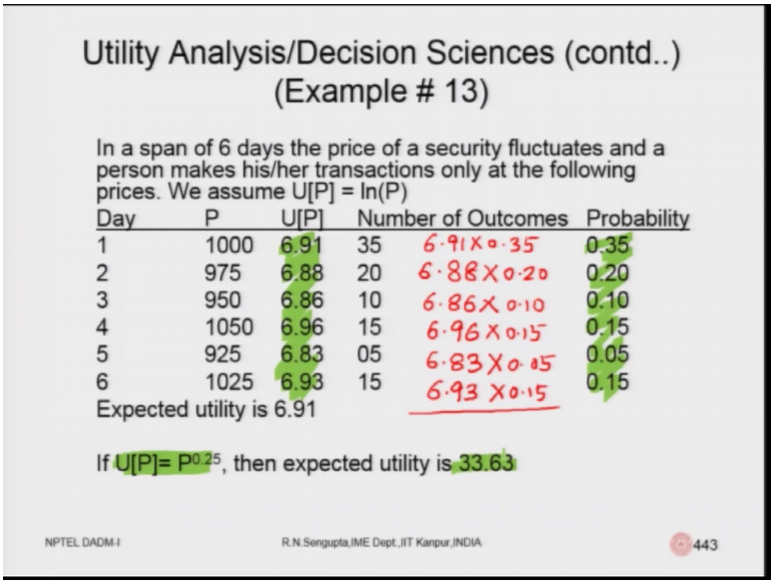

Let us consider the example 13. So, in a span of 6 days the price of a security fluctuates and a person making his or her transaction only in the following prices, she would making he or she. So, we consider the utility function is logarithmic and what we information which you have is that on the first column you have the day, second column you have the price, third column you have the corresponding utility based on the fact that the utility function is logarithm. Then the fourth column is the number of outcomes form which you will basically find out the probabilities and then the probabilities would be given in the last column. So, our main task would be to find out what is the overall expected value of utility.

So, given this what you do again? So, these are the values which you have. I will just mark it and again erase it. So, these are the outcomes. So, the total value of the outcome is hundred. So, the corresponding probability to utility of 6.91 which is the first value is 0.35 so on and so forth. So, if you go along the last column, the corresponding probabilities are 0.2 for 6.8, 0.1 for 6.86, 0.15 to 6 corresponding to 6.96, 0.05 corresponding to 6.83 and 0.15 corresponding to 6.93.

So, the probability; obviously, they should sum up to 1 and then if I need to find out the expected value, it will be like this. I will basically so, I am going to multiply these multiply the corresponding values. So, it will be 6 point my mistake, my mistake. The third column in the last column; so, these are to be multiplied. The values are coming out to be you will add them up that value comes out to be 6.91.

And if you have a utility function U to the power P to the power one-fourth which is here repeated and highlighted different one which is here. So, you basically change the utility functions which would change in this in this place. So, the corresponding changes would be here. I am removing this, removing this also for the time being. So, if I the values of the utility would change, probabilities remain the same. So, in this case the probabilities in the probabilities change multiplied this utility with the probability for P to the power one-fourth and calculate the value comes out to be 33.63.

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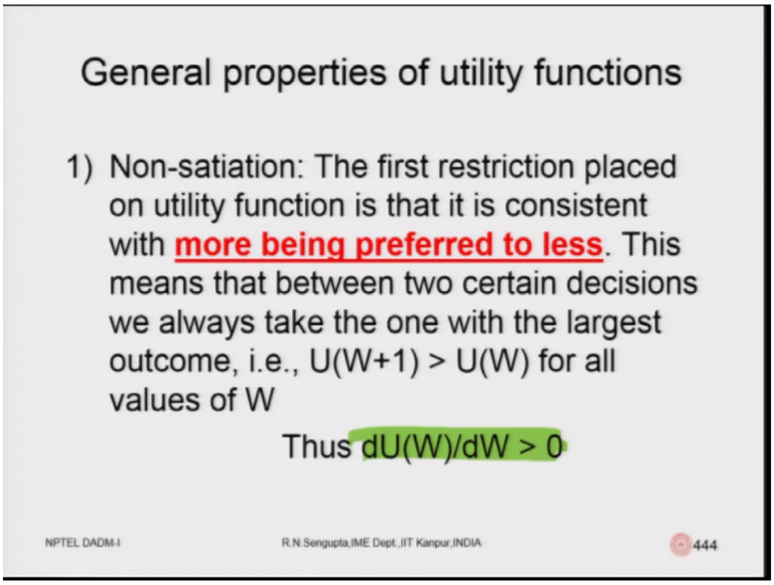

