Data Analysis and Decision Making - I
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Lecture - 26
Forecasting

Warm welcome, a very good morning good afternoon good evening to all of you; and this is the DADM which is Data Analysis and Decision Making one course under NPTEL MOOC series. And we are in the 26th lecture as you can see in this slide.

And this total course is for 12 weeks, 30 hours total number of lectures would be 60. And each week as you know we have about 5 lectures each being of half an hour. And this is we have just started the fifth week; that means, the fifth week has ended. We are basically is going to start on the 6th week today which is basic 26th lecture.

So, if you remember we have been discussing about different type of forecasting techniques. And in the forecasting techniques we did discuss that they can be the concept of weighted average and the concept of exponential weighted average. The weighted average exponential (Refer Time: 01:12) weighted average smoothing matters whatever the main crux of the idea is given the past data, you want to basically keep some weightages on the past data, whether high or low that will depend on what is your level of confidence for the past data is and you want to basically predict for the future.

Now, when you are trying to basically predict for the future for the simple weighted average or the you will basically consider some weights. But, the weights if you remember we have given them and equal proportions corresponding to how far you are from the mean value onto the left and the right such that the histogram formed is basically a normal distribution, because that is what we are aiming at.

So, they can be 3 moment month weighted average 4 months moving weighted average 5 month moving a weighted average. And, those 3 months or 4 months or 5 months could be further taken into sets such that you can find out 4 by 4 3 by 3 5 by 5 moving averages and then basically find out the averages of these averages

So, in the long run obviously the further they are from the mean from a central value where you are trying to predict lower with the weights closer they are higher with the

weights. And obviously the weights are symmetric in the sense, onto the right hand side and the left hand side of the mean value where you are going to basically predict the weights would basically be of equal numbers or equal values. And the and also remember another thing that the sum of the weight should exactly be equal to 1. And that is; what was the main the idea which we were trying to basically propagate.

In each and every discussion we were saying that the sum should be 1, such that the convex combination basically gives us the result we also considered and that was true for any type of forecasting method we have considered till now. We will consider further on is that the parameters which you want to basically find out those alpha beta gamma whatever it is those parameters should be found out in such a way that the sum of the squares of the errors should be minimized.

In the sense that we are trying to basically find out the minimum squared errors concept being utilized, such that we are fine able to find the alpha hats or the beta hats. And then basically utilize those alpha hats beta hats and gamma hats to basically predict for the future. That means, standing today which is t you would basically utilize the time period for t minus 1 t minus 2 t minus 3 in whatever ratios would be, the ratios are the weights and then utilize them basically predict for the t plus 2 would t plus 1-time period. Then once the t plus 1 actual values found out you basically stand there at t plus 1 and then try to basically predict for t plus 2 and continue in this way.

And I also did mention that the errors which was basically epsilon we have basically they would have a particular distribution. And that is normal distribution with a mean value of 0. And a standard deviation we will consider as sigma square or 1. So, the one would be the case for the simplest assumption and for the case for sigma square would with the m the variances of the errors based on the fact that we have some variance which is not 1 and basic will proceed accordingly.

To double check when you predict for t plus 1 t plus 2 t plus 3, if you add up all the errors technically the sum of the error should be 0. And the corresponding variance should be sigma square r 1 as it has been assumed before we start the problem. Now another thing we consider later on in the exponential smoothing concept was that we will try to basically utilize both the forecasted value as well as the actual value.

So, if you are standing at t, we will try to utilize the forecasted value of t minus 1 and the actual value of t minus 1. And then there is basically predict the forecasted value for the next time period; that means we are trying to basically predict for t, using the values of t minus 1. So obviously, the weights would be if there are only one forecasted value for t minus 1 one actual value 2 for t minus 1.

So, hence the weight should be alpha and 1 minus alpha such as the sum is 1. And then try to minimize as you have discussed, now you want to basically we find out that considering the fact that you want to basically give more weights for the past values. That means, you are standing at t trying to predict for t forecast for t and you will try to basically utilize the fact that the exponential weights you are going to give that was alpha and 1 minus alpha.

