Microeconomics: Theory & Applications
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Lecture – 47
Practice Session (Tutorial) on Monopoly

Hello, welcome back to the lecture series on Microeconomics. Earlier we have seen how to find equilibrium of a simple monopoly model and a third degree price discriminating monopoly model. Now we are going to revisit those models with the help of a numerical example. I hope this numerical example will help you to understand these models in greater detail.

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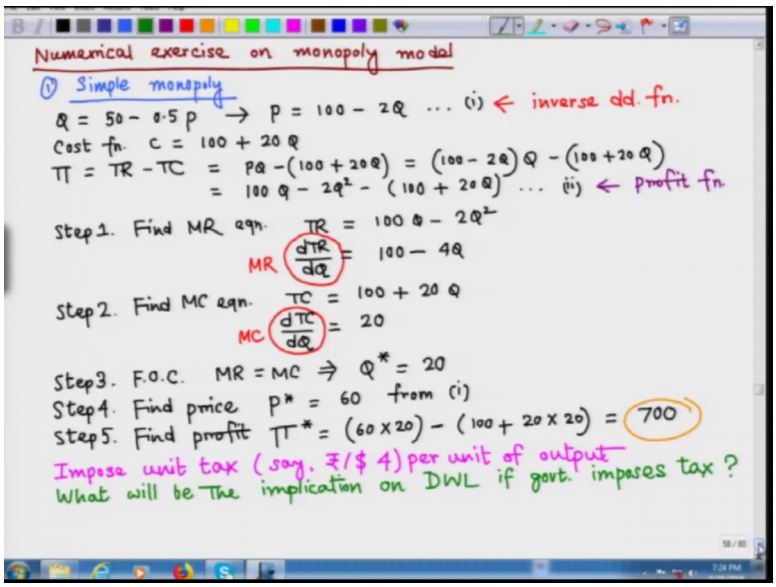

So, first we are going to start with the simple monopoly model, where the monopolist cannot divide the market into 2 or more homogeneous consumer groups. So, basically it assumes that the market see uniform. So, we start with a market demand function of the form, I must say all these numbers that I am using here in this numerical exercise are hypothetical in nature; so, if you change these numbers of course, the solution will change.

So, from this direct demand function, I can now get the inverse demand function which is equal to P equal to 100 minus 2 Q let me name this equation 1 right. So, this is basically my inverse demand function for the entire market ok. So now, let us also going

to have a cost function; so, the total cost function has the following form C equals 100 plus 20 Q right ok.

So now, from this demand function and the cost function one can note down the profit function of the monopolist could be written as total revenue minus total cost right. And how do we write the total revenue? The total revenue will be P times Q minus this 100 plus 20 Q that is the cost component right. Now we need to use the inverse demand function expression number 1 in this profit expression, and if we do so, we get alright. So, the revenue function if I expand it will become 100 Q minus 2 Q square and then of course, I have this 100 plus 20 Q cost component right. So, this is my profit function. So, let me use this equation number 2 to have my profit function, this is my objective function as well which the monopolist would like to maximize right ok. So now, we are going to study the steps which are required to get the solution to this particular problem right.

So, we will start with step number 1: and in step number 1 we need to find out the marginal revenue curve right. Because for the first order condition of monopolies equilibrium requires MR equal to MC right; so the first task at hand to find marginal revenue expression. How to get the marginal revenue expression? From the total revenue we can get the marginal revenue, right.

So, my TR is basically 100 times Q minus 2 Q square right. So, from here I can simply take derivative with respect to the decision variable, which is basically the output level and I get 100 minus 4 Q right. So, this is basically my marginal revenue right ok. So, now, what would be the second step? Second step is to find out the other part of the first order condition of monopoly equilibrium and that is basically the marginal cost right. So, again we start with our total cost function. So, that is basically 100 plus 20 Q right. So, if I take derivative with respect to quantity to be being produced, I get 20, right. So, that is basically my marginal cost. So, here we get a constant marginal cost in this simple example.