Now, we will come to the important property of non satiation. So, the first restriction placed on the utility function which we will be considering is that more would be preferred to less which means, the decision means that the that between 2 certain on 2 certain decisions or to certain decision the sense that there are 2 uncertain one, but these are the 2 ones which you have in front of you. We will take the one with the largest outcome and we will continue and that is 1 point 1. And point number 2 is that as you keep increasing the wealth technically we will assume that the utility would also increase. So, we will basically want more and more as the wealth increases.

Mathematically, when you solve obviously, it would mean that the first derivative would be would be greater than 0 as here it is. So, the concept of non satiation would be that the first derivative is greater than 0; that means, more I give you more you want. So, that would be one of the building block based on which you will be basically doing the calculations.

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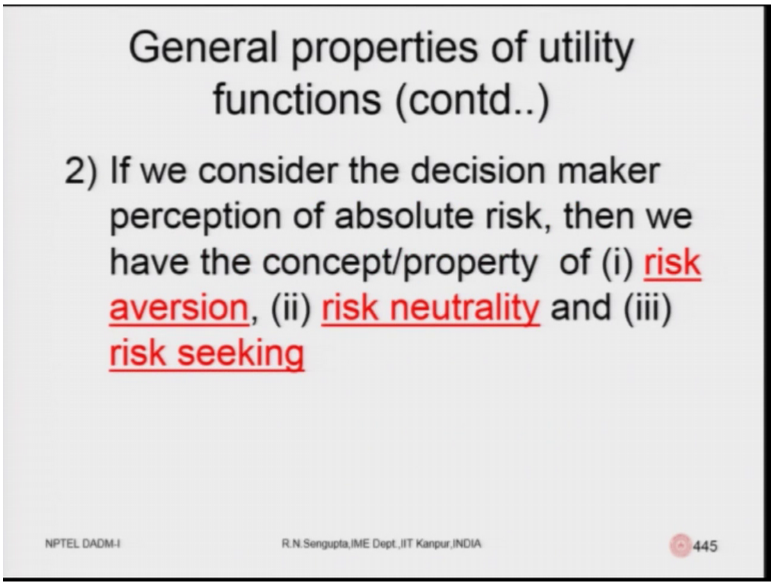

Second property; if you consider the decision maker perception of absolute risk. So, he is risk averse, she is risk averse, loves risk or hate risk whatever or indifferent, then we will basically club that human being having any one of these 3 properties which is risk aversion; that means, I want to basically run away from risk. Risk neutral means and indifferent rather risk is increasing decreasing, it does not bother me. Another case would be the third one would be where I run towards the risk in order to take more and more risk.

So; obviously, that would depend on what is my risk and return profile and that would come basically from the concept of the non satiation and the onset of risk property. So, let us consider a simple gamble which is known as the fair gamble.

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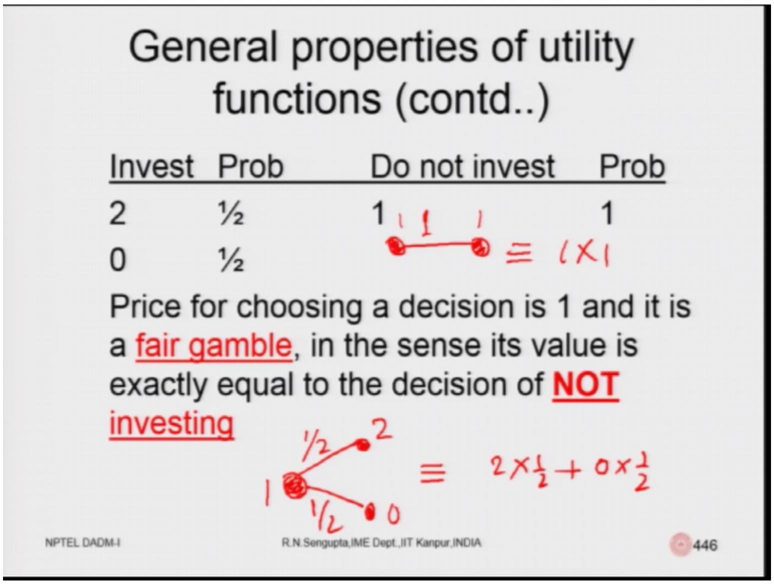

