Practitioners Course in Descriptive, Predictive and Prescriptive Analytics

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Lecture – 07
Overview of Probability

Good afternoon welcome to one more lecture of a analytics course applied analytics the practitioners approach on descriptive prescriptive and predictive analytics and as you have we gone through different basic aspects of it and we are trying to focus more towards the applied side of this course, today we are going to do quick overview about the probability. So, it hence today's presentation is the overview of probability.

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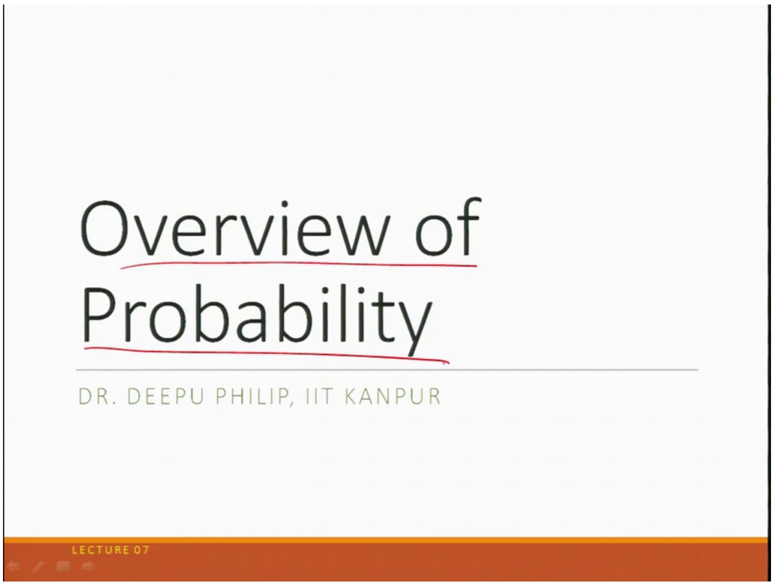

Know that for practitioners probability is kind of a tricky concept and quite a lot of things we talked about this.

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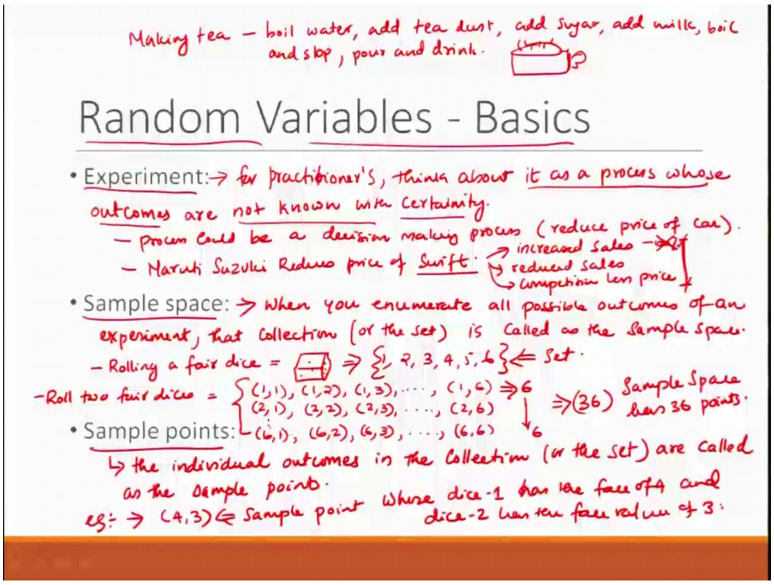

So, we will try to take it from the practitioners approach what it is. So, let us start with the definition of random variables, the very basic aspects of the probability and to do that the first thing we need to think about is this concept called experiment and the definition that we are going to cover here today is more about the what is an practitioners and an applied approach.

So, for practitioners the textbooks gives a lot of other definitions, practitioners for practitioners, think about think about it as a process, think about what experiment as a process whose outcomes whose outcomes are not known with certainty. So, a people talk about tossing the coin and all those kind of things, but let us talk about the experiment of making tea ok. So, we will boil water add tea dust, add sugar add milk then boil and stop pour and drink.

