

Microeconomics: Theory & Applications
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Lecture – 19
Price Change & Consumer Welfare (Part - 2)

Hello. Welcome you all to the lecture series on Microeconomics.

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Consumer's surplus

Assumptions :

1. Discrete good (x)
2. Utility function is quasi-linear
3. Fixed money income (M)
4. Constant market price (p)

Quasi-linear $u(x, M) = v(x) + M$

Reservation price (R)

The price at which a buyer/consumer is just indifferent between purchasing and not purchasing an unit of the good.

$R_1 \Rightarrow u(0, M) = u(1, M - R_1)$ for the 1st unit of x ... (i)

$R_2 \Rightarrow u(1, M - R_2) = u(2, M - 2R_2)$ for the 2nd unit of x ... (ii)

Assume $v(0) = 0$.

$v(0) + M = v(1) + M - R_1 \Rightarrow M = v(1) + M - R_1 \Rightarrow R_1 = v(1)$ (iii)

For the 2nd unit, $v(1) + M - R_2 = v(2) + M - 2R_2 \Rightarrow R_2 = v(2) - v(1)$ (iv) marginal utility

$R_3 = v(3) - v(2)$... (v)

$R_1 + R_2 + R_3 = v(3)$

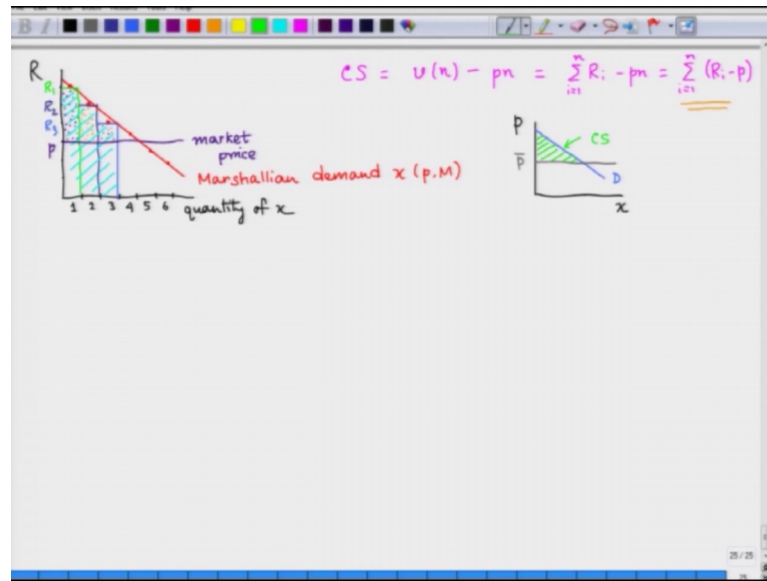
Generalized result :

$v(n) = \sum_{i=1}^n R_i$

Gross benefit from consumption of n units

In the last lecture, we have started our discussion on reservation price and consumer surplus.

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Now, we are going to draw a graph to illustrate gross benefit from consumption. So, along the x-axis we are going to measure those discrete quantities and here along the y-axis we are going to measure the reservation price R . Now, we are going to mark the units of consumption one by one. So, let us assume that for the first unit of the consumption R_1 is the reservation price and this is the highest level of price that the consumer say you to pay for the first unit of consumption. So, we can get a bar in the reservation price quantity plane.

Now, for the second unit of the consumption R_2 is the reservation price and the consumer is willing to pay this much for the second unit of consumption. Then for the third unit the reservation price is R_3 and so on so forth. I am not drawing for the other units of the good x . So, if I want to measure the gross benefit from consumption then basically I am talking about this shaded area.

Now, this good x is bought and sold in market at price P . So, now let us introduce that price and let us superimpose the price in the diagram. So, let us assume that this horizontal to x-axis line gives market price and that is P . So, in that case you see that although the consumer is ready to pay R_1 amount of money for the first unit of the discrete good x , actually he or she is paying only P to purchase that unit from the market. So, he or she is basically saving this marked area with small dots for the first unit of the

consumption he is making a net gain in terms of utility, right. Because earlier we have seen the reservation price can be expressed in terms of utility as well.

So, similar story happens for the second unit. So, for the second unit the consumer is ready to pay R 2 reservation price, but actually he or she is paying only P. So, this shaded area with these orange dots is basically the gain in terms of utility the consumer is gaining because he or she does not have to pay that amount R 2, he or she is paying a much lesser amount. And, the same story exists and we can tell the same story even for the third unit, right.

