Data Analysis and Decision Making – I
Prof. Raghu Nandan Sengupta
Department of Industrial & Management Engineering
Indian Institute of Technology, Kanpur

Lecture – 43
Utility analysis

Welcome back my dear friends, a very good morning, good afternoon, good evening to all of you and this is the DADM, which is Data Analysis and Decision Making 1 course under NPTEL MOOC. And as you know this is a 12 week course, total duration being 30 hours 60 lectures and each week we have 5 lectures each being for half an hour. And we are in the 9th week which in; that means, if you see the slide this is on the 43rd lecture. So, if you, if you and my name is Raghu Nandan Sengupta from IME Department, IIT Kanpur.

So, if you consider the concept of utilities and the decisions which I said would be a major chunk later part obvious we will come to the other things for Multiple Linear Regression Anova Manova accordingly. So, what we are trying to do is that we are trying to basically analyze that given for any decision or on any utilities, what is the net worth based on which you are trying to take the decision.

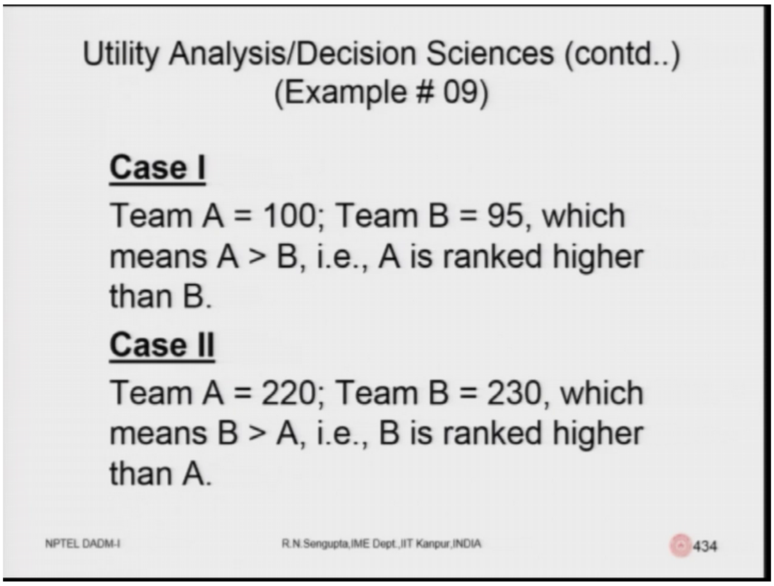
So, for the football match considering the teams had one sets of wins and one sets of draws, and yes a question the which would definitely be, rise from your side would be, what if the number of wins and losses are also different in both tournament 1 and tournament 2 which is case 1 and case 2, which is absolute possible. There is no question about that, but what I am trying to basically give you is an analysis that given that the wins and losses in numbers are same.

But if the points accrued for the wins and losses are different that how do you analyze the situation? So, they would definitely be as you can understand, they can be definitely two ways of looking at the problem. And one is in the case where the so, called net worth accruing for each and every decision is different, which is the main focus of this part of the discussion.

And another is basically the total number of outcomes accruing for each and every decision is each and every decision is different, which is the number of wins and losses, which you have which is also dependent on the so called accruing concept of the values which are going to come.

But the concepts of numbers which will lead onto probability would not be discussed in this set of lectures. Obviously, we will give fleeting examples for that we will give some conscious about that, but we would not be discussing in details using the so called concepts of probabilities and probability distribution and theory.

(Refer Slide Time: 02:58)