So, when you do all these kind of things steps then you finally, get a tea if you think about it you get a cup within which you have tea now you are drinking tea hot tea. Now, each time when you make the tea you actually would get not the exact tea, you will get some different versions of the tea. So, what type of tea you are going to get there is no guaranteeing of it you will get tea, but some different versions of tea.

So, similarly if you follow a process whose outcomes are not known with certainty you know that you will get an outcome, but which outcome you are get you are not going to be sure about it. So, this the process could be a decision making process decision making process for example, let us talk about reducing the price, reduce price of car. So, TATA motors try to reduce the price of the car. So, let us take for the timing let us take Maruti Suzuki reduces price of swift, let us say they reduce the price of their swift car model then what happens how is it going to impact well there is no guarantee that you can have increase the sales you can have reduce the sales ok. Then competition reducing the price, competition less price etcetera other cut a lot of options we are not sure what is going to happen you cannot say that this is going to happen 100 percent this is talk cannot say that. So, something will happen with some probability.

So, that kind of a situation where you are not sure about the outcome of the experiment or outcome which one will have occurs with certainty that is called as an experiment, the next our concept we need to talk about is the sample space. So, in sample space when you enumerate, when you enumerate all possible options all possible outcomes of an experiment, experiment that collection that collection or in a way the mathematical price for is it or the set that collection or the set are called as the sample points. Sorry as is called as a sample space or not are is called as a sample space let us clear this; it is the collection the set is called as the sample space.

So, if you are let us think about the example of you know rolling a die rolling a fair die dice ok. So, a die has a face. So, dice like a cube is all know about that just like 1, 2, 3 like this dots you rolling this the outcomes you are going to get is, you can get the face value 1, 2, 3, 4, 5 and 6. So, this collection of these outcomes this is what is called as a sample space this set is the sample space of rolling a fair dice fine.

Then there are many other ways we can think about doing this as well will give a take an example if you roll 2 fair dice, roll to fair dice then what is will be the sample space the sample space will be you can think about it this way the first die I will have a face of 1 second will also a face of 1 then the first die can have a face of one second can a face of 2, first die can have a face of 1, second die can have a face of 3 like this all the way to first die can have a face of 1, second die can have a face of 6.

Similarly the first die can have a face of 2, second die can have a face of 1, first die have a face of 2 second can also have a face of 2. Then 2, 3 like this all the way to 2, 6 similarly the [last] you can think about last option will be the first die can have a face of 6 second die can have a face of 1 ah. First die in have a face of 6, second die can have face of 2, first die can have a face of 6, second die can have a face of 3 all the way up to both dice giving 6 6 ok. So, this collection ok so, you know there are 6 options like here and there are 6 options this way. So, you have 36 different possible options ok. So, this sample space of rolling the 2 dice has 36 points ok.

So, which brings the next concept sample points what are sample points the individual outcomes the individual outcomes in the collection or the set as we called or the set are called as the as the sample points as an example in the rolling of the 2 fair dice the face of 4 and 3 this is a sample point sample point where dice die 1 has the face of 4 and die 2 has the face value of value of 3 ok. So, hope you guys understand the basic concepts of the random variables.

So, we are trying to get into the basics of random variables. So, these 3 concepts are important for us to get to the basics of random variables.

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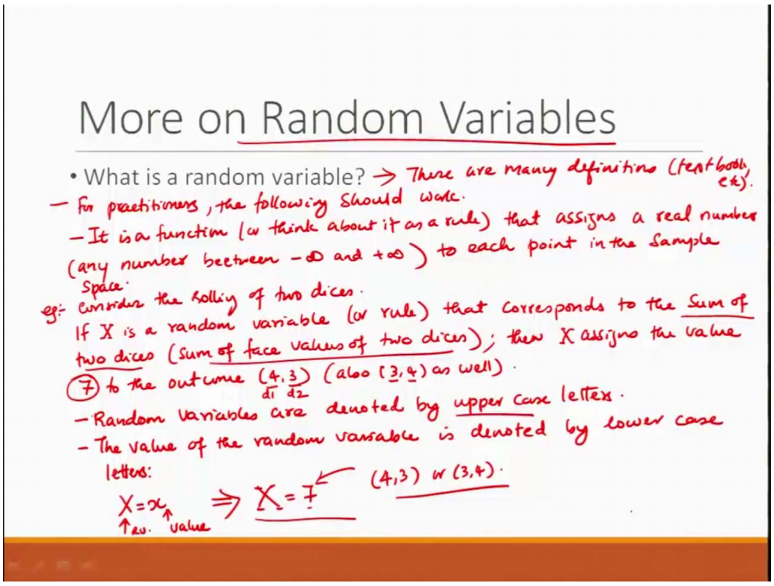

