

Project Management
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Module No # 2
Lecture No # 09
Project Risk Management Analysis I

Welcome back this is the ninth lecture for project management and if you remember the last slide based on which we ended the right lecture was related to the different techniques from the financial point which are there like IRR and interest rates, rate of return, expected value and so and hence so forth based on which you can take a decision whether the project is at all feasible and how you can compare different projects.

So and just before that two or three slides before that we consider the concept of decision trees and how the expected value and variance would be utilized. So now I will just try to analyze from the point of view of risk analysis. So our plan for this set of lectures and it is very specifically mentioning this important point of plan would be once I finish the risk analysis then I will try I will definitely do problems in related to three broad areas one with the decision trees.

One would be the different type of financial concept and then come to the concept of the risk analysis how it can be undertaken.

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$$\frac{1}{6} [1 + 2 + 3 + 4 + 5 + 6] = E(X)$$

Risk Analysis

For an outcome, which is random or probabilistic we denote it by X . Corresponding to this r.v. we have an average/mean of this outcome, which is termed as the expected value, denoted by $E[X]$. Simultaneously along with the average return we are also concerned about the cost involved in the outcome. This cost or the uncertainty we face regarding the outcome is known as the risk.

For an outcome be it financial or a project so our case in only a project if you are assured that is sure to give us a sign tangible benefit then we are not concerned about the change that will not happen and the corresponding probabilities would be very specific to that particular activities to say that whether there is ninety percent chance ninety five percent chance twenty percent chance whether the work will be done or not.

But in reality we know that maximum the cases the activities are the jobs or any decisions we have to face the game of chance comes and hence one should be aware of the probabilistic nature of the distribution and the return based on which you take a decision whether their activity or a job should be taken into action. So for an outcome consider any random outcome whether you are buying the project can be assignment I mentioned it can be anything building a bridge building a house taking a project to design a car whatever it is.

Consider that as X which is just a variable based on which we will do the calculation and we will use the concept of probability where X is a random variable. Corresponding to this random variable capital X we have average or a mean value. So this average or a mean value would basically mean that what is on an average the so called value you will get from X .

So if you if you remember if you roll the die if it is it is basically an unbiased die and there are six phases so the probability of either getting one or two or three or four or five or six is

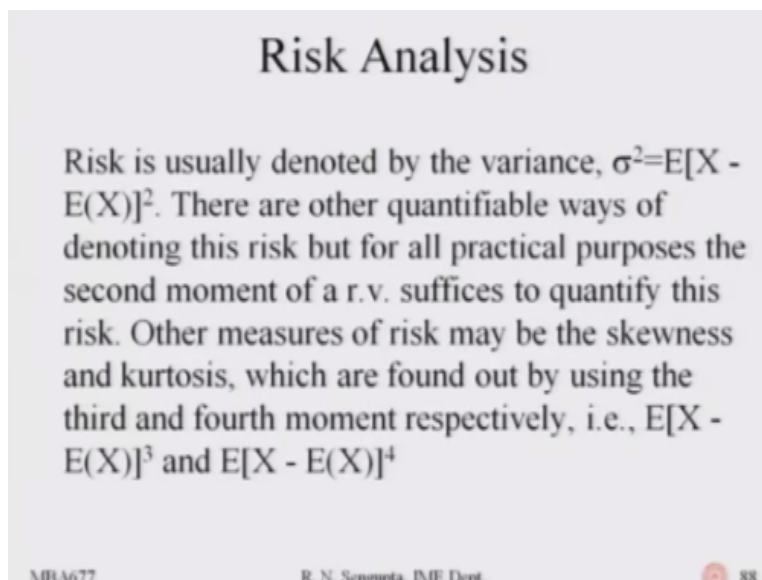
everything is 1 by 6 because 1 by 6 is added up e number of times is one which is the probability which we all know.

So if you want to find out the expected value of the outcome what we generally do as we all know is that multiply the outcome which is there which would be there one or two or three or four or five or six for the case of the rolling the die and multiply each term which is there which are the realized values of the random variable X and you will multiplies by them by 1 by 6 so this is done where you will basically have a six here and why I have put in the bracket because 1 by 6 is the probabilities which is common you can take it outside.

So technically it is 1 into 1 by 6 2 into 1 by 6 3 into 1 by 6 4 into 1 by 6 5 into 1 by 6 6 into 1 by 6 this is will give you the expected value of X and what is the formula based on which we are trying to do that I will come that within two or three slides.

So the cost for uncertainty for the decision if it is an expected value would be risk and I we remember I did mention risk time and again at one point of time we were basically discussion as the overall loss in the monetary sense but this basically risk basically means that for any decision which is probabilistic non deterministic of dispersion or variance coming into the picture.