And now they would not be alpha and 1 minus alpha they would be alpha 1 alpha 2 alpha 3 alpha 4 such that the sum of alpha 1 alpha 2 alpha 3 alpha 4 should be 1. And what are these are 4 1 alpha 2 alpha 3 alpha 4, they are the corresponding weights which we will if you will give for the predicted value for t minus 1. The forecasted value of t minus 1, the predicted value of t minus 2 and the forecasted value of t minus 2 correspondingly and you can basically make it to the 3 periods 4 periods depending on what your accuracy we want.

So, we were basically discussing a some small problem. In that case you had the data. Based on the data you want to basically find all the 3 month moving average then the 5 month moving average and we also mentioned that we will be using the weights which are basically now changing with respect to time. So, if you remember for the weights which was the alphas and beta alphas.

Let me, Indian not use the variable beta for the alphas basically we consider that some was one and they were fixed with respect to time, but what if the parameters themselves were basically changing with respect to time how do we do that. So, that was basically the discussion and we will try to basically do one very simple problem based on those lines.

Now, for the problem which we are discussing we did mention it was a 3 month moving average we have done that; now we are going to do the 5 month moving average.

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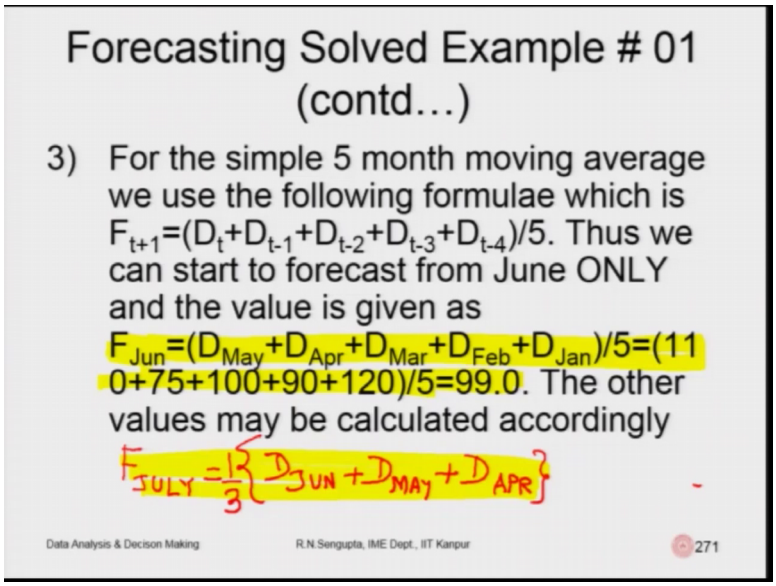

For the simple 5 month moving average we use the following formula which is the forecasted value for t plus 1. That means we are basically standing today trying to basically predict for tomorrow.

Then we will be utilizing the demand which used the actual value which is actually y, I have mentioned that very clearly in the slide in the last class that we will be considering y and d interchangeably without any confusion. So, if you want to basically find out the formula and utilize the 5 month moving average it will be addition of all the demands for the t period t minus 1 period t minus 2 period t minus 3 period t minus 4 period; that means, 5 months sum them up divided by 5, because you are considering the weightages to be one-fifth for each and of these values.

Thus, we can start to forecast only from June as the data's were given and the value would be given for the month of June as the forecasted value of a June which is here this value let me define it. This is the June which we are going to forecast depending on the values of January.

So, this is January one month, February 2 months, March 3 month, April 4 month, May fifth month, add them up divide by 5. Hence the total weights for each of them is one fifth. So, it becomes 110 which is for May 75 April 100 march 90 February and 120 January. So, if you add them up divide by 5 it comes out to be 99. So, other values can be calculated accordingly. So, 99 is basically the 5 month moving average.

Now, if I want to go to the month of July. So, July again 5 months demand for June, demand for May, demand for April, demand for March, demand for February. So, 1 2 3 4 5, 5 months done you divide by 150. So, this is the 5-month equation for the month of July; if it is 3-month equation.

So, in that case with the problem which you have already done in the last class; so I come to that; so you will remove this you would remove this also. It would be one third. So, you will take April May June and them up d divided by one third and basically do the calculation. So, you will come to the detailed calculation we have only highlighted one thing. So, we basically consider these values. So, this is for June and you will consider for July accordingly. Again I am not used those values similarly for august and continue in this way.

