Project Management

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Module No# 08
Lecture No # 40
Q-GERT: Queue Graphical Evaluation and Review Techniques and Theory of Constraint

Welcome back my dear students. And this is the last lecture for this project management course. We have tried to cover a lot of topics, but I will try to wrap up GERT. I know it is a little bit theoretical but we will try to basically give it an example more as a trying to understand problem rather than trying to solve them. But generally the concept which we have used I am repeating whatever I have said in the last class. And I may have been repeating many of the concepts time and again.

But trust me the concept which we dating in PERT, the CPM, the crashing, the leveling concepts, the decision trees, all can be extended for the GERT. Only that the, logic concept is coming in a very big way such that you can consider the concept stochastic probability the probability that the path would be taken and the looping concept. So if you remember we, in the last slide, we discussed about the w-function and whether in the series and the parallel concept how they can be either added or multiplied depending on what concept, which you are trying to follow.

And w-function was basically probability into e to the power the functional value of time. It is basically something to do with the generating function. If some people are aware of the concept of the generating function, moment generating, characteristic functions, in from the points of view of probability. That is not very important but I just thought I will mention that.
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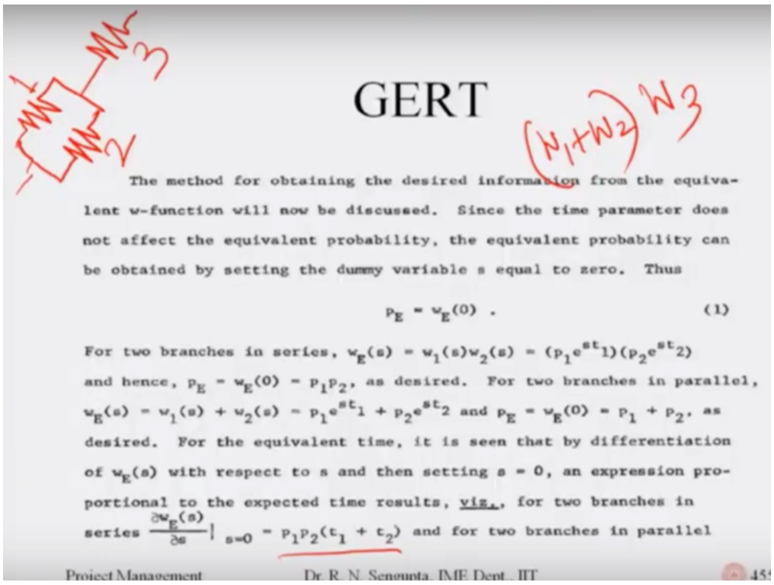

So the method for obtaining the desired for information from equivalent w-function will now be discussed. Since the time parameter does not affect the equivalent probability, the equivalent probabilities can be obtained by setting the dummy variable is equal to zero. So the variable based on which we are trying to calculate that S value, that can be set to zero. And you can differentiate that, put it to zero that actual differentiation function, and then find out the overall characteristics of that function. Not the characteristic function, but the characteristics of that function.

So the for two branches which are series, I told you, what do you do? You just multiply them. Now if the question is if they are in parallel, what do you do? You just add them. If the question is now, third question from your point is, if 3 of them are series, what do I do? Multiply w1, w2, w3.

If they are in parallel, what do I do? All of them are parallel, just add them up w1, w2, w3. Now if your fifth question is that if two are parallel and both of them are in is series with the third one. So for that let me go back to the so if so consider this I will consider as 1, this is 2 and this is 3.

So the logical answer is, if you ask a question that how do I find out the w equivalent function, it is very easy. For that first two, 1 and 2, they are in parallel so if they are in parallel, what would you do? You will add them. So it will be basically w1 plus w2. Find the equivalent form.

And if it is 3, so what you will do, you will multiply w3. So whatever the complicated structure is you just follow the concept of for the series one you multiply, for the parallel one you sum. So again continuing the discussion so I just went 1 step forward in order to make you understand so but just again tracing back the discussion.