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Risk Analysis

Risk is usually denoted by the variance, $\sigma^2 = E[X - E(X)]^2$. There are other quantifiable ways of denoting this risk but for all practical purposes the second moment of a r.v. suffices to quantify this risk. Other measures of risk may be the skewness and kurtosis, which are found out by using the third and fourth moment respectively, i.e., $E[X - E(X)]^3$ and $E[X - E(X)]^4$

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So risk is usually denoted by the variance given by sigma square is equal to the expected value of in the bracket X minus e of X is X is already expected value of mean know the whole square. So there are other quantifiable ways for denoting the risk but for all practical purposes the second moment so this is the second moment based this is not the second moment this is not the second moment we find out the concept of sigma square.

So the second moment of the random variable are utilized which suffices to quantify this risk other measure the risk could be can skewness which is the third moment for calculated from the third moment it can be kurtosis so on hence so forth. So these formulas are given for the skewness if you see it is given E X cube X 4 and so on and hence so forth. But we will concentrate on risk concept of which is the second moment.

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Risk Analysis

If we have X as a discrete variable then we have

$$E[X] = \sum_{\forall i} x_i f(x_i) \quad V[X] = \sum_{\forall i} \{x_i - E[X]\}^2 f(x_i)$$

If we have X as a continuous variable then we have

$$E[X] = \int_{-\infty}^{+\infty} x f(x) dx \quad V[X] = \int_{-\infty}^{+\infty} \{x - E[X]\}^2 f(x) dx$$

So if you have X there as a discrete variable then we have the formula which is given that if it is discrete then the expected value would be found by summation of X I's. So X I's are the other realizes value multiplied by the corresponding probabilities and then you sum them up. So if the problem which we just did very fleetingly trying to find out what is the expected value of the outcome when you role the die was basically 1 by 6 into 1 till 1 by 6 into 6.

So these 1, 2, 3, 4 are the real as value 1 by 6 is the probability and if you want find out the variance use the formula which is basically summation of that square term X which is the realize

value minus the expected value and the whole thing multiplies by the corresponding probability. If X is continuous non discrete then you miss rather than using the summation sign we use the concept of integration.

So this is basically the X effects or or in this case we can use D of capital F of X considering the distribution function is given while this small F of X is the PDF while capital F of X is the CDF value which we all know and must have done all these things all of you must have done these things in class either twelve or in basic engineering and probability and statistics.

So similarly for the continuous variable the variance is calculated with the inside term remains the same just you basically integrate it from minus infinity to plus infinity which are the limits for the minimum value of X and the maximum value of X which you have.

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So consider this in the risk return framework so this of the word I am using for the first time and where the return on the expected value and the risk of the variance or the standard deviation now what we mean by standard deviation again am sure all of the candidates who are taking this course all the students are well aware. Standard deviations basically mean the square root of variance or the risk.

So we are trying to measure the risk using standard deviation or variance along the X axis along the Y axis we have the return and this rather than using any dots I have just highlighted using the squares of the rectangles which are different in colored. So what we have is project A, project B, project C and project D. So what we want to find out is that what is the overall risk for these individual projects and if a company consider like L & T.

L & T is building up of bridge in west Bengal say for example building a big government office in Bombay L & T is also say for example trying to build up a high way say for example state of Karnataka. So in these cases these are all projects and if L & T as a company wants to basically find out what is the overall risk for the set of projects.

Then it will basically try to very simplistically find out what is the risk and the return framework what is the risk prospective as well as the risk perspective and they basically combine them as a as so called consolidated portfolio or a consolidated set of projects such that you can take a decision based on the overall risk and overall return for all the projects combined together which is shown here in the blue rectangle which basically the combination ABCD.

So you question would be that do we combine ABCD in the same proportions I consider L & T has limited budget so you it wants to basically find out whether it can invest in all the four projects in some proportions. So that is the proportion based on which L & T will make a decision so those proportions have to be found out using simple concept of optimization. I would not go into the details but I will just give you a flavor that how it can be done.

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before what you are trying to do you are trying to basically put them into different buckets or baskets as that the overall risk is basically minimize the maximum possible extent for understanding consider the very simple hypothetical case when there are N number of such projects N Number of assets.

So if you go if you remember the slide where the X and Y axis was there which was return and risk. So these points which are there for different projects so there are Nth number of dots which are there which basically signify n number of projects. So for each project there would be return and risk which I am trying to basically draw using the vertical one and the horizontal one vertical one basically would mean the risk this this is the standard deviation if you remember and this is basically the returned lord the unexpected value.