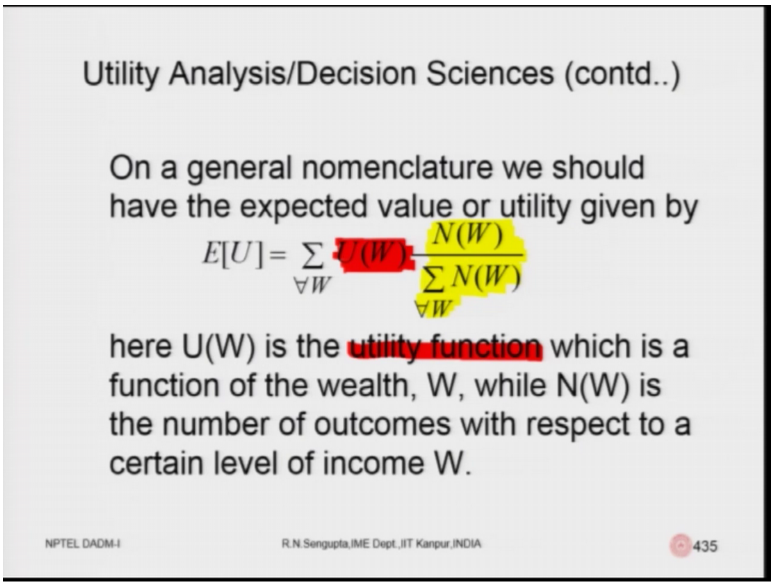
So, continuing the discussion for case 1 tournament 1, team A has 100 points; that means, based on the fact that you multiplied by the, the points into the number of wins, then multiply the points with number of corresponding points with number of losses, multiply the corresponding point with number of a draws, sum them up the value comes out for A is 100.

Similarly, the values which comes out for B is 95, which means that A would be ranked higher based on the point scale. So, A would basically the winner considering that A and B were competing for the, the price consider tournament 2. Consider for this case, if I multiply the corresponding points which is for the wins, loss and loss and draw are different from case 1 tournament 1. And; obviously, we are keeping the number of losses number of wins number of draws for A and B are same which I did mentioned very categorically before I just I, when I started this lecture.

This is been the same then the number of points of A would be 220 number of points accrued to from point team B would be 230 which means that now the ranking system would be reversed, B would be ranked higher than A. Hence, A would be the winner. Now, the picture which is coming out is that if I change the O overall accrued points for any decision if that total points level is changing; obviously, it would mean that the ranking system would change and; obviously, it would also imply that intrinsically the corresponding so, called numbers would also be dependent on the point systems or the scaling systems or the scoring system, yes it will be true.

But as I mentioned just before this discussion has just started I will also mention it once again that we would not consider the so called numbers based on the outcomes as being dependent on the accrued points. And I am using the words accrued points A number of outcomes all this thing interchanging the, but basically I will come to what is the actual term which is used in our analysis.

(Refer Slide Time: 05:20)



Now, here it comes, on a general nomenclature, we would have the, that the expected value or that could the so called the multiplication, what we do for the points into the numbers sum them up.

So, that is basically utility which is given on the net worth. So, the expected value utility given would for N decision would basically be multiplication of two terms, one is the functional form the utility which is UW. So, that is what the points were; that means, for a win you win 5 or a for a loss you basically lose minus one whatever it is. Similarly for the case when the concept of ranking was there between the in the first example you gave 15, 10, 15 and all this things.

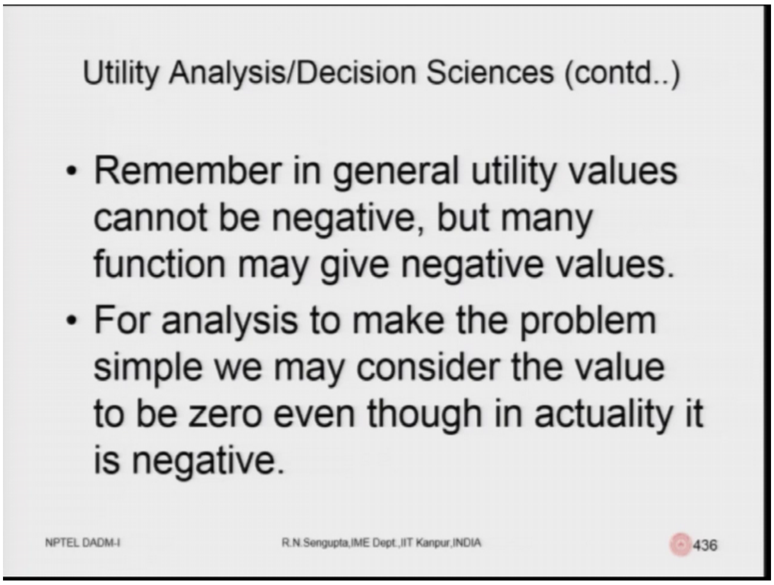
So, those what basically the expected value the value based on which you want to find out. So, here the UW would be those points of 15, 10, 15 and so on and so forth and the second will value or the term which you find here. So, this second term is basically the probability, the first value is basically the utility. So, if you remember I mentioned that how many such wins how many such draws will be there for X 1 Y.