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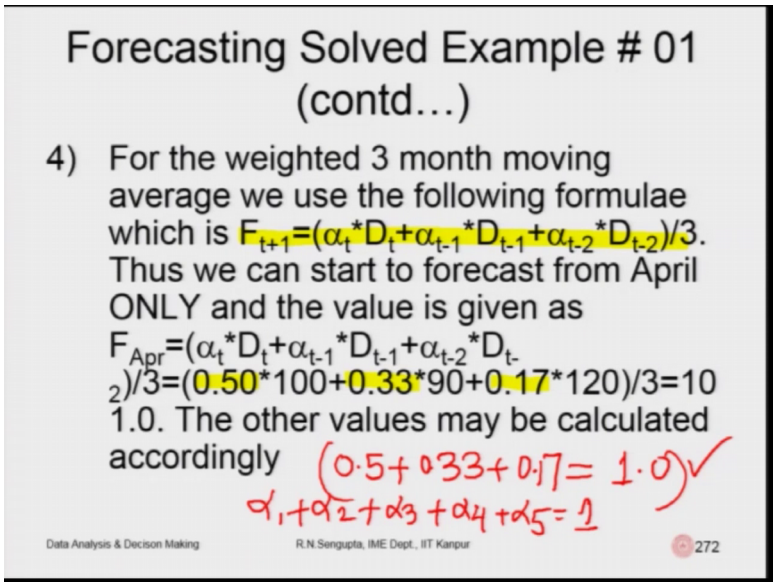

Now, we are trying to find out the weighted 3 month moving average and we will use the weights accordingly. The weights are given as I will highlight them 50 percent 33 percent 70, 17 percent. So, what is the weight? So, the weight is one. So, that should be the case the addition of all the weights for whatever time period you are doing giving it is 5 time periods, if it is signs 5 time periods. So, in that case it is 5 alpha 1 alpha 2 alpha 3 alpha 4 alpha 5 add up to one here it is 3 time periods. So, they add up to one.

So, for the weighted 3 month moving average we use the following formula. So, the forecasted value for t plus 1 would be equal to the weights multiplied by the

corresponding demand or y value for the time period of t minus 2, t minus 1, and t in order to help us to predict for t plus 1. If it is basically 5 month moving average as it is 3 here we change is to 5. So, in that case it will be not starting from I am talking from the right hand side.

So, it would be not be t minus 2, it will be t minus 4, t minus 3, t minus 2, t minus 1 and t demand values each of them multiplied by the corresponding values of the alphas. So, alphas would be here corresponding to d of t minus 4. It will be alpha into t minus suffix t minus 4 then the d value when it is d suffix t minus 3 the alpha value would be alpha suffix t minus 3. And the corresponding values later on late later on means forward we are coming would be dt minus 2 I am this suffix is I am a word I am not using. So, d t minus 2 into alpha t minus 2 then on next one would be d t minus 1 or 4 t minus 1 and the last one is d t alpha t.

Thus we can only start to forecast from month of April only and the values are given as follows. So, add them up and divide by 3. So, you are basically going to take 50 percent of 100, 33 percent of 90 and 17 percent of 120 that value which comes out to be for such weights for the values of d t d t minus 1 d t minus 2 r 100 and 90 and 120 the value comes out to be 101.

The other values May be calculated accordingly similarly if it is the next month May. So, in that case the alpha t, alpha t minus 1, alpha t minus 2 would be for the corresponding just 3 months behind May.