So the vertical would basically mean the risk in the return sorry and the horizontal would basically mean what is the value of the risk the prices of the assess of the projects are moving in such a way that they are the respective prices are all are for the time being consider there are uncorrelated. So they are correlated what are the consequence I will slowly go into that the return of his asset has an average which mean value of M for this example which is very simplistic.

What is the general formula I am going to come to that and try to explain that the variance of the risk of the project or the asset is given by sigma square that means for each of them are the same which may not be true in actual practical purposes. Similarly the return of each asset project being equally being as same as M may not be between the practical sense and also the word that they are uncorrelated exactly may not be the same in practical sense.

So what is the general sense formula I will just put it on the slide for the ease of understanding of the of the students. And finally weights if you remember I mentioned that L & T has three project Calcutta, Bombay and Karnataka that bridge, the building and the highway. So in those cases if they want to invest if they have hundred crores they want to invest some proportions. So that it is basically what is signifies the fourth point.

Here we assign the weights for each assets or project considered in the portfolio are of equal proportion. So if you have N numbers of projects obviously to mean one by n for each again it is a very simplistic assumption. So now what we have in in consideration if I remember I just mentioned that I will write the force the general formula.

So if you consider the overall risk of the set of so called portfolio of asset of portfolio of projects which you have it would mean that it is W_i , W_j signa IJ where $I = 1$ to n well $J = 1$ to n . Now this W_i and W_j are the corresponding ways which is being invested for asset I and asset J. While signa IJ is basically the covariance structure the covariance value which is existing between asset I and asset j. Now if I expand that and obviously remembering the fact that $I = 1$ to n and J is called also $1 = n$.

So they can they can be values where I and J are exactly equal so if they are equal and if you try to basically draw this concept or the formula which have written using the concept of matrix. So this matrix would be of a size n cross n YN because if I try to find out the value of the covariance of the first to the second so on and so henceforth. So they would be plotted along the top most row similarly if I have the second asset again or the project on the second row.

And similarly for the Nth one on the last row corresponding to the fact if I again delineate or draw the first asset along the first column the last asset along the last column. So if you understand this is the n cross n matrix with the principal diagonal if you consider so I will just try to utilize a different yes this highlighted one green one this yellow one which you have this is the principal diagonal where the corresponding 11 element 22 element 33 element in the case of N and element basically means the covariance of the first asset of the project to itself.

Covariance of the second asset of the project to itself till the last term which is the N position which is the covariance of the NF asset to itself which means basically the variance of the first asset variants of the second variants of the second asset in the variance of the nth asset. While of the diagonal element which you have so again i will try to change the color so these element or this element they are exactly opposite to each other if you consider the principal diagonal.

So the element which is on the second column first row and the element which is in the first column second row is basically the covariance of the first asset or the project to the second one or vice versa. So if I write it down it basically would mean σ_{12} is equal to σ_{21} = 2 row which is the correlation coefficient multiplied by σ_1 multiplied by σ_2 .

So I will use the suffix one two which means the correlation coefficient existing between first and the second now this σ_1 and σ_2 are the corresponding standard deviation of asset one or project one and standard deviation for asset to a project to which is basically the square root of σ_1^2 , square root of σ_2^2 while this row is basically correlation coefficient. So if you consider the principal diagonal technically it would be the first one element or say for example the NN element it is σ_{11} multiplied by σ_1 multiplied by σ_2 .

So row one one basically means the correlation of the first two itself which is one similarly the correlation of any asset or project with itself would always be one while this one sorry this is one this would mean that this is the standard deviation the first multiplied by that is the standard deviation the first X the first X the standard deviation of the first again which is again if you consider it comes back to the variance.

So all the principle diagonal or the variances of diagonal variance element of the covariance's so off diagonal element on the top half is exactly the replica of the second half because the covariance's of 1 to 2 or 2 to 1 or exactly the same. Now if you see this formula if you expand that and then consider these four points 1, 2, 3, 4 and the simplistic assumption which I mentioned in the practical sense one is not two, two is not 2, three is not two, four is not two what are those.

Uncorrelated means there is no correlation coefficient which means the correlation coefficient for the first to the second or second to the third whatever are all zero which means the of the diagonal n elements none of them exist. So only you have basically is the principle diagonal is the variance of first, second, third, fourth that means there are n such elements. Second one being

the point is that which is also simplistic the return of each asset project has an average value which is M .

Which means that if you are trying to find out the return is basically calculated by summation of WI and this σ is the standard deviation of the return. So average of return I will consider as R \bar{I} , \bar{I} means the average this R is the return which is the same concept which we just discussed in few slides back about the IRR and how the internal rate of return and the rate of return of the banks which basically pays to you if you deposit your money or for the project returns are basically R only.