Now, the next concept is now when we talk about experiments sample points and everything, then what are random variables actually how do we define a random variable ok. So, the first thing is there are many definitions, many definitions textbook etcetera, but since it is a practitioners course let us see what the practitioners think about it. For practitioners, practitioners the following definition the following should work ok.

So, this definition is more meant to practitioners it is a function or thinks about it as a rule, think about it as a rule a rule of thumb or something like that. A rule that assigns that assigns a real number that assigns a real number that is real number means any number between minus infinity and plus infinity.

So, the entire scale any real number to each point in the sample space each point in the sample space ok. So, let us consider for example, let us consider the, consider the rolling of 2 dice. We already mentioned in this example if you are rolling the 2 dice then what are we going to do say, if x is a random variable x capital x is a random variable is a random variable or rule ok, that corresponds that corresponds to the sum of 2 dice 2 dice or sum of face values of 2 dice if that is the case we are looking at dice.

Then x assigns the value 7 to the outcome 4, 3 also 3 4 as well so; that means, if you roll a die and if the random variable is the rule that says the sum of the 2 dice, the rule is some of the face values of the 2 dice. If that is the roll then if you roll 2 dice and you get the die one as face value 3 and die 2 as the face value sorry die 1 as face value 4 and die 2 as face value 3 then this 7 is assigned to the face value of this. Similarly 3 or 4 also the dice will change, but the same value gets assigned to it ok.

So, always remember random variables, random variables are denoted by uppercase letters ok. So, typically you use capital letters uppercase letters to denote the random variable the value of the random variable, the value of the random variable is denoted by lowercase letters. So, mathematically if I say x equal to little x, then this is the random variable and this is supposed to be its value or another way to think about it is if I say x equal to 7 and then x this capital letter denote the rule some of the face values and then 7 is that the sum of the face value is coming to be 7.

So, this is the rule that assigns the real number to the point in the sample space and the point in the sample space can be 4 3 or 3 4 ok. There can be other points as well, but for the time being as an example this is what we are going to deal with hope you guys understand this part of the concept. So, we will get into the next one then.

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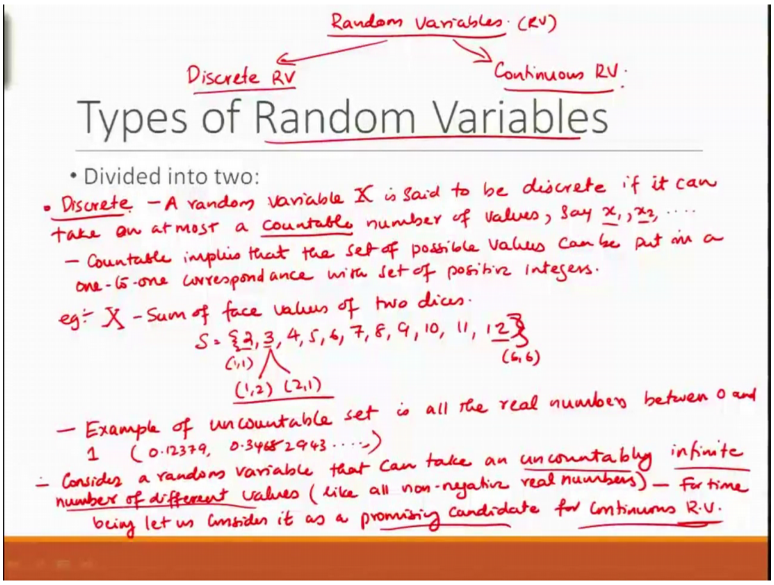

The random variable for us is typically divided into 2 there are many ways people look into it and for. So, let us think about it does this way, random variables as I said earlier they are divided into 2. So, the first 1 is what we call as discrete the r v r v is called random variables and continuous r v random variable.