For two branches in parallel, the equivalent w-function would be the sum of them and if want to find out that add the case when the dummy variable is zero. So obviously, e to the power whatever value which you have it that is multiplied by ts value which is zero so obviously it will be 1. That is e to the power zero is 1 and based on that when we find out the w-function it will be p1 and p2. So for the series one it would be multiplication and for the parallel one it would be sum.

For the equivalent time, it is seen that by differentiating that equivalent function w which you find out for whatever complicated network which you have. If you differentiate with respect to zero, an some expression will come. For two branches when you differentiate, the expression which comes is p1, p2 and then bracket t1 plus t2. So obviously let me point it out, when you differentiate, the exact value you want to find out would be the case when the equivalent dummy variable is put to zero.

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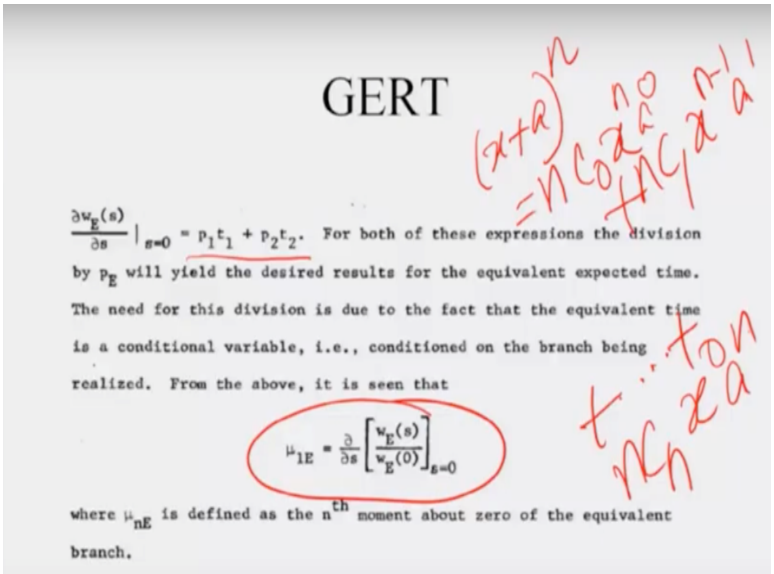

And for the two branches in parallel, the function is P1 into P2 plus P2 into p1. For both of these expressions the division by PA will yield the desired results for the equivalent expected time. The need for this division is due to the fact that the equivalent time is a conditional variable. That is, conditioned on the branch being realized. If it is not realized obviously that value would not be coming.

From the above it is seen that the equivalent expected value would be for the case, I mean you will want to find out the average value. It will be the differentiation, partial differentiation obviously for the equivalent value we. Where the dummy variable is not zero and the other value being the we when the dummy variable is zero. Where mu suffix ne is the defined as the nth moment so it can be the first moment, second moment, third moment and the whatever it is.

So if you remember very simple class eleven and twelve algebra, or simple arithmetic of the series expansion what we had? We had x plus a to the power x plus a to the power n, was basically given by nc zero x to the power. Let me write it down its equivalent, na to the power zero plus nc1, x to the power n minus 1, a to the power 1. Plus dot, dot, dot. Plus the last one would be ncn, x to the power zero, a to the power n.

And that can be done for whatever series expansion we you have, depending on what is the domain and the range of x, of a and whether n is an integer, whatever it is. And you can basically have different type of expressions. Now depending and this why this is important is that, when you come to the moment generating function and the characteristic function. Basically they will give you the properties of a probability distribution.

So in the similar way when I take the first moment, second moment, third moment and so on and so forth for the nth moment. And when we expand or put the value for the dummy variable s equal to zero, they would give me the characteristics of the function based on which I want to study what is the average equivalent time and the corresponding probabilities for the network, GERT network.

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GERT

Further exploration shows that

$$\mu_{nE} = \frac{\partial^n}{\partial s^n}\left[\frac{w_E(s)}{w_E(0)}\right]_{s=0} \qquad (2)$$

and, hence, $\frac{w_E(s)}{w_E(0)} = M_E(s)$ is the M.G.F.* of the equivalent time, t_E. It is convenient at this time to define the n^{th} cumulant, K_{nE} (for $n \le 3$ the cumulants yield the moments about the mean directly), which is given by

Furthermore, when we put find out the nth moment, so hence this is the moment generating function of the equivalent time and that time would be given by t suffix e. E is basically the equivalent time in the similar way when you have the expected time in the PERT network. It is equivalent at this time to define the nth cumulative, which is kne for any values n guess less than three and which is given by this formula.