Now let us go to step 3 and here we now concentrate on the first order condition. So, we need to equate MR and MC expressions that we have got and if we do so, then basically we get a value of Q for which these 2 are equal, and that value is basically monopolists profit maximizing output level and this is here Q star equal to 20.

Now, once we have obtained the quantity, optimal quantity or production or output level of the monopolist the next step is to find price. And if you remember from our discussions on monopoly, one has to go to the demand function and plug this value of optimal quantity in it to get the optimal price that the monopolists sets in the market right.

So, if we plug this Q star equal to 20 quantity or output level in my expression number 1 which is my inverse demand function, then basically I get P star equal to 60 right from 1 the inverse demand function ok. Now the next important thing although optional is basically find the monopolist profit ok.

So, to find profit, we have to make use of the profit expression which is basically my expression number 2 here in this example. So, as I am plugging the optimized values of Q and P in the profit expression, I write optimized profit or maximum profit pi star as 60 times 20 minus 100 plus 20 times 20 right and we get a profit level 700 ok. So, we have obtained the profit figure of the pure monopolist in our example. Now there is another interesting case that one can discuss in terms of this numerical illustration and that is the case of tax imposition by government.

So, now we are going to impose unit tax say rupees or dollar whatever we have not assumed any monetary unit per unit of monopoly output right. And basically the interesting question that we are going to also address is the following. What will be the implication on dead weight loss, if government imposes tax any type of tax we are going to study the implication of unit tax only so.

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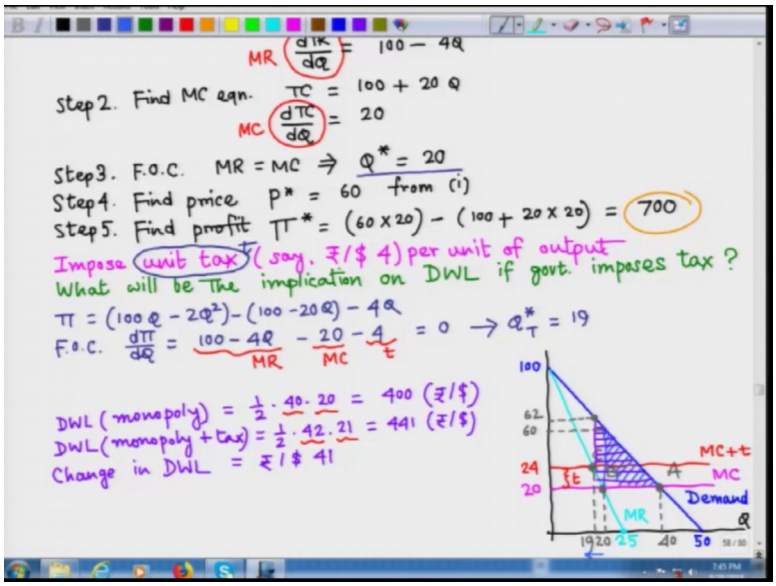

Now, if we assume a particular value of unit tax, then how profit equation of the monopolist is going to change. So, we are going to rewrite the profit expression. So, it is going to be; this is the revenue part right then cost will be deducted right and then the tax that the monopolist has to pay to the government shall also be subtracted from the revenue right and that is 4 Q.

So, if we now look at the first order condition for monopolists profit maximization problem we get what? We get d pi d Q equal to 100 minus 4 Q right. So, that is basically the marginal revenue part right if you remember this part is coming from the marginal revenue part and then there will be this marginal cost; so marginal cost. And then finally 4 will come and that is basically the tax element right. So, this is basically the marginal revenue and then this is basically the tax right ok. So now, we can set that equal to 0 and then from there if we solve for Q from these first order condition, we get what? We get Q star and I use T subscript to denote the equilibrium output of monopoly after tax imposition, so that basically becomes 19 ok.