So, if you join this shaded area with the small dots that basically represent the gain in welfare of the consumer because the consumer is not paying the highest amount that he or she is actually willing to pay. So, that is basically the reservation price we are talking about. So, with this graphical illustration now it will be easier for us to define consumer surplus which is abbreviated consumer surplus is abbreviated by CS and we write that to be $v_n - p_n$. If the consumer is purchasing n units of the discrete good, this is the result. We have already seen the result earlier that v_n is basically nothing, but sum over i equal to 1 to n reservation prices.

Alternatively, one can also write; we used this very last expression to graphically measure the consumer surplus. Now, one can assume enough units of the discrete good x , so that this length of these bars reduces and if they reduce then we can approximate these bars by a point and if we join these points then we get so, suppose I get thinner bars. So, if that is the case you know let me assume one point for one bar like this. Similarly, there will be something for the fourth unit, then fifth unit and then sixth unit as well.

And, if I join these red dots by a line it may not be a straight line. But, for simplicity let us assume that we can join them by a straight line then we get what is known as the Marshallian demand, right. So, Marshallian demand is denoted by $x(p)$. So, one can say that consumer surplus can be simply represented in a price quantity plane through a demand function as well.

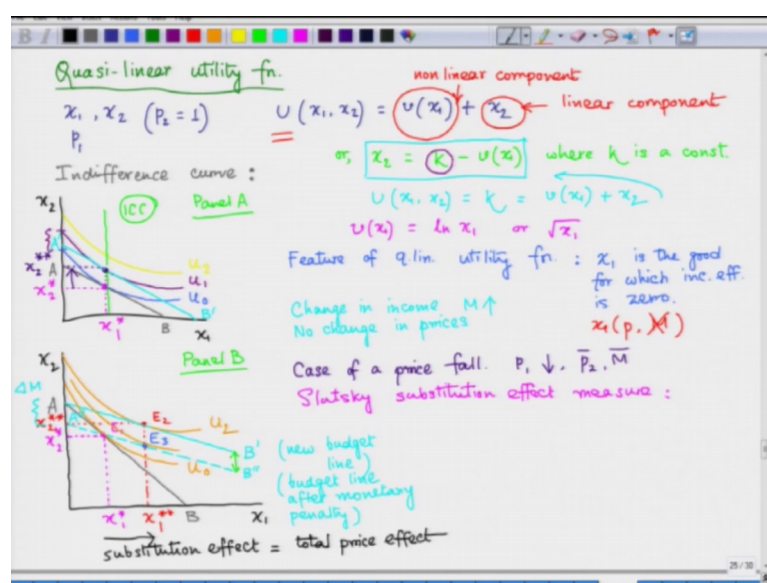
So, let us assume that we have a demand function, let us assume that we have a straight line demand function with a positive intercept along the price axis. Let us denote this demand function by D and if we assume there is a constant market price coming from perfectly competitive market or some other form of market is denoted by say \bar{p} . Then

consumer surplus is basically given by the area of this triangle. We have derived consumer surplus in the case of a discrete good.

Now, let us discuss how a consumer finds its equilibrium in the case of this discrete good. So, remember for each unit the consumer has a reservation price and there is a market price for the commodity. So, at the unit or for the unit the consumer finds its reservation price equals to the market price that is the number of units that the consumer is going to purchase and consume.

Now, let me go back to the idea of quasi linear utility function. We have assumed quasi-linear utility function to derive reservation prices and finally, consumer surplus. But, we have not talked at length about that utility function. It is a new type of utility function from the previous ones that we have studied now, what so interesting or important about quasi-linear utility function.

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Quasi linear utility function is a very popular functional form used in microeconomic analysis. So, here we assume that we are working with two goods; x_1 and x_2 . Last time we have seen x_2 as money, but now this time we assume x_2 is all other goods whose price is p_2 and that we set equal to 1 and p_1 is basically the price of the commodity 1. Now, the utility function in this case can be written in the following manner. So, there is a non-linear component and there is a linear component. As this utility function U has

two components, one non-linear and one linear this is called quasi linear utility function means partly linear utility function.