So, we are going to talk about discrete random variables and continuous random variables ok. So, let us talk about the first one discrete ok. So, we are now talking about discrete, a random variable x a random variable x is x uppercase x x is said to be discrete if it can take and at most or take on at most a countable, most countable number of values, values let us say x 1, x 2 etcetera lowercase is used to denote the values.

So, when you say countable this face countable what does it means, countable implies that the set of possible values set of possible values, values can be put in put in one to one correspondence, one to one correspondence ok. lease check my spelling because I am not very good with this corresponding the one to one correspondence with set of positive integers ok, this is what the discrete random variables is talked about ok. So, if we can connect it. So, if you think about the random variable x.

So, let us think about an example we talked about the random variable x where x is the sum of the face values x sum of face values of 2 dice if that is the case then the sample space of x is equal to 2, 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 sorry this is a set when you set 2 this implies the face value of one and one and 12, the face value of 6 and 6 all other possibilities are mapped into different values in this ok. So, if you have upper to 3 it will be mapped to 1 and 2 and 2 and one. So, both will be mapped by this random variable 3 ok. So, these 2 outcomes are mapped by 3 in this case alright. So, if that is the case then we talked about uncountable set what is an example of a countable set an count then what is an example of an uncountable set.

So, example of uncountable think that you cannot count, an uncountable set is all the real numbers, all the real numbers between 0 and one 0 and 1. So, here you can have values like 0.12379 then there is another value called 0 3 4 6 8 2 9 4 3 like this.

So, you can have infinitely large, infinite numbers between just 0 and 1 ok. So, if that is the case consider a random variable, a random variable that can take that can take an uncountably, uncountably infinite number of values number of different values different values for example, like all nonnegative in real numbers.

So, think about a random variable that can take uncountably infinite number of different values ok. So, for time being let us consider it as consider it as a or promising candidate promising candidate for continuous random variable.

So, what we are doing here is that the countably infinite number of different values they are promising candidate for continuous random variables, we are not defined continuous random variable it, but for the time being let us consider it that way if that is the case then let us look into a the probability.

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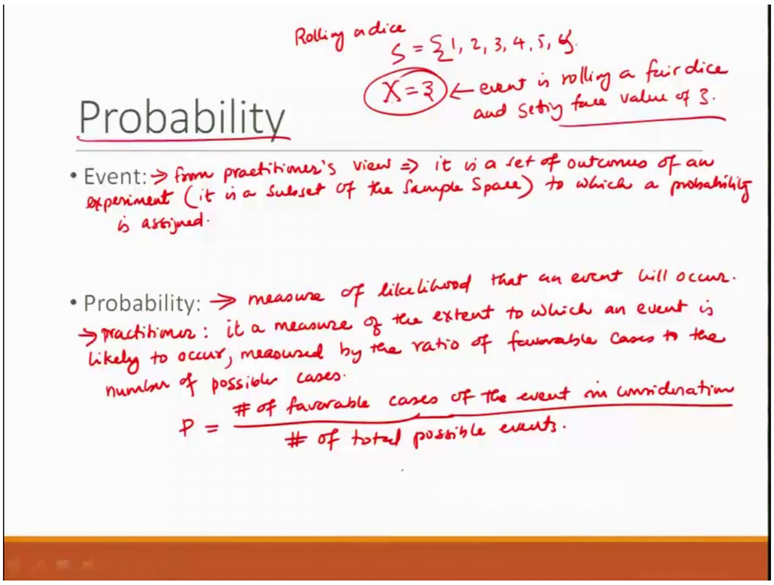

So, it is easy to describe probability in terms of the discrete random variables and then we will move to the continuous random variables little the down roll. So, for that we need to look into the concept of event first ok.