Note that last time we have obtained Q star equal to 20 that is the pure monopoly output without taxation. So, let us now draw a diagram to understand, what is exactly happening here ok. So, this is my monopoly output Q and I plot my marginal cost and marginal revenue etcetera here along the vertical axis as usual. So, let us first plot the demand function ok. So, here the demand function has an intercept of 100 along the MR MC or

along the vertical axis right. So, let me first have that and given the demand equation, we know that its horizontal axis intercept will be 50 ok. So, let me create some room here for numbers 100 fine.

So, then comes the marginal revenue curve. So, we know that marginal revenue curve will start from 100. And as there is this relationship of a linear demand function we know from there, that the slope of the marginal revenue curve will be exactly half of the demand curve from which it is derived. So, by following that logic we know that the quantity at which marginal revenue will become 0 is 25.

So, let me join these 2 points and then I get my marginal revenue curve right. Of course, there will be a negative extension, but we do not want to write because this is a relevant for all practical purposes. So, this is my demand right ok. So, now, let us plot our marginal cost and the marginal cost is basically a constant number which is 20 ok. So, let me plot that this is my marginal cost curve MC and the value is around 20, right. So, the competitive market equilibrium is basically obtained at this intersection point A, where the marginal cost or the supply function intersects the demand function right. And we can solve for the perfectly competitive equilibrium output, because that is required for dead weight loss calculation. So, and we find that number to be 40 ok

And now the intersection of MC and MR will be denoted by this point B and that is basically my monopoly equilibrium and we have found that this monopoly equilibrium quantity level is 20 right ok. Now what happened? There is a tax imposed by the government right. So, the tax is 4 unit. So, let me assume that there is a parallel shift of the marginal cost curve and this gap is basically my tax and the intercept will be then 24. So, this curve is now MC plus tax ok. So, another monopoly equilibrium will be found here at this intersection point of new shifted marginal cost plus tax curve and MR curve, say point C and we have also solved for the output level which corresponds to this new equilibrium of monopolist firm and that is 19 right.

So, this is more or less the picture. Now how to compute the dead weight loss: that is the question. So, now, let us see; what is the dead weight loss area in the monopoly case. So, we need to compare the equilibrium A and equilibrium B and we know from our definition of dead weight loss, it is given by this triangle shaded by this blue lines. So, that is basically the dead weight loss due to the pure monopoly. Now if the government

imposes a tax to penalize monopoly, monopolist finds its new equilibrium and at this new equilibrium, it actually produces even less units of output. So, actually the market level of output declines from 20 to 19. So, basically the dead weight loss area increases and now the increased dead weight loss area is given by this triangle shaded by this purple lines right. So, after we found out the equilibrium quantity levels we have to figure out the prices right.

So, in the case of monopoly, we have seen that the market equilibrium price charged by the monopolist was rupees 60, it was set here. Now after the tax imposition basically we have to move up from this point C and we have to read the price from the demand function and if we do so, then we get market price 62 which shall be charged by the monopolist right ok. So, now, we are ready to do our dead weight loss calculation. We will basically compare 2 dead weight loss numbers and look at the measure the difference. So, component number 1 will be the dead weight loss due to presence of monopoly in the first hand and we know how to compute, we have to use the simple formula of area of a triangle that is half times base times height right. So, we can apply that formula to get 400 rupees or dollar whatever dead weight loss.

So, these 40 is basically coming from the difference between the perfectly competitive market price and the monopoly price. And these 20 is coming from the difference between the perfectly competitive market output level and the monopoly equilibrium output level right ok. So now, let us go back to the dead weight loss calculation this time for monopoly plus tax case right. So, we have to use the same formula, and this time it will be 42 times 21. So, this 42 is coming from the difference of perfectly competitive market price and the price of the commodity charged by the monopolist after tax imposition by government. And this 21 is basically the difference in the monopoly equilibrium output level after tax imposition and the perfectly competitive output level. So, this comes out to be 441 rupees or dollar ok.

