Microeconomics: Theory and Applications

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Lecture – 31
Firm's Optimization Problems (Part-2)

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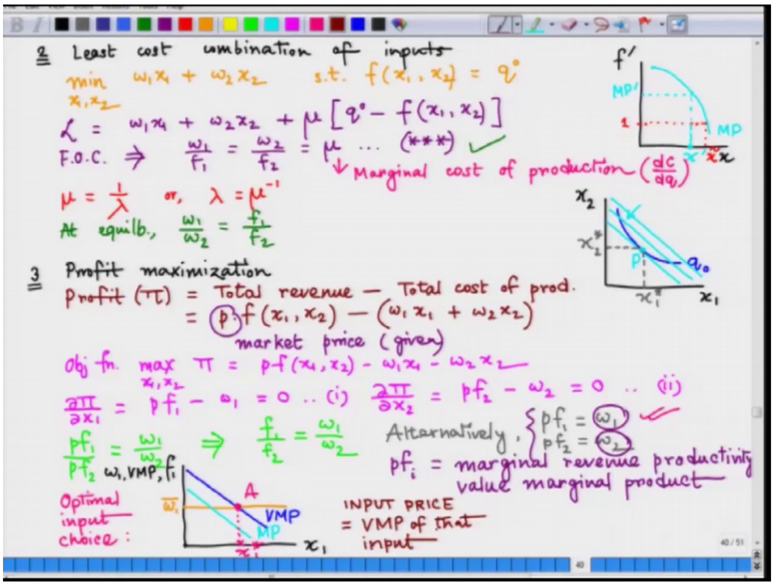

Hello, welcome back to the lecture series on Microeconomics. We have been theorizing the production behavior of a firm; let us move on to the third and final problem of a firm which is a problem of profit maximization. So, we will first start with the expression for profit, which is generally denoted by symbol pi in economics and that is defined as the difference between total revenue of a firm and the total cost of production.

So, what is total revenue? Here in this model of long run two variable input production function, that will be the market price which is denoted by symbol p times the output level, which can be alternatively shown in terms of the production function and the total cost will be w 1 x 1 plus w 2 x 2 the expenses made on the variable inputs right.

So, here we introduce another parameter in the model, which is market price; let us assume that the firm faces this market price and the firm has no control over it so that is given. Now note that we have shifted from the constrained optimization problem to an unconstrained optimization problem. So, our objective function that we need to maximize is basically this profit expression pi which is.

Hence the decision variables are of course, input levels right and what do we need to take what? And then we have to take derivative of course, this is no wonder you have to take partial derivatives to find out p times the marginal product of the input 1 minus the price of the input 1 that shall be equal to 0.

Similarly, we can find the other one which is p f 2 minus w 2 equals to 0. So, this and this are my first order conditions of this unconstrained optimization problem. Now let us rearrange 1 and 2 to see whether I can infer something interesting. So, I can always rearrange these first order conditions to write, and what do we get? Surprisingly we get the same condition back that we have derived in the previous cases. The slope of absolute value of the slope of the isoquant shall equal to the absolute value of the slope of the iso cost line.

Now, a profit maximization behavior of the farm can be told from a different angle and that could be through the lens of optimal input choice and now we are going to discuss the same. So, alternatively we can write p f 1 equals w 1 and p f 2 equals to w 2 directly from 1 and 2 right. How do you interpret these things here? Of course, this is the input price w 1 and w 2 we have already seen. Now, how can we interpret this entity which is denoted by f p times if i; i is basically denoting either input 1 or input 2. So, this is basically what? F i is marginal product. So, this is basically if we increase the use of input 1 by one unit, then how many units of output we can get extra in return? And if we multiply this by the market price p then basically that is an increase in the total revenue for the firm.

So, this can be interpreted as marginal revenue productivity of a particular input. If we assume that the firm has no control over price as we are assuming in this case, this can also be called value marginal product as you can see in different textbooks. So, now, let us go for a graphical illustration of this alternative representation of first order condition.

So, here let me draw a diagram. So, here along the horizontal axis I measured the input use of input 1 and here I measure the marginal product, and if I multiply this marginal product of the market price of the final output then I get the value marginal product right. So, these two variables I am measuring along the vertical axis. We are going to assume without any loss of generality that we have a straight line marginal product curve for the input.