Why fair? Because you have, in one case you have the unbiased coin probabilities are half enough and another case you have a coin where if the probability is 1 biased coin. Now, you consider that if you invest 1, 1 unit and that 1 unit, 1 rupee, 1 dollar, 1 yen, 1 dirham's, 1 Canadian dollar, I am not going to that detail that we will come later on. You investing 1, I will basically get in the case when there is a probability when the head comes, I get an outcome of 2 in that scaling of 2 units and if your tail comes I do not get any return. And case 2 if I which is a biased point, if I invest 1 or basically an input 1 unit I get the same input back 1 unit and I am not talking here anything about to do with the utility, considered is intrinsically already there.

Now, if and when I am trying to basically compare these 2 and decisions so; obviously, my main task would be to compare them considering the expected value. So, if I have the expected value what I would do? I will find out the expected value for the case I and compare with the expected value the case 2. What is the expected value for case I? Case 1 expected value would be 2, considering as the utility for the first term into half that is the corresponding probability for that utility outcome plus the value of the utility of the second term which is 0 multiplied the corresponding probability of the term which is again half. So, if we 2 into half plus 0 into half you will basically have a total output expected value as 1.

And in the next case, in the case 2, if you do not invest; obviously, the corresponding probabilities would be do not invest in the sense that I invest 1 I would be get back the same amount in that in that sense; probability is 1. So, in that case I invest 1 and get back 1 the expected value is 1. So, in case 1, I am I am initially investing 1, the outcomes are 2 and 0. So, it is like this. So, invest 1, probability of half I get 2, probability half I get 0.

So; obviously, the expected value would be 2 into half plus 0 into half. And case 2 when I have invest 1 I get back 0 invest 1 and I get back 1 probability is 1 the expected value is equal to 1 into 1. So, both the expected value is same. Now, if I am indifferent which means, I am I am risk neutral. If I want to prefer the risk, I will technically mean that I want to I am a risk lover person if I am want to basically take the deterministic event. So, which means, that I am basically trying to run away from the risk, which is a risk avoider.

So, risk neutral person risk taking person is neutral risk avoiding. So, risk I want to take risk means I will go for the uncertain event. Indifferent means, I am indifferent I am indifferent to any of these 2 decisions. I can either take the first one or the second one or vice versa and in the third case if I am want to basically be sure I want to reduce my risk I will basically take the second decision where the probability is 1.

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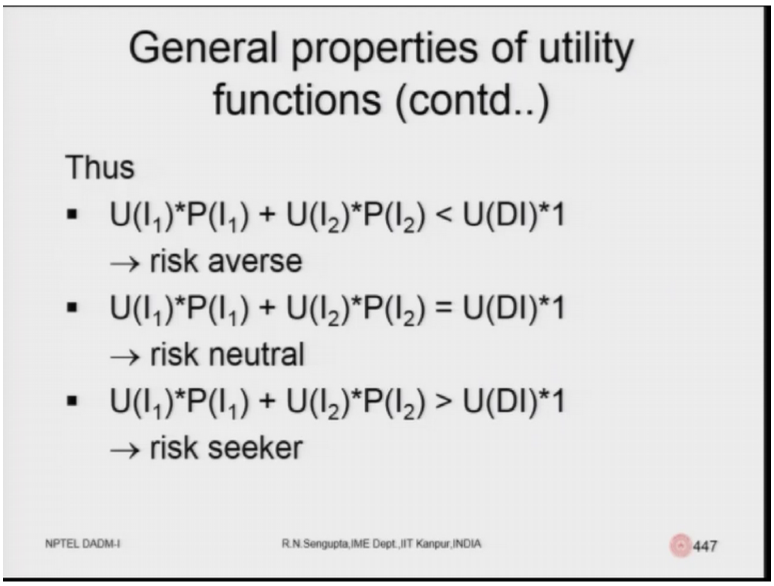

Now, the general properties of the utility is considering the functional form. So, when I am basically risk averse it would technically mean that the corresponding probabilities for the expected value remains the same. What is more important to note is that the perception based which the decision is changing would depend on what type of person he or she is. Whether he wants to love, whether he or she wants risk is indifferent to risk and basically he is wants to basically take risk.