So, basically the change in dead weight loss is about rupees or dollar 41 right. So, we see that there is an increase in dead weight loss following tax imposition by the government. It is because that anyway we had some market imperfection to start with due to the presence of monopoly, on the top of that government imposed the tax and that created another market imperfection, because the perfectly competitive market assumes zero intervention by the government, in the working of a perfectly competitive market. So,

this is the way we have found the monopolist equilibrium quantity level price and the profit.

Now let us look at another numerical example, which talks about the third degree price discriminating monopolist. We know that a third degree price discriminating monopolist actually can successfully discriminate between various consumers. So, the monopolist for sure knows that there are homogeneous, subdivisions of the consumer population, and each subdivision of consumers has different price elasticity. And now if we assume that this is indeed the case: how the monopolist goes about profit maximization.

So, next we are going to look at that through the numerical example ok.

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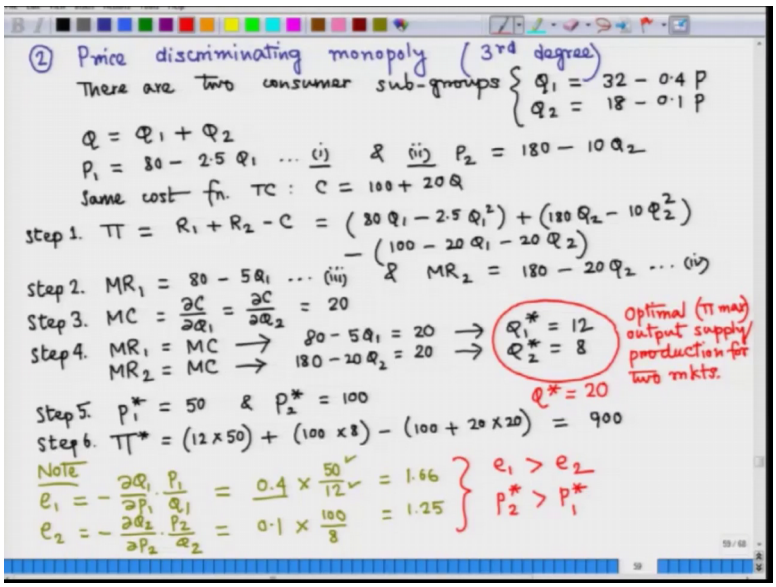

So, in this case let us assume that there are two consumer sub groups with different price elasticity's right.

So, for these two different sub groups we will have 2 different demand functions then and they are given as Q 1 equal to 32 minus 0.4 P and Q 2 equals 18 minus 0.1 equals P 18 minus 0.1 times price ok. So, now, note that I have chosen to sub market demand functions in such a way that Q is Q 1 plus Q 2.

So, if I add these 2 sub groups demand functions, I get the original demand function with which I started my first numerical example. So, I can now find the inverse market demand functions for these 2 consumer sub groups right. So, if I do so, then I get; as the

monopolist will discriminate and charge different prices in different markets, now I am going to use the suffix 1 and 2 for my price and quantity levels right compared to my previous example.

So now, we are going to assume the cost function. So, we take the same cost function and there is a reason for choosing the same cost function, so that we can compare the profit levels between the pure monopoly model and the price discriminating third degree monopoly model.

So, the same cost function means that we have this TC is 100 plus 20 Q right ok. So, now, let us proceed step by step. So, first we are going to write down the profit expression. So, profit will be the revenue from market 1 plus revenue from market 2 minus the total cost of production. So, we have seen how to come how to derive the revenue function. So, I am not repeating each and every step, I am just giving you the final outcome and then finally, I have to subtract the cost right ok.

So now, step 2 would be to derive the marginal revenue curves right. So, here there will be 2 marginal revenue curves because we are talking about 2 different markets right and each market has a different demand curve. So, I give you the final expression of the marginal revenue curves in these 2 markets and I name this number 3 and MR 2 equal to 180 minus 20 Q 2 that is my number 4 expression ok.