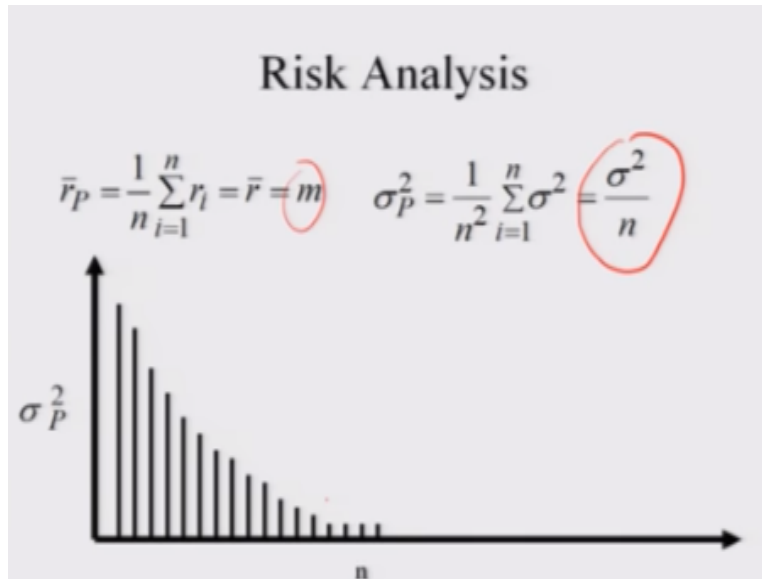
So it make sense in order to find out the average value of the return so if I want to find out the average value of the whole set of projects would basically be the corresponding rates multiplied by the average return for each project or asset which you have and then sum it up. Because if there are n you have to basically sum them up for all the ends.

So in this case what we are assuming for the port this is for this is for the whole portfolio. Now if you see this second point it means that for each of them are same that is the return of the each project this $R\bar{I}$ are exactly the same. So that means each of them gives you the same return of M . The variance of each asset project is σ^2 so again coming back to the first point we saw that we had only the principal diagonal of the diagonal elements are 0.

So again to take it one level further we consider each element in the principle diagonal are exactly the same. So we do not σ^1 σ^2 σ^3 those are not there only thing is that each variance is σ^2 Y by itself or for the variance for the first project or the first asset second project second asset third project third asset is same. Finally if you go to the fourth point which is weights of each assets are exactly the same it means that WY which you have.

As if you remember if you have such N projects we are considering that each of them investment is being done in any asset or the project is exactly equal proportion. So if you add up one by N n number of times is exactly becomes one which should be the case.

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If you draw this risk return framework for these four simplistic assumptions for the formulas you will see the return of the project is M because you are adding up our eyes n number of times and all of them are the same value. So basically they cancel out and this is m and the variance if you want remember you want to find out the using the concept of the of probabilities concept of statistics the overall portfolio for the set of portfolio for the project basically becomes sigma square by n .

Because the square term comes hence it becomes n which means that if you draw on the risk return framework the value of the return. So here I am not drawing the return to the risk I am trying to basically find out the risk with respect to the number of such assets which I have or number of projects which I have in my portfolio. So overall risk of the total set is given on the y axis number of such investments of other projects is given on the x axis.

So if you consider as N increases this will decrease as shown in this diagonal so this is just an hypothetical example two under to make you understand that as the value of n increases for such a portfolio theoretically it is possible to make the overall risk is zero.

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Introduction to Portfolio approaches

In order for diversification to be most effective we must assume:

- Projects in the same industry tend to be correlated.
- Projects in different industries are not correlated.

So introduction of portfolio approaches are there so in order to diversification to be most effective we have we should have projects in the same industry tend to be generally correlated. So if there is there a there is an two industry based on say for example coffee and tea. So consider the our products for which for which if the coffee price increases we intend to assume that a tea price should decrease because the consumption basically for tea then increases for coffee decreases so again vice versa.

So in these case price movement are not positive correlated so the first point basically means that in the same industry like if they are the car and the steel industry where car industry uses a lot of steel. So if they are in the same sector or say for example petroleum products and car industry and car industry. So if they are the same type of sector so tend to basically move into same direction projects are assets in different industry are not correlated.

So they move in opposite directions and they can be some cases where the projects or the assets movements are totally irrelevant to each other that means their overall dependence structure is zero that means the correlation coefficient is zero.

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CAPM

In order to do away with the enormous amount of calculation which is required when you are combining a large number of financial assets, we use a proxy which subsumes that the expected value of any particular financial asset can be replicated by the market. By market we mean the market index.