As we see that there is a linear component in the quasi linear utility function; that means, that if we start with one particular indifference curves say for utility level u_0 then all other indifference curves are just vertical translations of the original indifference curve. So, it implies that all indifference curves representing higher level of utility are basically vertically shifted versions of the original utility level u_0 represented by the original indifference curve we started with.

So, let us have a diagrammatic illustration of this concept. Suppose, u_0 gives the initial level of utility and this is the indifference curve. Now, note that there is a intercept along the x_2 axis which is very peculiar. We have not seen indifference curves before where it touches one of the axis, but here it does. So, we can write or rewrite this utility function as x_2 equals some k minus v of x_1 where k is a constant and that constant changes from one indifference curve to the other. So, that means, that the height of each indifference curve is some function of x_1 and the constant k .

So, let us draw the other indifference curve; suppose, we now draw the indifference curve for a higher utility level u_1 . So, what exactly is happening here? So, we set our utility function at a particular value. So, this is the utility level and then that is equal to v of x_1 plus x_2 the original utility function form and from here we can get back to the equation. Now, what can be the example now v of x can take the popular forms like $\log x$ or square root of x , these are standard forms you will encounter in a typical microeconomics textbook.

Now, there is an interesting feature of this quasi linear utility function and that is one of the commodities which is part of this utility function has zero income effect. So, the way we have drawn our graph here x_1 is the commodity for which income effect is zero. What does that mean? It means that demand function for commodity 1 does not depend on income level. So, the demand function for commodity 1 is dependent on only the price and not the money income.

Now, let us check this out through the graph. Let me now draw let me now assume that there is some price ratio and if we know the price ratio we can definitely superimpose the budget line in this diagram. So, this is the initial budget line say AB and of course, this

budget line will make tangency to the original utility sorry, this budget line will make tangency to the indifference curve original indifference curve at some point and at that point there will the optimal level of choices determined. So, we will have x_1^* optimal units of good 1 consume and optimal units of good 2 consumed, right.

Now, if there is a change in income then let us see what happens. So, let us assume that change in income takes place. So, if there is a change in income takes place, but prices are fixed. So, change in income say my money income has gone up, but no change in prices. So, then the slope of the budget line will not change it will only shift parallelly upward, right. So, we will get a new budget line say A' B' and of course, there will be another new tangency point where the consumer equilibrium is determined.

And, at this new equilibrium what we will observe that the demand for commodity 1 or the optimal choice for good 1, has not changed. Whatever money income whatever units of money income increment took place in the budget equation of the consumer is reflected through the consumption of commodity 2 only, which is all other commodity. So, there will be a rise in commodity 2's consumption or all other goods consumption say it will go up from x_2^* to say x_2^{**} . But, x_1^* the optimal choice of commodity one will remain unchanged.

So, then only we can see that the change in income does not play any role in the demand of commodity 1. So, we have seen that a change in money income does not play any role on the demand of commodity 1. Now, for that commodity 1 we can say income effect is 0. Now, let us this is very peculiar, right. So, let us try to find out some practical real life examples where this kind of consumer preferences may arise.

We can think about some goods like say no simple goods pens, pencils, salt etcetera. So, these commodities have some importance in our life, but it is not heavily important such that if money income increases we are going to spend a lot of money to purchase more and more units of this commodities. So, what I mean to say I mean to say that although they are required, but their presence in our consumption basket our entire consumption basket is less is very low and that is why probably the income effect is 0 for these kind of goods.

Now, let us link this discussion on quasi linear utility function to our previous discussion of total price effect decomposition into income effect and substitution effect. So, we have

seen in this case of quasi linear utility function for one good income effect is 0. So, let us see how our discussion on Hicksian and Slutsky substitution effect is going to be adjusted if the consumer has a quasi linear utility function or quasi linear preference. This time also we are going to do the analysis through a graph.

So, in this diagram what we get a vertical Income Consumption Curve or ICC. So, first AB is the original budget line and now we assume that there is a price fall. So, of course, there will be a new budget line which is much flatter and let us assume that B prime AB prime is the new budget line, right. So, now, let us superimpose the indifference map in the case of the quasi linear utility function. So, let us first assume that we have the indifference curve which finds tangency with initial budget line. Let me denote the utility level by u naught the starting level of utility.