Now, the next step is to find my marginal cost of production right. Now note that I can write. So, we get the same constant marginal cost back ok. So, next step what to do? We have to equal to this individual marginal revenue curve expressions to the marginal cost and hence we get the equilibrium. So, we first equate MR 1 to MC and if we do so, we get 80 minus 5 Q 1 equals 20 if we do so, then we get a Q 1 star value of 12, if I now equate my marginal revenue curve from market 2 to the MC then I get and of course, I get some value of Q 2 we solves this equation.

So, note that I got my optimal means profit maximization output supply or you can say production for two markets right. Now note that interestingly if I now sum them up I get the total market supply which is Q star equal to 20 and this was indeed the case in my example 1 where we discussed the case of a pure monopolist ok. So, now, let us move on. We know that the monopolist is a third degree discriminating monopolist. So, we

know that our monopoly firm is a price discriminating one of third degree. So, the next step would be to find different prices to be charged in different markets.

So, once we get these optimal levels of Q 1 and Q 2 we can plug back these numbers in the individual inverse demand functions. So, we can plug these numbers Q 1 star and Q 2 star into my inverse demand functions, which are given by equation 1 and equation 2 right to obtain the optimum prices to be charged in two different markets right. And if we do so, then basically we get ok. Now the last step which is required to show you that if the monopoly is able to discriminate, then it actually attains higher profit compared to a normal monopoly model, so at that we need to compute the profit right. So, last, but not the least we have to compute the profit level. So, let us now plug the optimum values of P and Q and compute the profit level right.

So, if we compute it is going to be 900. So, if you remember our theoretical discussion on monopoly, you may remember that there is a relationship between the own price elasticity of demand and the prices to be set in individual markets, if there is price discriminaty price discriminating monopoly model is in operation so, that is what we are going to check.

So, we have got two different prices and two different quantity levels for two sub markets, let us check whether indeed the price elasticitys are different or not. So, now, let us compute this price elasticity. So, e 1 is basically the own price elasticity of demand for the market 1 and we know how to compute this is not a very difficult task right. So, this is the formula right similarly we can write ok. Now these expressions are to be evaluated at the equilibrium point in each market right. So, if we consider the market number 1 to start with then these are the numbers. So, this is basically the negative of the slope of the demand function and this is basically the optimal price and this is optimal quantity level right ok.

So, the final number that we get is around 1.66. Now let us come to e 2 and similarly we take the negative of the slope of the demand function times the optimal price divided by the optimal quantity alright. And if we compute this is coming around 1.25. So now, we have seen here that elasticity of demand at the equilibrium in market 1 is greater than the price elasticity of demand evaluated at the equilibrium in market 2 and we see that P star 2 is greater than P star 1.

So, here we see that price charged in market 2 is greater than price charged in market 1, but note that in market 1 basically the price elasticity of demand value is higher ok. So, with this numerical illustration, we are done with our discussion on monopoly models. So, we have seen so far 2 extremes, where we have large number of sellers that is the perfectly competitive market model and the other extreme is basically the monopoly model where we have only one seller.

So, in the first case one sellers strategic move to produce some units of output more or less does not have any influence on the market outcome that we have seen. And in the other case we have seen that if there is only one seller; then there is nothing to worry about competition because you are the king in the market. But there can be intermediate cases where there could be 2 or more number of sellers, but the number could be less than 10.

So, basically each and every individual firm or seller has some significant portion of the market to serve, and in that case there are chances of strategic interdependence. So, if firm one thinks that if he chooses a strategy a, then the opponent firm is going to choose a strategy b and if that happens then firm 1 is going to get some monetary reward x.

Now, there is also a possibility that if the first firm chooses our strategy c, then the opponent firm will choose a strategy d. And hence as a result, the first firm will get some monetary reward y. So, this is basically the field of oligopoly, where oligopoly means few sellers and next we are going to discuss these types of model.

So, we will continue our discussion on imperfect competition market in the next lecture.