So, this is basically what I you need to find out that numbers of wins and number of draws this technically should be in the terms of a probability. Hence, we would basically have a probability distribution on us and a cumulative distribution which for the time being we will ignore that it will be a priori given it would not be dependent on the utility as such. So, this is the utility whether marked by red. So, what I want to find out is basically multiply the utility with the corresponding utility with the probability of the utility. And sum them up and basically that is what is basically be given by the expectorated.

Here U W is the utility function which is a function of the wealth total amount of money which is given. Now, in the example of say for example, when you are taking a decision when the values are 15, 10, 15 for the first example, I am considering the wealth as W and the corresponding utility also coming out as W; that means, U W is equal to W even though in the actual practical sense it may not be true, I will come to the examples later on.

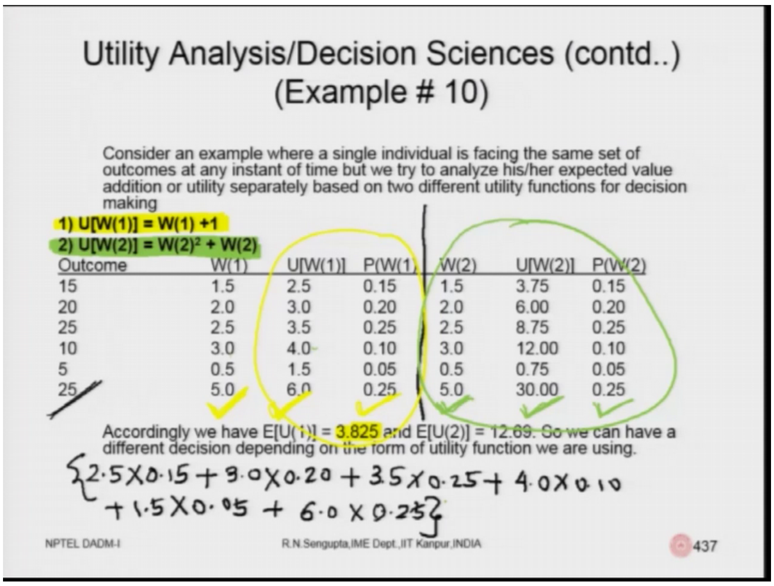
So, here UW is a utility function which is a function of the wealth while N W which is the numerator for the right hand side is the number of outcomes which with respect to a certain level of the income and if you see and you divide by the summation of a N W that would basically give you the probability. So, these are multiplying a value with this corresponding probability and summing them up to get the expected utility.

(Refer Slide Time: 08:09)



Remember in general utility values cannot be negative and that means U W cannot be negative. But they may be examples where they can give get negative value for our analysis to make the problem simple we may consider the value to be 0 even if the values are coming out to be negative. So, if it is a minus 2 will commit, commit consider as 0, if it is minus thirteen we will consider at a 0 even though for actual explicit problem formulation and solution, we will consider the negative value also.

(Refer Slide Time: 08:49)



So, let us consider another example. So, consider an example where a single individual is facing the same set of outcomes at any instant of time, but we would try to basically analyze his or her expected value of addition or of a utility separately, based on the fact that there are two different utility functions which is applicable for him or her. So, what are the utility functions? In the first case let me change the color for the first case. It is basically a linear utility function which I am marking by yellow. For the second case it is a quadratic utility function which I am marking by light green.

If the person has two different utility functions for due to different situations, it can change and based on that I am trying to find out that how would that same person with two different utility functions rank the decisions. So, let us consider the problem as such, the outcomes are given on the leftmost column. So, these outcomes are what already explain them they are in 15, 20, 20, 5, 10, 5 and 25. And the second third and fourth column which I will just marked with the so the, I will mark in the second the third and the fourth. They are all corresponding to the utility function that I should be using a different my apologies it would be better if I use the second, if you can see it the third and the fourth.

They are with corresponding to the first utility function which is also marked in yellow and is a linear the last 3 columns fifth, sixth, seven are corresponding to the utility function two which is also marked in green. So, the first column basically outcomes remains same and I will come to that why they are same later on. Now, basically it means that if let us go column wise. So, let us first consider the second column. So, these are the values of the wealth I am taking arbitrarily.