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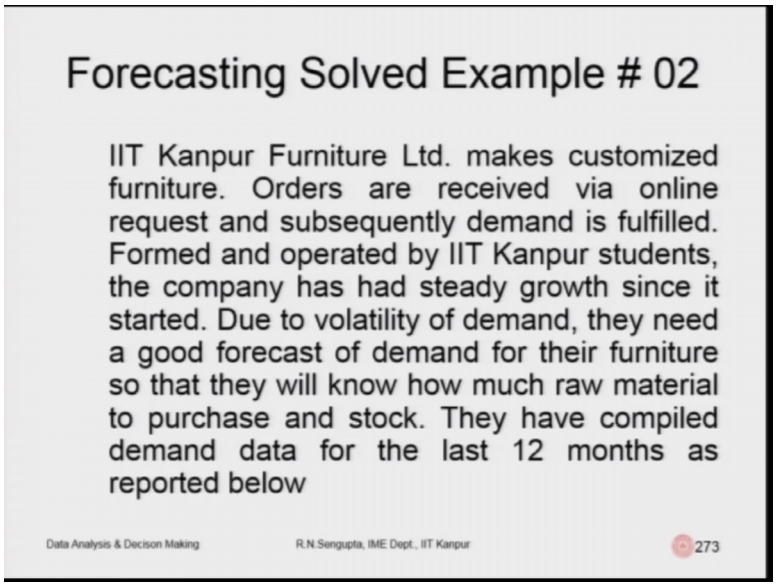

Now consider another problem IIT, Kanpur furniture limited makes a customized furniture orders are received via online request and subsequently demand is fulfilled formed and operated by IIT, Kanpur students.

The company has started has had started growth since it still since it is inception are started due to the volatility of the demand they need a good forecast of demand for their furniture; so that they will know how much raw materials to purchase and stock up for their production. They have combined demand data for the last 12 months and it is reported as follows.

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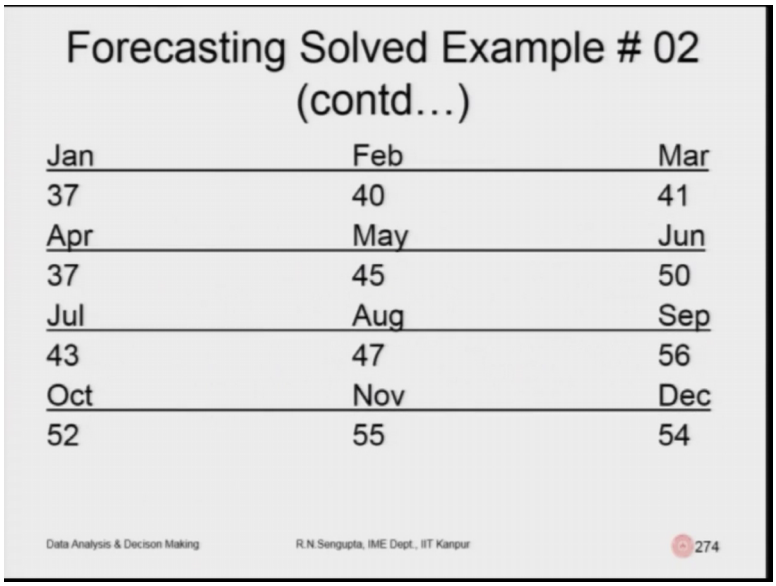

So, the data for the corresponding months 12 months for January it is 37 February is 40 march is 41, then for the month of April it is 37 May June corresponding values are 45 50 July august September is 43 47 56 respectively, and October, November, December; 52, 55 and 54 respectively.

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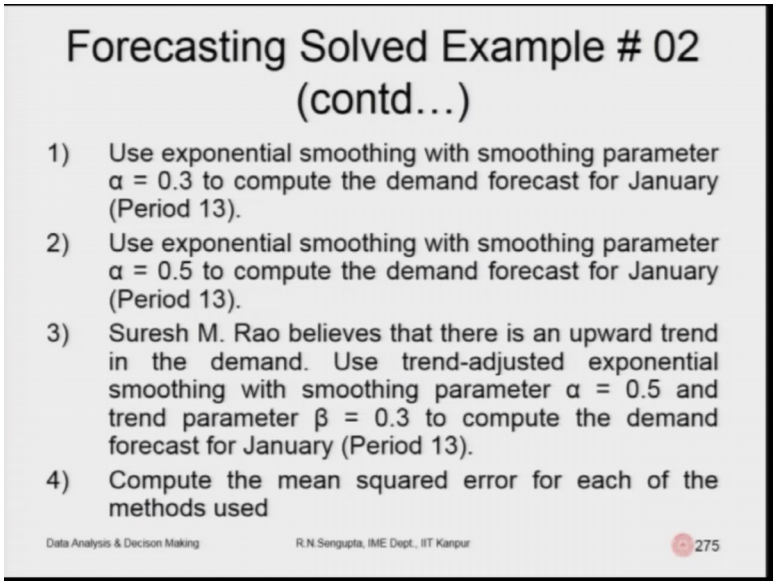

So, what is the problem? What we have to solve use exponential smoothing with smoothing parameters alpha is equal to 0.3 to compute the demand forecast for month of January which is January of the next day which is the 13-time period.