So, in the case when I want to basically avoid the risk it would mean that the so called expected value of the sure event that general then return for the person who wants to run away in that risk would be much more than the case when the event is basically a non deterministic one; which means, the utility of investment 1 multiplied by its corresponding probability plus and there are 2 arms remember; utility of investment 2, multiply if the property of investment 2. In this case the expected value from here from the arm which is non deterministic that total value to the person who is a risk averse person would be less than the certainty event.

So, technically the expected value is may be same, but the overall judgment based on which he or she is trying to find out that which value will give higher returns which value will give lower returns. He or she will be more attracted towards the sure event. That is why she he or she is a risk averse person. In the case when the second bullet point when the place is basic play person he basically indifferent whether take the certainty value sorry my mistake; whether take the sure event, whether do not take this sure event and what is the expected value, so on and so forth.

So, if the person is in this in this case, the person is inclined indifferent to both of these decisions, which would mean that the expected value of the arm which is giving me the deterministic event which is on the right hand side that will be U of the DI which is the deterministic investment into 1 would be equal to the expected value of the non deterministic event which will be U 1 into P 1 plus U 2 into P 2 and that value should be exactly match with respect to the sure event. So, that is that is the perception, the person is this neutral.

And finally, for the third case, if the person wants to take the risk the fact would be than the expected value of the non deterministic values, so called intrinsic expected value would be more than the expected value of the sure event. Hence, the person would be attracted to take the decision which has non deterministic concept into the picture.

Hence, the person would be seeking the risk because trying to basically take the risk he or she thinks that in the long run the average return would be much higher. So, the first person who is risk averse is thinking of only on the downtrend. Second person is indifferent and the third person is mainly thinking of the upward trend.

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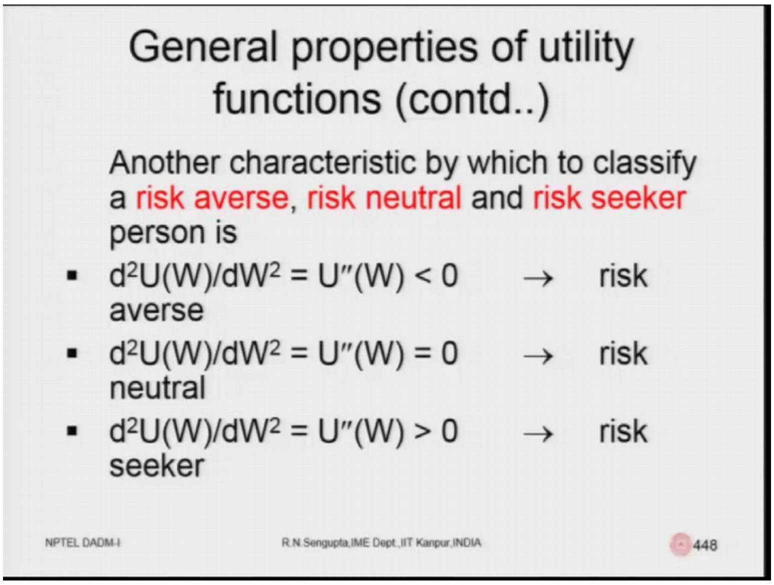

Another characteristics by which to classify risk averse risk neutral and risk seeking person is by this. So, now what you when you want to analyze in the first case, the non cessation is always true whether you want to love the risk, whether indifferent to risk you are hating risk.

But the actual property base to in which you can analyze whether the person is a risk averse person, whether the person is a risk neutral person, whether the person is a risk loving person would basically come from the second derivative; so, the second derivative of the utility function, if it is so they can be 3 things from the marginal rates. The first rate, so the rate can be increasing at an increasing rate, rate can be increasing at it as a constant rate and a rate can be increasing at a decreasing rate.

So, based on that when I am basically want to do the start studies then obviously, I will come to the second moment. So, the second moment if it is positive it means that is increasing at an increasing rate. So, hence the person if you see here is a risk seeker because more I give, more you want risk. If the second derivative in increasing at a constant rate then obviously, the person is risk neutral. I will come to these details in very simple diagrams and the third person the third case would be the person basically wants to avoid this to his overall decision would be I will increase my overall utility, but I will increase it as a lower it; that means, it is increasing at a decreasing rate. In that case the person would basically be risk averse.

So, I will basically continue discussing this in the last class of the 9th week and I try to slowly basically give examples in a much better concept such that utility analysis is clear to you; such that we can basically proceed for the point of dictation and other methods which are there under the multivariate statistics. With this I will end the class, have a nice day.

Thank you very much.