Now, if I multiply the market price and if I assume that market price is above 1, then I can find the value marginal product of the input and that can be given as this blue line VMP right. Now let us assume that we are also measuring the input price which is given by w 1 along the vertical axis. So, now, let us superimpose the input price in this diagram, and as this is a given number constant represented by say w 1 bar and that is basically parallel to the input axis.

Now, this alternative representation tells us how a firm optimally chooses input level. So, now, let us look at the diagram to understand this. So, let us now focus on the intersection point denoted by this point A. So, this is the point where our alternative first order conditions are satisfied. So, we are interested only in the case of input 1. So, let me raise this.

So, this particular condition is satisfied at point A; and if we now come down on the input axis what do we get? So, the coordinate value gives me the optimal level of input 1 which maximizes the profit for the firm. Similarly we can draw another diagram for input 2. So, we can represent this alternative route to profit maximization of firm as input price shall equal to the value marginal product of that input.

So, we have seen how a firm maximizes profit and what are the required first order conditions. Now, let us quickly look at the second order conditions, if we do not assume that we are working with well behaved isoquants which are smooth differentiable and convex to origin.

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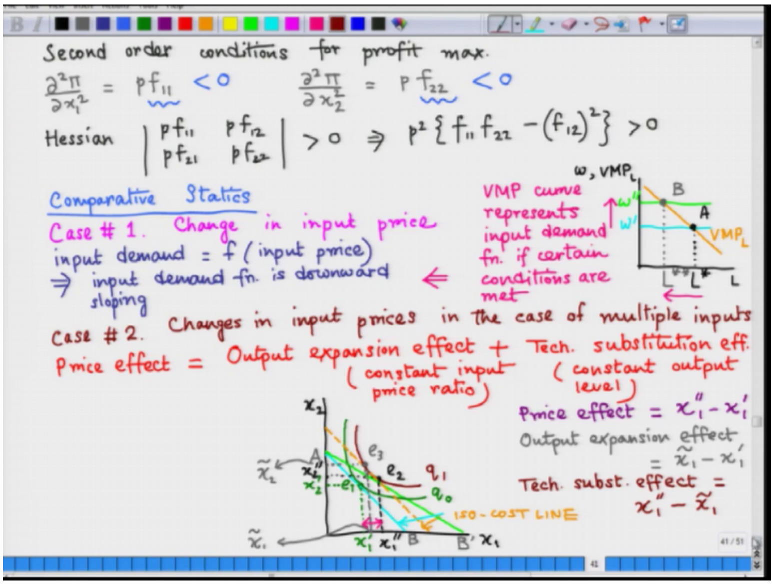

So, in that case we have to take the second order derivatives right. So, we will take the second order partials like this, there are two decision variables. So, of course, we have to take two double partials and we can assign some signs to these second order partials.

Note that due to diminishing marginal product of a factor input f 11 and f 2 are negative right. So, this is like this represents downward sloping marginal product curve. So, overall these derivatives take negative sign, but this is not end of the story. As we have two variable inputs here we have to construct the Hessian matrix right. So, we can now construct a Hessian, the determinant of the Hessian would be p f 11, p f 12, p f 21, and p f 22 right.

And if you remember our lecture on basic differential calculus and optimization, this needs to be greater than equal to 0 for profit maximization. So, we can simplify this by writing p square basically we are now computing the determinant value right. Now the second order conditions require that both the marginal products decrease as more of them are employed ok.

So, we are done with finding equilibrium of a firm through first order condition second order conditions, now let us look at another interesting angle of theory of firm through the analytical technique called comparative statics. We have studied comparative statics analysis, the tools and their uses in consumer theory in great detail. So, let us now see how this concept is applied in theory of firm.

So, here in our model of profit maximization, what are the parameters here the parameters are the input prices and the output price right. So, if these parameters change, then how that is going to impact my firms profit maximization behavior or the optimal value of the input that is going to be of interest. So, let us look at a comparative analysis, where we change the value of input prices to see how the model is going to react to that change.

So, first let us talk about a simple model where there is only one input being employed to produce one output. So, say that input is labour. So, we know how to find the optimal quantity of labour input, we have to construct and draw the value of marginal product curve. And then we have to equate that to the current input price which is wage rate in labour market to derive to find out the optimal use of labour input right. So, let us have a diagram and then we will see how a comparative statics analysis can be useful to find more details from the graph.

So, along the x axis, I am measuring my single input labour and here along the vertical axis I am measuring value marginal product L say stands for labour and I am also measuring wage rate say W right. So, let me tell you that this can be done with respect to capital or any other input as well right without loss of any generalization. So, let me assume that we have a straight line value marginal product curve right and let us also going to assume perfectly competitive labor market, which has some wage rate given by W prime.