Use exponential smoothing with smoothing parameters alpha is equal to 0.5 to compute the demand forecast for January which is basically 13-time period again. So, in this case you are using smoothing parameter of 30 percent one case and another case you are using this smoothing parameter of 50 percent. One person by the name of Suresh M Rao believes that there is an upward trend in the demand use trend adjusted exponential smoothing with smoothing parameters as alpha is given to 50 percent and beta is equal to 30 percent or point 5 point 3 to compute the demand forecast for the month of January which is the 13th month.

And now you compute the mean squared error for each of these methods and technically you have to basically comment intelligently.

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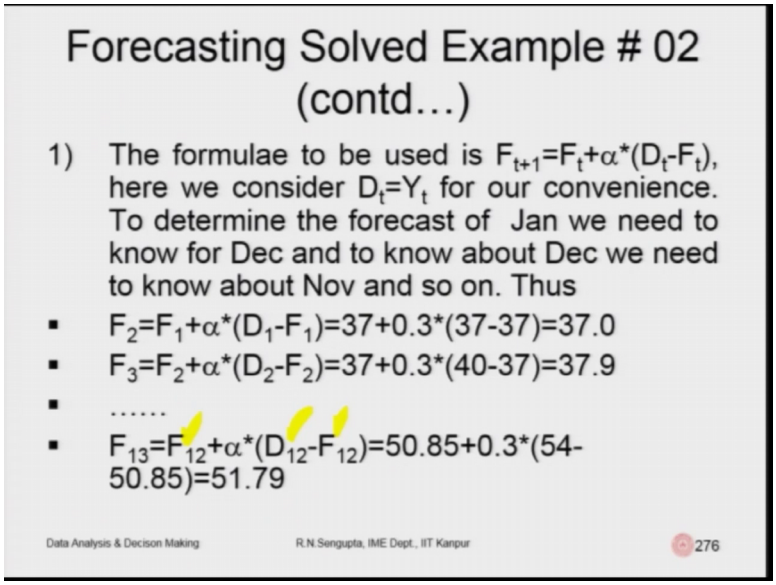

Now, the first problem, which is basically if you remember the alpha values are given as 30 percent 50 percent.

The formula to be used in this case is that I am standing at t plus 1, I am trying to forecast as t plus 1, but I am trying to bit basically utilize the data for the time period of t. So, the forecasted value of t plus 1 would be F t which is the forecasted which has happened in the last time period. And you will basically consider some alpha value depending on the accuracy or the more or less the values is and multiply it by the difference of the actual value and the forecasted value which is d suffix t minus F suffix t that should be basically multiplied by alpha.

So, in case if you consider the alpha value for any forecasting method is 0 in that case. Obviously, you will have the value of the forecasted value as equal to just equal to what has happened in the last period which is forecasted of the last period. Now, we will generally assume that the demand or the value of y are the same as in the same way we have considered in the last problem this I just wanted to mention. So, there is no confusion because in the formula we are writing initially y and then we are trying to basically replace that with d. So, in order to avoid that to we want to basically determine the forecast of January.

We need to know basically for December and to know about December we do not need to know about November. So, basically we will go one step at a time. So, thus the formulas are like this, when you are basically projecting for F 2 that is time period 2 F 2 is equal to F 1 plus alpha and the difference between the demand for time period 1 minus the forecasted value of time period 1 which is alpha in the bracket d 1 minus F 1.

So, that when if you take corresponding the data which is provided. So, in this case F 1 is 37 alpha as you know we have taken as 30 percent and technically we will consider when we are trying to do the this type of trend analysis we will consider the demand of time period 1 is equal to the forecasted or time period 1 which is basically the forecasted value which is whatever has happened in a in time period 1 has been able to basically give the exact value without any loss of efficiency the actual demand for the of the time period 1.