So, what is event? So, again it is a practitioners definition from practitioner viewpoint practitioner or practitioners view what is it, it is defined it is a set of it is not defined it is a set of set of outcomes of an experiment. So, which means it is a subset of the sample space ok. So, subset of the sample space to which a probability assigned to which a probability is assigned is assigned ok. So, what we are basically saying is that if you say if you are rolling a fair die.

So, if you say the experiment is rolling it die and then you get the sample space to be values 1, 2, 3, 4, 5 and 6 ok, if I say that the random variable x is equal to 3 which means if x the random variable is getting a face value of 3 then the probability this even this rolling it die the event is rolling a fair die and getting face value of 3 ok.

The random variable is like the face value event is getting the face value equal to 3, if that is the case then what is probability, probability has many measures or many definitions and the typical textbook definition is the measure of likelihood, likelihood that an event will occur ok. You are measuring the likelihood that that event will occur this is what the book definition says from our viewpoint plus for the practitioner what is it? It is a measure it is a measure of the extent to which an event these likely, an event is likely to occur measured by the ratio of favourable cases to the number of possible cases.

So, what we are saying here is that probability plus denoted as p is equal to the ratio. So, it is a fraction of the number of favourable cases how many of these are the favourable cases, for the thing that you are trying to measure for the favourable cases of the event in consideration if that is the case then to the total number of number of total possible events, if this ratio if you calculate this ratio is what we call as the probability ok. So, if that is the case. So, let then let us see how to do an example.

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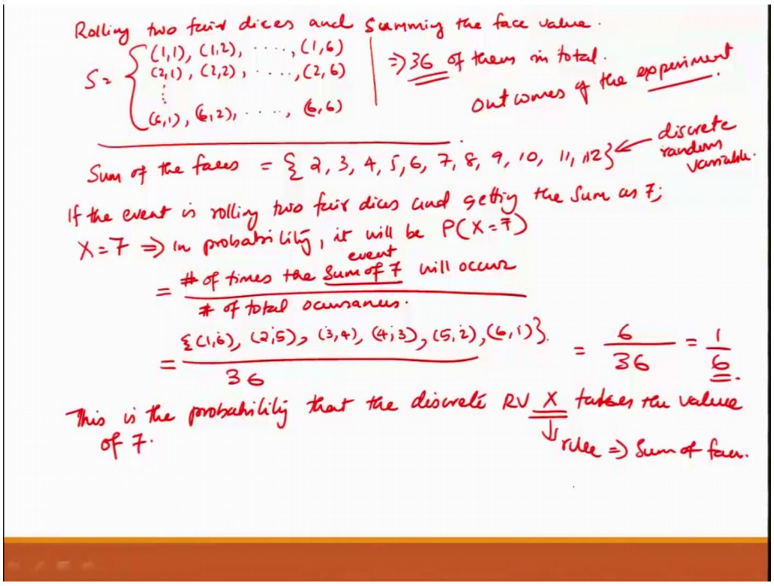

So, I said earlier rolling 2 dice rolling 2 fair die dice and summing the face value summing the face value, if that is the case the sample space we had denoted earlier cause 1 1, 1 2 etcetera all the way to 1 6 then 2 1, 2 2 all the way up to 2 6 then similarly 6 1, 6 2 all the way up to 6 6 ok.

We said these there are 36 of them, them in total we also saw how to calculate the 36 this is the rolling of the fair die this is the outcomes of the experiment the experiment of rolling to fair dice ok, the experiment is rolling to fair dice and the sum sample space of the sum or some of the phases can take the value soft 2 3 4 5 6 7 8 9 10 11 12 that is what we said these are the face values it can take and we said this is say discrete random variable because this is countably finite or countably discrete this is a discrete random variable.

So, if I say that if the event is rolling 2 fair dice and getting the sum and getting the sum as 7 if that is the case then the random we seen already it is equal to x equal to 7. So, in probability it will be probability, it will be probability of x equal to 7; that means, rolling fair dice and getting the sum to be equal to 7 what would that be. To calculate that this is number of times the sum of 7 this is your event ok, even will occur by number of total occurrences this we have already seen it as 36 we see the 36 as a total occurrences. So, that will become its equal to 36 over something what are those over values.