Now, if that is the case we know our equilibrium will be obtained at this point say A and at this intersection point first order condition for profit maximization is made and the firm goes for L star units of labour right. Now what if there is a change in wage rate in the labor market suppose it has gone up for some reason we do not know why, but suppose it has gone up let us see what happens then. So, now the new wage rate in the market is w double prime right.

So, now the firm will definitely try to look for another equilibrium. So, that it can maximize its profit in the change situation, and that will be obtained at a new equilibrium point say B, and from this intersection we can come down to the input axis, and we can see that the new units equilibrium level of labour use will be L double star. So, in essence what do we observe we observe that as my wage rate in the labor market or the input price has gone up, we observe a downfall in the hiring or purchase of labor input right.

So; that means, that there is an inverse relationship between the input price and the quantity demanded of a factor input. So, basically we are saying that value of marginal product curve represents input demand function, if certain conditions are met right. So that means, that if we have input demand. So, we have input demand as a function of input price and input demand function is downward sloping.

Now, let us move to the case where we have two or more inputs, and let us see what will be the impact of input price change. And you will be surprised to see we are going to use the same tools that we have used earlier under the heading of consumer behavior and we are going to talk about the price decomposition effects etcetera. So, let us have a look at this issue quite quickly, because we have already analyzed this issue at length before.

So, now we are going to study case number 2, which is changes in input prices in the case of multiple inputs right. So, let me write directly that we are talking about a price effect that can be broken down into output expansion effect plus a technical substitution effect right. So, let me quickly draw a diagram to give you a glimpse, this will be a recap of what we have done.

Now, let us focus on these two components output expansion effect and technical substitution effect. The first component output expansion effect assumes constant input price ratio and the second one technical substitution effect assumes constant output level right. So, let us see how we can differentiate between the two again through a diagram; x 1 and x 2.

And suppose we start with an initial isocost line displaying some input price ratio and suppose next there is a fall in input price for input x 1 and we get flatter isocost line this time and let me name them. So, the original isocost line is represented by A B, and the new one is the shifted isocost line or tilted isocost line that can be called B prime right.

Now let us look at the tangency point. So, let us find out firms equilibrium in the first place. So, that can be obtained by figuring out the tangency between the old isocost line with an isoquant and suppose the output level is q naught ok. So, this point e 1 is basically the firms equilibrium right and then basically at this point, we get some levels of input use, optimal input choice of the firm right and we assume that we have x 1 prime and x 2 prime as the initial input choice.

Now, the input price has changed and we want to find out the new equilibrium of the firm. So, for that we have to now find out another tangency point with this new budget line or new isocost line A B prime with another isoquant level. So, suppose at this tangency point e 2, the firm finds the new equilibrium and accordingly we can figure out the optimal input choice of the firm and we get x 1 double prime and x 2 double prime right. Now let us see that this increase in the input demand for input 1, how can we decompose this into the output expansion effect and technical substitution effect?

So, the trick is that, we have to now fix the old input price ratio and find out another tangency with another isoquant. So, now, let us focus on how we can decompose the total price effect into output expansion effect and technical substitution effect. What happens when an input price falls? Suppose a firm starts with fixed resources some money given to it and that can be donated by c naught right. So, if there is an input price fall. So, then the firm finds some extra cash saved with the current input level. So, this extra cash it can spend on purchasing more units of both inputs and if that happens, then with this extra units of both inputs firm can produce a higher level of output.

So, basically the farm reaches a higher isoquant level. So, let us draw that in the diagram so, here in the diagram note that, the firm has a higher level of output that it can reach with a fall in the input price. Now suppose I want to see, what is due to the output expansion only. So, in that case I have to now fix the input price at the initial level and I have to now draw another isocost line this broken orange line.

This broken orange line is then isocost line as well right and this has the same slope with the old isocost line. So, we find that there is tangency obtained at say a point e 3 and new equilibrium could be obtained at point e 3 and the corresponding input choice would be given by the coordinates of this newly found point e 3.

So, here we can name this input level as x 1 tilde and we can level this one x 2 tilde, we do not have space for names there. So, now, we are interested in the input 1. So, the output expansion effect is given by x 1 tilde minus x 1 prime. So, the technical substitution effect is given by x 1 double prime minus x 1 tilde right. We will continue this discussion on optimization behavior of firm in the next lecture.