. So, in this case it will be 37 plus 30 percent into the difference between d 1 and F 1 the difference being 0. So, actually the F 2 value comes out to be the same value which is 37. When we go to F 3, F 3 would be F 2 plus alpha into the difference between d 2 and F 2. So, in that case the value the difference is basically 40 and 37 the difference between them multiplied by 30 percent or point 3 the value comes out to be 37.9; we in this way we go to the third value 4th value 5th value so on and so forth.

The last value which is for the forecasted for 13th time period would be equal to the forecasted value of twelfth time period as rightly mentioned here I will just marking it. Plus the alpha value which is 30 percent into the difference between the forecasted than the demand for the twelfth period which is true here and that value comes out to be 51.79. Now remember one thing that whenever we are calculating the forecasted value

the alpha value always remain the same it is not dependent on time. So, we are going to change that later.

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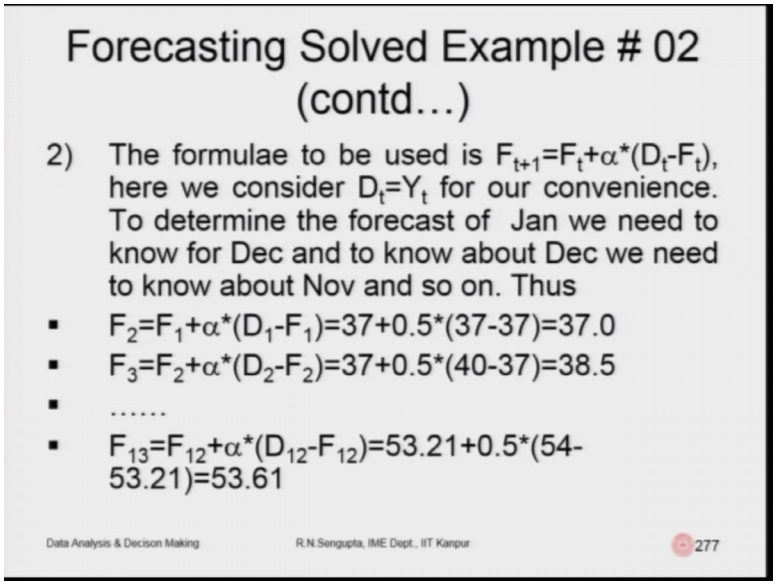

Now, you want to basically calculate in the same concept being utilized, but we will basically put the weightages alpha as 50 percent which is 0.5.

The formula to be used again is same F t is equal to F t F t one at t plus 1 is equal to F t plus alpha into the difference between the demand of just the last time period which is d t minus F t. And I also we will consider at the beginning that d 1 is equal to F one. So, this one suffix is basically a nomenclature it does not mean that it has to be one it can be is a 0-time period it can be minus 1-time period whatever it is.

So, in this case we will consider d t is equal to y t as per that convention in order to make the problems clear to you, because in the formulas formulation if you remember we utilize it y and in this case we are using d. So, let us be not be confused let us make things clear. So, determine to determine the forecast of January we need to know for December and to know about December we need to know about November and so on and so forth.

So, thus the forecasted value of time period 2 is equal to the forecast period of time period 1 plus alpha into the difference F 1 and d 1. Similarly, the forecasted period of value for time period 3 is equal to forecasted value of time period 2 plus alpha into the

difference between the demand of d 2-time period 2 and F of forecasted value of time period 2. So, in this we will continue till the last value which is basically that the forecasted value of time period 13 is equal to forecasted period of time period 12 plus alpha. And in that bracket you have basically the difference between the demand of twelfth time period forecasted value of twelfth time period.

So, the corresponding values starting from F 2 till F 3 can be calculated I have just stated here F 2 value is 37. Now remember because why it is 37 it is exactly equal to F 1, because the difference between d 1 and d 2 and d 1 and F 1 is 0, because 37 minus 37 is 0. So, the value of F 2 is 37 value of F 3 is 38 point 5 so and so forth till the last value which is F of 13 is 53.61.