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GERT

$$K_{nE} = \frac{\partial^n}{\partial s^n}\left[\ln M_E(s)\right]_{s=0} . \qquad (3)$$

Thus, the second moment about the mean, the variance, can be obtained directly as K_{2E}. Equations 1, 2, and 3 hold for all branches with the subscript E replaced by the subscript of the branch under consideration.

The w-function was developed based only on the series and parallel basic networks. However, it can be shown that any network is a combina-

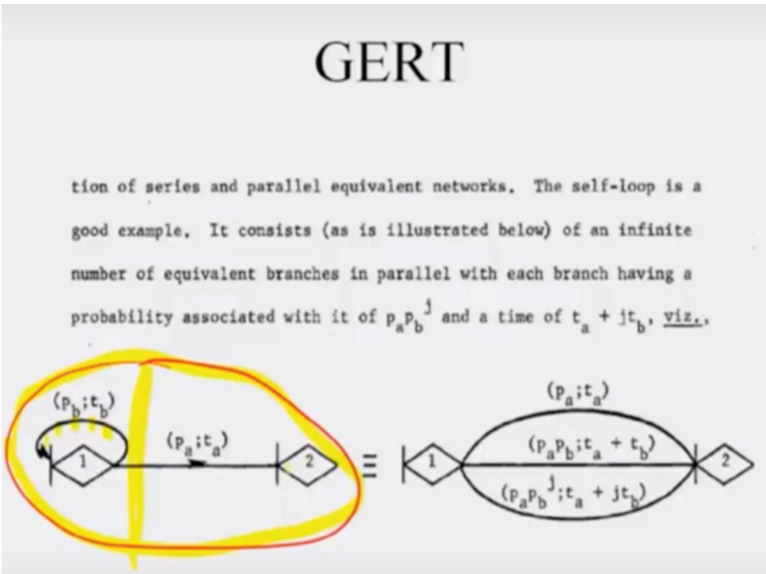
So what we need to find do is basically try to find out the log of this function. Now this log of this function would have some implication. I know it is a little bit difficult for people to understand. But there is a concept of maximum likelihood estimation concept which is very heavily used in statistics. So we are trying to basically bring some resemblance with the concept as you are trying to find out the cumulative values with respect to the maximum likelihood function.

So maximum likelihood function, very simply is that, you find out the log of the probability function. Then differentiate that log of that function, with respect to the parameters which you want to find out. And then put that value is equal to zero and find out the estimated values of these parameters. So, if there are two parameters you will do, partial differentiation twice considering.

And for the first case you will different partially differentiate the actual log likelihood function with the first function considering this is say for example alpha, considering beta is constant. In the second case we will again differentiate the log likelihood function with respect to beta, considering alpha is constant. And find out this alpha and beta values, which would be estimated values.

So in the similar way, we can do using that same concept thus the second moment about the mean, the variance. So second moment is basically to do with the variance the third moment is to do with the skewness fourth moment is to do with the kurtosis. So I am trying to bring a simile to the concept of probability.

So using these equations, which hold for all different type of branches, the values of the of subscript e can maybe replace by the subscript of the branch under the consideration and you can expand this. The w-function was developed based only on the series and the parallel network. For the loop hence however it can be shown that any network is a combination of series and parallel equivalent. So it can be extended to any case.



The self-loop is a good example so if you consider the self-loop, so if you have the self-loop here. Let us consider the here, which I am now highlighting let me try to basically use a different color, so it is yellow now. So the color has changed now if you consider this self-loop, the first part, onto the left where I am highlighting, there is a loop. See this is the loop which I have and the right-hand side there is no loop.