So, one way to get 7 will be in the first case if you have one and 6 it will be 7 second case it will be 2 and 5 that will also give you a 7 then there will be 3 and 4 then there will be 4 and 3 which is the first die will give you a 3 second die will be for this 4 and 3. Then the third one then the other case will be 5 and 2 in the last case will be 6 and 1 if this is the cases then you see this is 1, 2, 3, 4, 5, 6. So, there are 6 possible ways you can get 7 and there are 36 possible ways. So, your probability 6 over 36 which is over 1 over 6, so this is how you call the probability calculate the probability of this particular event ok.

So, this is the probability that the discrete random variable discrete random variable x takes the value takes the value of 7 what does this actually denote x is the rule and the rule is sum of faces. So, this is called as a probability of the discrete random variable or discrete event the event is that you are rolling a die and rolling 2 fair dice ok.

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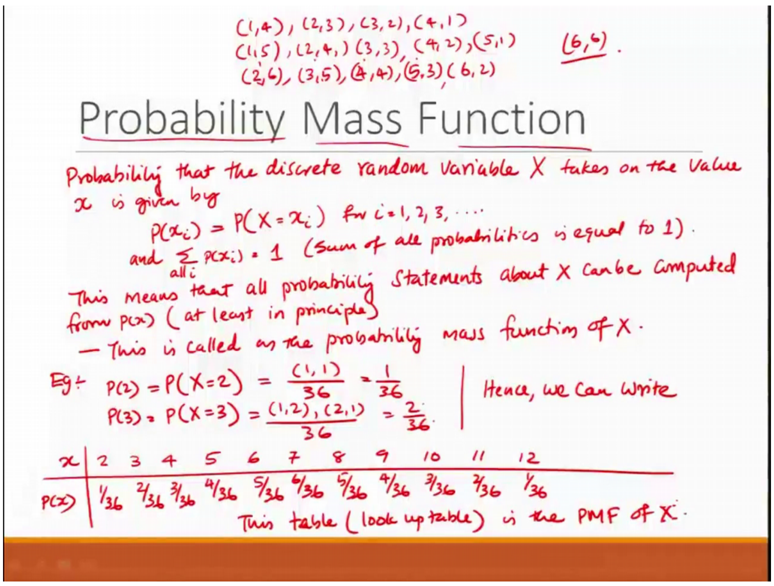

So, then from here now we are going to move into the new concept called probability mass function ok. So, earlier as we said that we said that probability, probability that the discrete random variable, discrete random variable probability that the discrete random variable x takes on the value takes on the value little x is given by is given by probability of x I which is equal to probability of x equal to x I for I equal to 1 2 3 etcetera.

So, this is the notational way of writing the probability that the random variable x takes the value x little x and for each individual case 1 I for one 2 3 4 I is an index it will take different values and summation of all I p of x I is equal to 1. That means, all these probabilities if you some of them sum of all probabilities is equal to 1 something like this ok. So, this means that, this means that all probability statements probability statements about x x can be computed can be computed from p of x or at least in principle.

So, in principle you can calculate all probability statement x about x. So, this is called as the probability mass function of x, mass function of x ah. So, if you have say that lets think about the example of rolling 2 fair dice then if we say probability of 2 which means probability of x equal to 2 when you are rolling 2 fair dice and looking at the sum of the values, then this is equal to we are saying the only one where to get this is phase 1 and a 1 over the 36 of it which is 1 over 36 like this.

So, if I say probability of 3 then we are talking about probability of x equal to 3 which means you are all to fair dice and get the values or 2 phase values and sum them and those values sum equal to 3 that is 1 2 and 2 1 over 36 which is 2 over 36 number of favourable to total number of outcomes.

So, if that is the case hence we can write and we can write little x, the value of little x we can write kind of a table where it starts with 2 then let us say you have 3, 4, 5, 6, 7, 8, 9, 10, 11 and 12 these are the possible values and you have the p of x the probability of x. So, this means probability that the x takes the value of 2 we already have calculated 1 over 36 3 via calculator 2 over 36 4, 4 is equivalent to 1 and 3, 3 1, 2 2. So, that will be 3 over 36 5 will be what are the values of 5 it will be 1 4, 2 3, 3 2 and 4 1.