So now we will consider the asset pricing model even though it is basically applicable for finance first we will try to utilize it here in order to do away with enormous amount of calculation which is required when you are combining a large number of financial assets or project we use a proxy which subsumes or assumes that the expected value of any particular financial asset can be replicated by the market that means what we have there is a market or so called theoretical set of portfolio of the of the projects.

And each of the project which is being undertaken by you as a company or say for the examples from second party third party parties are basically different players who are there in the market. So all of the projects individually or as a portfolio for them or as an individual for you are a portfolio for your case everything is related to the market. So market is basically a theoretical a hypothetical concept which basically considers the portfolio of all the assets or all the projects which can be taken up at any point of time.

This is basically the market like in in consider for the finance you have the BSE sensex or the NSE. So which means that it basically gives you a picture how the overall financial market is doing and all the assets which are there the individual stocks like Tata steels, Tata motors Reliance, Bajaj, Godrej whatever they are they are individual assets or projects and the prices of all of them basically somehow related to the overall market.

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CAPM

$\sigma_c = \beta_i (\sigma_m)$
 $y = ma + c$

Considering that there is only one variable which regulates the price movement of any particular asset we consider that risk to be denoted by β_i ($i = 1, 2, \dots, n$) for any particular i^{th} asset, such that.

$r_i - r_f = \beta_i (r_m - r_f) + \varepsilon_i$
 $\varepsilon_i \sim N(0, 1)$

Assuming normal distribution for the returns and also considering $\varepsilon_i \sim N(0, \sigma^2_{\varepsilon_i})$, we have, after taking expectation

$\bar{r}_i - r_f = \beta_i (\bar{r}_m - r_f)$

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So this is the concept of CAPM in general considering that there are only one variable which regulates the price movement. So this one variable is basically what I meant was based the market ah the risk can be denoted by beta suffix sign. So you are seeing beta for the first time till now we have been discussing that risk is basically sigma or variance sigma square and so on and so henceforth.

But beta is basically the overall risk when you consider the capital asset pricing model to be true such that the relationship between the price movement of the i^{th} set of the i^{th} project with the market is given by the this simple formula which means the difference between R_i . R_i is basically return from the eye at a set of the projects minus R_f . So R_f basically you are seeing the first time is basically the risk free interest rates.

So if you are considering say for example the project so you have hundred rupees to invest in a project and you want to basically compare that with the investment if you do if you keep that amount of money in the bank. So what you will be there you will be basically understand whether the project would give you positive return or negative return positive in what sense. If the bank interest rate fixed interest rate or the risk free interest rate is arbitrary consider ten percent.

So you will try to basically understand whether the positive will give us a return greater than ten percent. If it is not obviously then the returns for the project would be with comparison to the risk free interest rate would be negative. So what we want to find is that that as we are comparing the return of the project or the asset with the market both the market as well as for your decision for the asset of the project would basically be pegged against the risk free interest rate R_{FF} .

So in the left hand side you have the difference between your project returns and the risk free interest rate and on the right hand side what you have is basically the risk factor for your corresponding project on asset with respect to the market multiplied by the overall excess return on extra return which the market would basically have over the risk free interest rate which is R_F .

And this last term basically the white noise this is basically the ϵ so generally if you consider the very simple equation which you have done in the mathematics $Y = MX + C$ this bears some resemblance to that the only fact is that R_I and R_M are stochastic hence ϵ which is the white noise is also stochastic. So if you take the expected value on both the side R_F being risk free interest rate the value remains the same.

So what is interesting to be noted is that this ϵ which is generally we consider has a normal distribution with zero mean and one standard deviation. So if you take the expected value on the left hand side you will basically have $\bar{R}_I - R_F$ because R_F is constant and on the right hand side β will be considered again outside the expected value you will have $\bar{R}_M - R_F$.

\bar{R}_M is basically the expected value and the last term expected value is basically zero as per the assumption. So if we consider this and this it basically means that there is a straight line this is the value of the intercept see this is the tan of the this angle the tan of the angle is M and based on that you try to basically analyze the problem.

So assuming normal distribution for the returns and also considering this so here either you consider one or sigma square whatever it is the main factor is X should be zero. We have the final equation as this which I mentioned has some resemblance with the concept of $Y = MX + C$.

So with that I will end on this lecture and in the lecture we will try to cover the first set of problems from the decision trees and the concept of different of a financial ratios not ratios this interest rate all this things how they are used in order to basically understand the concept of project and later on after those two small problems in the area of decision trees and the financial things we will try to cover the AHP and ANP in slide details with the simple problem thank you very much have a nice day