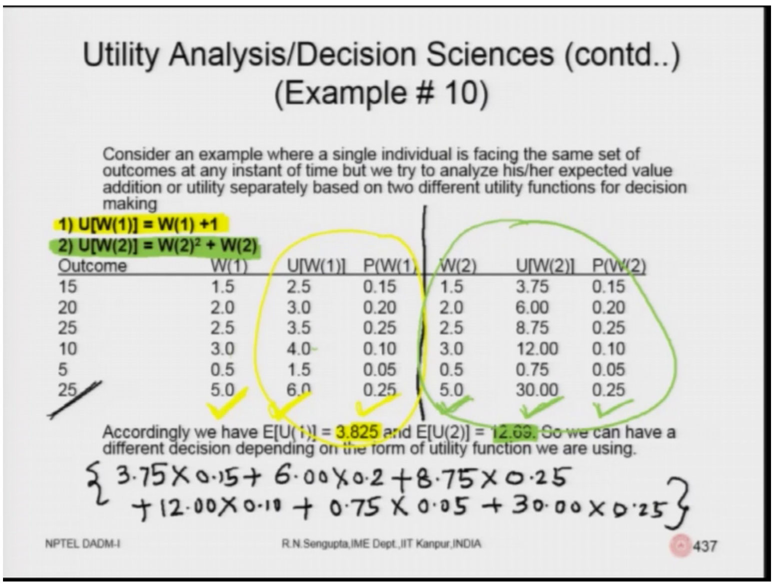
So, I am considering wealth's of 1.52, 2.535 then it should basically be I am considering values of 0.5 and 5. So, this value there is some mistake I will come to that. So, technically this value would be 0.5, ok. It would be 0.5 depending on my apologies, it will say it is right. So, how do I find out these values, these values are. So, if we use the first utility function this is W 1 plus 1 so, though you wealth values are given. So, the wealth values are, are 1.5 plus 0.1.

So, that comes with 2.5 which is right 2 plus 1 is 32.5 plus 1 is 3.5, 3 plus point plus 1 is equal to 4.5 plus 1 is a 1.5, 5 plus 1 is 0.6 is 6. Now, I need to find out the probabilities. So, the probabilities are based on the outcomes. So, if I basically consider the total outcome which is the first column. So, the total value is 100 which technically means out of the 100 such sort outcomes which I have, if I find out the corresponding probabilities for the base case, where my wealth is a 1.5. If we consider the first row utility is 2.5, then the corresponding probability for that utility which is 2.5 will come as, as 15 by 100 which is 0.15.

Similarly, if I consider say for example, the 25 outcome which is the last row, in that case, the wealth is 5, the utility is 6 and corresponding to a utility of 6 in the number of times, it will basically be supported, would definitely be 25 by 100 which will be 0.25. So, once I have all the values corresponding to utility and probabilities, I basically multiply 2.5 into 0.5, 0.15 plus 3 into 0.2 plus 3.5 into 0.25 plus 4 into 0.1 plus 1.5 into 0.05 plus 6 into 6, 0.25; I add them up. So, this is what I do for the first, which is 2.5 into 0.15 plus 3.0 into 0.20 plus 3.5 into 0.25 and let me put a bracket plus 1 2 3.

So, 4.0 into 0.10 plus 1.5 into 0.05 plus 6.0 into 0.25 this, this is the utility for the first case which basically comes out to be and highlight again is 3.825. Similarly, let me go to the next set corresponding to the second utility. So, second utility sets this so, I if I let me, so this value comes out to be 3.0 this value which I am just highlighting on marking. So, this comes out to be 3.825. Now, I remove this. So, this is basically a calculation based on first utility.

(Refer Slide Time: 15:25)