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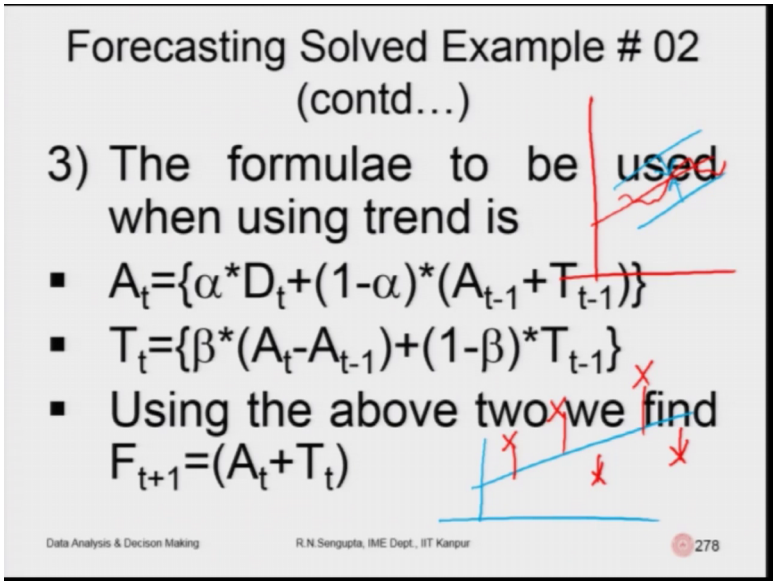

Now, we will utilize that if you remember the smoothing concept and adaptive smoothing values we will try to utilize that the formula to be used when using the trend. So, we have some trend means maybe some smoothing whatever it is, that trend concept is 80 and the smoothing value which we will basically try to find out it will adapt means as time progresses this value should basically increase or decrease. So, you will have basically 80 and equal to alpha.

So, alpha obviously those values of 80 would change depending on the time period. So, that is very important to understand. So, it will be alpha into demand for time period t and we are basically finding out for the time period t only a and 1 minus alpha because

the convex combination I will (Refer Time: 24:24) plus the sum of the weights to be one. So, it 1 minus alpha into 80 minus 1 into t minus 1; so that means, we are trying to basically find out the weights such that we are able to calculate them in the best possible manner such that there is no such error.

In the sense the error would; obviously, there, but the fluctuations are taking consideration with the trend and the seasonality. So, the trend value would now basically be a new variable beta. So, beta into the difference between 80 and 80 minus 1 plus 1 minus beta would be t of t minus 1. So, what you are basically trying to do is that there is a trend line and that trend line would obviously increase, but you will also add a plus and minus movement along the trend line based on which you will try to basically predict.

So, if I am able to draw it. So, this is there would be a trend there is; obviously, a fluctuation. So, what we are trying to do. So, any fluctuation which is happening in between you will try to basically push it towards the trend. So, plus minus 1 minus beta which you are trying to do is basically trying to bring it more closer to the trend line which we have here.

So, using this formula we will try to basically forecast and the forecast would be basically the trend plus the variations which you have. So, generally what it is something to do with the concept wise intuitively has something to do with the multiple linear regression. Because in the multiple linear regression if we remember we have a straight line which is the mean value and all these values actual values and these are the errors.

So, technically this central line would well consider the average value which is the trend. And any fluctuation which is happening, I am trying to basically push them from the above, and push them from the below towards the basic the mean value average value such that so called variance is reduced to the maximum possible extreme.

Now, if I am talking on the variance, it would mean what I am trying to utilize the weights alpha beta whatever it is in such a way that the sum of the squares of the error should be minimized with respect to alpha and beta. Now sum of the square is what basically find out the difference between actual value of y minus predicted value of y that is the error square it sum them up.

Now, when you are trying to minimize minimizing the expected value of the square is exactly equal to the formula of the variance we try to utilize. So, hence when you when I am saying that we are trying to basically find out the minimum square in the sense, find out the square of the errors differentiate with respect to the parameters put it to 0. This is exactly what we are trying to do in the OLS system or the ordinarily square which we have discussed in details to the diagrams in the multiple linear regression this is exactly what it will be done in the trends also. So, there is a central line plus and minus any fluctuation they are brought back to the mean values as soon as possible.

With this, I will close this 26th class and continue more discussion related to this problem in the 27th class accordingly. Have a nice day, and thank you very much for your attention.