So, these are the values of 5. So, that will be for over 36, 6 will be 1 5, 2 4, 3 3 then 3 3 either way it is the same you cannot distinguish between each other 3 3 then there is 4 and 2 and then 5 and 1 there is no 6 0. So, if you look at this 1, 2, 3, 4, 5. So, it will be 5 over 36 7 we already calculated 6 over 36, now what will be the value of 8 how many ways you can make 8. So, you cannot make it with the one. So, it will be 2 and 6 then; obviously, then with 3 you can make it as 5 then you have 4 and 4 then you have 5 and 3 and 6 and 2 so, 1 2 3 4 5.

So, it will be 5 over 36, similarly 9 will be for over 36 this will be 3 over 36, 2 over 36, and 12 will be 1 over 36, 12 will be just the case of 6 and 6. So, this think that this is this table or some people called as a lookup table ok, this lookup table is the p m f probability mass function of x random variable x which is about rolling to fair dice and taking the face value.

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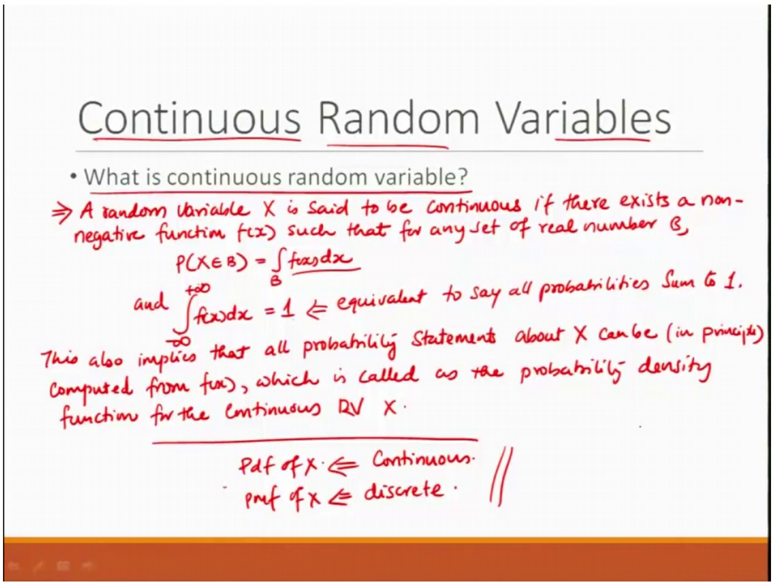

So, now, with this it is time for us to come to the definition of continuous random variables. So, what is a continuous random variable, we have earlier mentioned that if a random variable can be given the accountably infinite values between 2 limits a and b and all the real numbers countably infinite real number values between the random between the limits a and b then that is a possible candidate for a continuous random variable.

So, now let us formally define because now we know what is a probability mass function and other things of using which we can now define a continuous random variable. So, what is a continuous random variable? A random variable, a random variable this is again the practitioners definition x is said to be continuous, to be continuous if their exists non negative, non negative function a non negative function f of x such that such that for any set of for any set of real numbers or real number any set of real number b probability of x element of b that random variable is element of b is given by integral of b f of x d x ok. So, you keep integrate this function within that particular b any real number set that will give you the probability and integral of minus infinity to plus infinity f of x d x is equal to 1 if you integrate it across the entire set of real numbers this is equivalent to say all probabilities, probabilities sum to 1 ok.

So, this also implies, this also implies that all probability statements or probability statements about x x can be or at least in principle, principle x can be computed from f of x the f of x is the function that we talked about which is called or which is known called as the probability density function, density function, probability density function for the continuous random variable x continuous and random variable x ok.

So, in this particular case continuous random variable is not necessarily just a assigning of real values it is the assigning of real values in such a way that there exists a positive function or a non negative function f of x that will define where it will allow you to find the probability of x for any given set b and if you integrate this function from minus infinity to plus infinity the value will be equal to 1, if that is the case then this is called as the probability density function p d f of x ok.

So, p d f is typically used for continuous and for p m f is of used for discrete all right. So, using this you can find the probabilities at any given point any particular probability statement can be calculated. So, let us give an example, let us take an example ok.