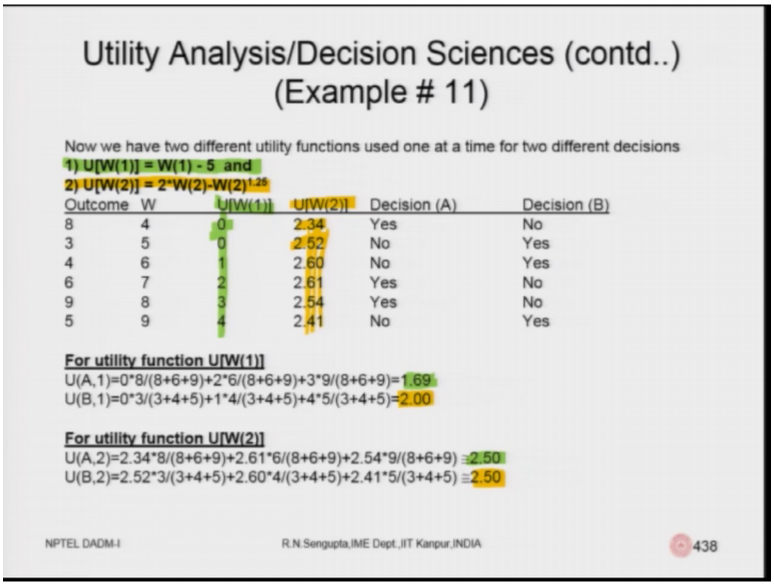


Now I will go to second utility. So, they would be 3.75 into 0.15 plus 6 into 0.2 I did skip it sorry for that. So, if I want to basically have a, a 3.75 utility what I would do based on the quadratic utility function which is the green one, it is a 1.5 whole square plus a W 2 ; that means, 1.5 whole square 2.25 plus 1.5 would basically come out to be 2.75.

Similarly, when I go to the second one, it will be 2 square which is 4, 4 plus 2 is 6. Then if I go and go to the value of 3 where I am basically pointing my, from this pointer, so it will be 3 square which is 9, 9 plus 3 is 12. If I go to the last one, it basically be 5 square into plus 5 which is 30. So, if I do it, so, I get the second the last column. And now I am basically I am doing the calculation exactly as I did 8.75 into 0.25. So, plus the next value would be 12 into 0.1 plus 0.75 into 0.05 plus 30 into 0.25 that value I find out the actual value would come out as 12.69 which is I am mark light green.

So, if I want to compare the same type of decisions outcomes for the same person, but two different nuclei functions, you can find out the utility function two would basically give him or her that person a higher set of points, based on which the ranking may change. So, we can have a different decision depending on the norm form of a utility functions, we are using and you can have values as shown.

(Refer Slide Time: 17:40)



Let us consider an 11th example say different one, now we have two different utilities use one at a time for two different sets of decisions. So, if you remember I am, did mention that the utility values functions are changing the various probabilities are changing.

So, basically this considering both of them together so, again I will basically mark it colored. So, the green one is basically for the linear utility function and the orange one is for the so called non-linear utility function. So, linear value is W I minus 5 and the non-linear one is 2 into W 2. So, W 2 is A W 1, W 2 are the values which would basically be the same, but I am just using a nomenclature in order to make you understand or a give us a better picture. So, W 1 and W 2 technically, they are not different they are the same variable.

So, a utility function 2, would be 2 into W 2 minus W 2 into 1.25. Now the outcomes are given like this which is the first column. So, outcomes are given as a 8, 3, 4, 6, 9, 5. And the wealth values which is also I am considering arbitrarily are given by 4. So, I should not be highlighting it sorry R 4 5 6 7 8 9 now I do not need to do the calculation. So, A basically I mark them as a green and orange. So, I will do it accordingly.

So, for the utility one, the first utility based on wealth is W, W would be W 4 minus 5 is minus 1, but as for the convention we will take as a negative being 0. So, these are basically the values which I have; for the second value it will be 5 minus 5 which is 0, third one will be 6 minus 1 which is 6 minus 5 is 1 is given 7 minus 5 is basically 2 is given 8 minus 5 is 3 is given that 9 minus 5 is 4 is given.

Now, what I do is that I basically analyze for the decision making E, where the utility is the one which is green in color which is linear. Now the decision has for 8, I will mark it also, so and I will basically combine them accordingly. So to have a comparison even though I am marking it as a green it would be utilized for the second utility function also remember that.

So, I will keep changing the colors. So, please note it as I do it so; that means, if the things become much clearer. So, for decision A the consider utility as 1 which is green one, it is yes, no, no, yes, yes, no which means are the sort of go, no go gate where the decision is, is allowed and the decision is not allowed. So, similarly for decision B which will and also can be used for both for the green and the orange one which is utility 1 and utility 2, it will be utilized.

So, let me basically highlight, this because it will be utilized. Now, I am based on the green one, I am going to do the decisions and I will mark it accordingly. So, let me go one by one, for decision A utility A 1. So, it will be if it is yes. So, the total outcomes would be 8 would be supporting it and total number of such supports would basically come yes, yes, yes. So, the values would be 8 plus 6 plus 9 would go into the denominator as it is.