So, at the tangency point the initial choice level is denoted by this consumers equilibrium point say E 1 and we see that the consumer consumes x_1 star units of commodity 1 and x_2 star units of commodity 2. Now, as there is a price fall the consumer now makes another tangency and let me draw that indifference curve with which the new budget line AB makes a tendency. Let me assume that utility level is u_2 and this is utility that is given by the higher level of indifference curve which becomes now tangent to the new budget line and this is the point of equilibrium. So, this is E 2, right.

So, we see that at equilibrium E 2, the consumer consumes x_1 double star units of commodity 1 and x_2 double star units of commodity 2. Now, let us assume a Slutsky substitution effect measure. What will happen under this measure, as we observe that due to price fall the consumer has gone. So, how we are going to enact the Slutsky substitution measure? As we observe that due to a price fall the consumer has gone up the indifference map and he or she is currently at a higher level of indifference curve representing a higher level utility.

We have to penalize the consumer for this gain in terms of the utility. So, we have to take some money out of the consumers wallet. How we are going to penalize the consumer? We are going to take out money from consumers wallet such that the consumer is forced to come down to the initial consumption bundle which is x_1 star and x_2 star. So, basically we have to force the consumer back to the equilibrium E 1 point from equilibrium point E 2. So, now we have to penalize the consumer, right through some

monetary penalty. So, the monetary penalty takes place in such a way that the consumer is able to purchase the original consumption bundle that he or she was consuming before.

So, now, we are talking about another budget line A double prime B double prime. So, the gap between A and A prime is basically the penalty, that is basically ΔM . These many this much of money has been taken out from the consumers wallet, right. It is a reduction in money income. But, as it is only a change in income prices do not change and the budget line after monetary penalty has the same price ratio as of the new budget line AB prime which is basically after the price change right.

So, now, see with this current A prime A double prime B double prime budget line how consumer finds its equilibrium. So, we know that income effect is 0 for the commodity x_1 from our previous analysis say panel B the analysis that we have done in panel A we know that for the commodity 1 income effect is 0. So, now we can trans we can use the knowledge that we derived from our panel A diagram in our panel B diagram. So, here also the income effect for commodity 1 will be 0, right.

So, that means, that as there is a change in money income this gap it will have no impact on the demand of or demand for commodity 1. So, basically what we are saying is this. So, the new equilibrium will be somewhere here, right. So, that means, that we are talking about another indifference curve which will make a tangency at this new equilibrium E_3 and at this new equilibrium we see that the same level of x_1 consumption takes place. But, the consumer has reduced its consumption of commodity 2.

So, what is the take away? The take away from the exercise is this; the journey from x_1^* to x_1^{**} , this rise in consumption of commodity 1 due to a price fall is entirely due to substitution effect and not income effect. So, here the total price effect is equal to the substitution effect. What is so interesting about these findings that income effect is 0 in the case of quasi linear utility function and the total price effect is basically due to the substitution effect only; be it Slutsky or you know we can also show the same thing under the Hicks substitution effect scheme also.

So, if we assume quasi linear preference for a consumer then our life becomes easy, right. So, if there is a price change and we want to trace out a consumers demand function for a commodity then we know we do not have to worry about whether we are

no dealing with Hick's substitution effect or a Slutsky substitution effect. Or we are going to or we have to work with the Marshallian or the Slutsky or the Hicksian demand function, the compensated demand function that we have studied earlier.

So, in the case of quasi linear preference there is no difference between a compensated demand function and a Marshallian or ordinary demand function. So, that has a huge implication in applied economic research because then you do not have to work with ordinary ah. So, in that case you if you assume a quasi linear preference then you do not have to work with the compensated demand function which is theoretically more robust. Actually it is very difficult to find compensated demand functions in reality.

So, if you assume a quasi linear preference then you can straightaway work with the Marshallian demand or the ordinary demand function. And, as there is no difference between the Marshallian demand and the Hicksian demand in such cases, then the consumer surplus the welfare change measure of consumer is also identical in this case. But, if you do not assume quasi linear preference then the consumer surplus measure will change because there is a difference between the compensated demand function and the ordinary demand function. So, it will matter which type of demand function you are assuming when you are measuring consumer surplus.

So, with this we have concluded our discussion on consumer surplus and typical topics in consumer theory, but let us do a bit of digression. Here we are talking about price change of one commodity and we are interested to see what is happening to that very commodity whose price is changing. But, you probably have seen that there is an impact of price change of a commodity on the other goods demand as well.

In the next lecture, we are going to study a concept which will help us analytically model that or measure that.