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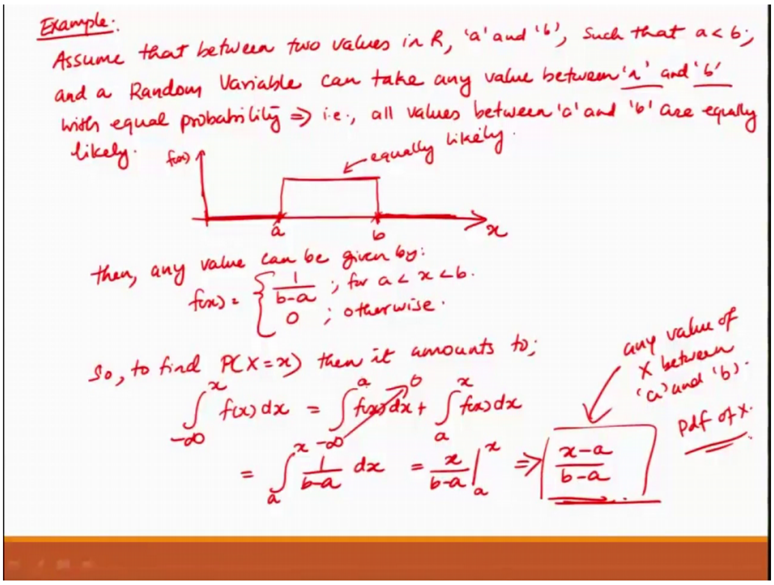

Example to denote this collect assume that assume that between 2 values, 2 values in r r is a set of real numbers a and b get these are the 2 values such that a is less than b ok.

So, the 2 values a and b and then such that a is less than b and a random variable random variable can take can take any value any value between a and b, random variable can take any value between a and b with equal probability, probability any value can be taken with equal probability which implies or that is all values between a and b are equally likely, equally likely.

If this is the random variable random variable that any value can be taken between a and b with the equal probability, all values are possible with the equal probability then you can think about it this way if you have a graph you want to draw a graph this is the numbers going in next, your set of real numbers and in which somewhere you have a and somewhere your b where a is less than b if that is the case and if you think about this was the f of x the probability of this.

So, between this a and b all of these will have the same value ok. So, up to here it is 0 there is no probability, at a it will have one probability and the same probability will remain until you reach b and after b there is no probability 0 probability. So, between a and b any random variable to take their probability is one and the same this denotes the equally likely aspect or same with equal probability, equally likely alright.

If that is the case then any value any value can be given by given by f of x equal to can write a function they say 1 minus 1 over b minus a. So, if you take this difference this is b minus a take the difference and find the ratio of this then this will give an equally likely probability for all values between a and b take any value between a and b this is the equally likely probability or 0 otherwise ah.

So, for all other places this probability is not defined. So, if that is the case if you want to find. So, to find probability of x equal to little x that is the case then it amounts to amounts to integral minus infinity to x f of x d x which is equal to integral minus infinity to a f of x d x plus a 2 x f of x d x this is integration by parts and we know that within these limits it is 0 because the function is not defined.

So, this goes to 0 you only need to do is this integral which is equivalent to integral a to x, 1 over b minus a that is your f of x d x which is equal to x within the x over b minus a which is equal to a and x ok. So, if you do this is x minus a by b minus a. So, this equation this closed form equation that we write here this tells you what is the probability of any value any value of x between a and b, this can be called as the p d f probability mass function is for discrete as the p d f of x alright.

So, with this we kind of look at the our discussion on probability, group discussion on probability for the practitioners come to an end and I know there is so much more associated with this about hypotheses and all those kind of things which you will discuss in the later classes to go.

But the most important part of this is everybody should realize that the concept of probability can be addressed in this particular fashion and this is the basic building blocks of the probability and from here we can actually build the larger theorems and these probabilities are important especially when you are doing analytics because when you are having a hypothesis and we need to try to do hypothesis testing. You will be dealing with different probability distributions and other aspects are such. So, thank you for your patient hearing and we will see you in the next class.

Thank you.