So, the corresponding value of a utility 0 multiplied by 8 divided by 8 plus 6 plus 9, this is the first term, next term where it is, yes, it is utilities 2. So, it will be 2 into yes. So, it will be 6 in the and the numerator and in the numerator and in the denominator all the yeses which is 8 plus 6 plus 9 and lastly for the last value to be yes again for utility is 3, 3 multiplied by 9 in the numerator and in the denominator, it will basic be the ratio to convert into the ratio it will be 8 plus 6 plus 9 as it is given.

So, hence the value comes out to be 1.69. Now, I will pause here, I will basically go to the utility function 2 because I want to basically compare this. These are rather than comprising of the utilities are of comprising of that of the decisions. So, what I would do is that, let this color which is there on B remain and remove this color because now I will basically analyze it with the second utility. Now, the second utility color is basically orange, done.

Now, consider the second utility for the second utilities basically a function of 2 into W minus W to the power 1.25. So, the outcomes are given as, as already mentioned 8, 3, 4, 6, 9, 5, I use that in the formula and get the utility values corresponding to 8, 3, 4, 6, 9 5 as. So, the corresponding decisions for A, now you will see yes, yes for corresponding to the values of A wait, ok. So, these are not corresponding to B my. So, I basically so, these values are no, yes, yes, no, no, yes I want to find out the corresponding probabilities, it will basically be multiplied by the corresponding utility for the utility values will change no.

If let me take one value is to how did 2.34 come it is basically 2 into W 2 minus W 2 to the power 1.25. Put W as A W 2 as 4, the value comes out to be as 2.34. If I consider say for example, wealth of 9. So, the utility would be function will be 2 into 9 minus 9 to the power 1 to 0.25, but the value comes out to be 2.41.

Now, when I basically take such values utilities have to be multiplied by the probabilities. Now these are corresponding to the decision B. So, the decision B has no, yes, yes, no, no, yes. So, hence the corresponding probability which will be coming out for the case would be for utility 1 and 2 basically, first highlight utility 2 because I am comparing 1 with A 2 with B and again combine them. So, the utilities would be for decision B decision B should have been the different color my mistake because coloring scheme they are not fine, it is quite confusing.

Now, I combine them. So, basically it will be 2 point for the value of decision B would be basically be 2.25 which is this multiplied by yes becoming here yes number of time is basically 3 divided by how many such cases are there is 3 plus 4 plus 5 as it is, it is 2.52 into 3 divided by 3 plus 4 plus 5, then for the next one, where it is? It is 2, 2.6, 2.6 multiplied by the number of outcomes is favoring which is 4, 4 divided by 3 plus 4 plus 5.

Similarly, for the last one, it is 2.41 multiplied by 5 divided by 3 plus 4 plus 5 and if you add up the values come out to be 2.6, 2.50. Now, basically let us consider both the decisions A and B are there for utility 1 and 2. So, consider decision B for utility 1. So, let me again change the color. So, now, for utility 2 which is B for decision B and then I am basically considering the utility as linear.

So, the linear functions would be it is 0 because for the first case it is for considering the utility as the second one, it will be 0 for the utility for the first one using the decisions B wait, wait it will be in the case, it will be 0 multiple because let us say yes multiplied by the corresponding frequency, it will be 3 divided by how many such yeses are there, it will be 3 4 and, and 5. So, it will be divided by 3 plus 4 plus 5.

The next one would basically be a utility of 1, 1 multiplied by the outcome which is 4 divided by 3 4 5. Similarly, for the last one, it is basically 4 multiplied by 5 divided by 3 plus 4 plus 5, the actual value comes out to be as say for example, 2. So, when I comparing the utilities U A and U B which is for decision A and decision B only considering the linear one, then in that case level or the points accrued to be would be higher as it is 2 with respect to 1.69 for 1.

Similarly, when I compare basically the corresponding utilities only based on the second quadratic, utility function combining both the decision A and decision B, the values comes out to be say for example, mark it with the green for utility 2 considering a decision A, it comes out to 2.5 when I compare both decision A and decision B for utility 2, the value comes out to be 2.5 in both the cases. So, with this, I will end this lecture and continue the discussions further on related to the utility and the decision based on is how the accrued points would be given and also the probabilities can be found out have a nice day.

Thank you very much.