Now when I do this self-looping, it basically means, the loop is basically a simple concept of a parallel circuit. So if it is a parallel circuit, obviously you will have the concept of the probabilities of the time. Use this probabilities and the time with respect to the w-function. For the series, what we did? We basically multiplied the w-function. For the parallel one, we added up the w-function, differentiated with respect to s, that is the dummy variable put it to zero, and then found out the values.

So you very simple proceed in this line find out the equivalent values of the n and nth moment for a zero very simple loop looping circuit. And when you have the looping circuit, you can basically find out the equivalent probabilities and the time, as shown here. So whatever complication which you have, you can simply convert into a series and a parallel, depending on the what the concepts of the GERT are, or the GERT is and then you can convert them accordingly.

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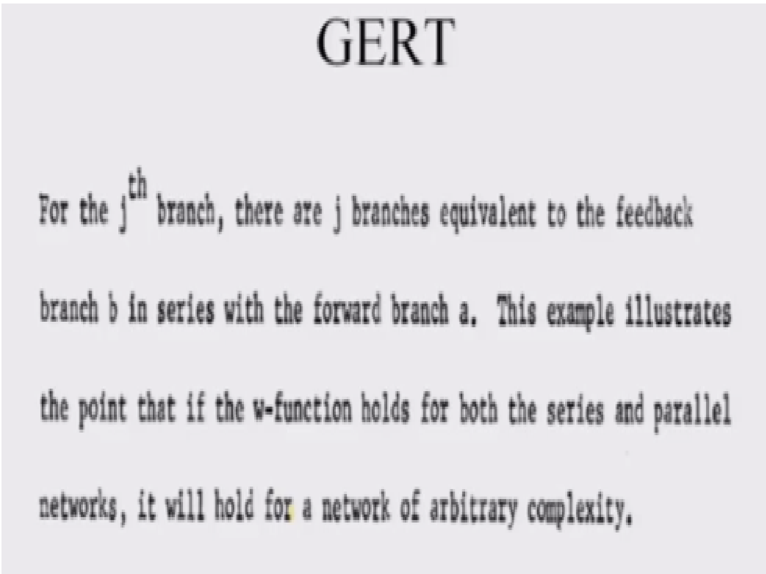

For the jth branch, there are j branches equivalent to the feet branch in the in series. This example illustrates the concept that if the w-function holds for both the series and the parallel, it will hold for the whole network. And the overall network probabilities and the time can be found out for the whole network as such. Corresponding to the fact that each of the arcs have their equivalent w-function. Which have been calculated using the concepts of series, which means being multiplication, parallel being addition.

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GERT

Network type	Equivalent function, $w_E(s)$	Equivalent M.G.F. $M_E(s)$
(a) Series	$p_a p_b M_a(s) M_b(s)$	$M_a(s) M_b(s)$
(b) Parallel	$p_a M_a(s) + p_b M_b(s)$	$\left[\frac{1}{p_a + p_b}\right][p_a M_a(s) + p_b M_b(s)]$
(c) Self-loop	$p_a M_a(s)[1 - p_b M_b(s)]^{-1}$	$(1 - p_b) M_a(s)[1 - p_b M_b(s)]^{-1}$

Fig. 3--Reduction of networks with stochastic time intervals

So if you have the chart, so in the first column which you have the network time the they are series, parallel and loops. Loops can be of any complication parallel can be of any complication. Series can be any complication. But given the concept, the formulas which you have is for only a and b, it can be extended to any level, as I mentioned that there in in three or four slides before. The equivalent w-functions are given for series, parallel and the loop. And the equivalent moment generating functions are given.

So basically using this concept if there are say for example one series and one parallel. You will basically take a, b combine them in any concept which is there. If so say for example, there are two parallel so you will if there are put two parallel, you will first use the concept which is given in the second row, which is b element here. Find out the equivalent w-function find out the equivalent moment generating function. Then see if there are two series, then you use the concept which is given in the first row which is with the bullet point a.

Find out the equivalent moment generating function. Find out the equivalent w-function. Then check what the relationship of the series and the parallel is. If both of them are a parallel, then again you use the parallel concept if there is a looping, use the looping concept if they are in series, use the series circuit and proceed accordingly.

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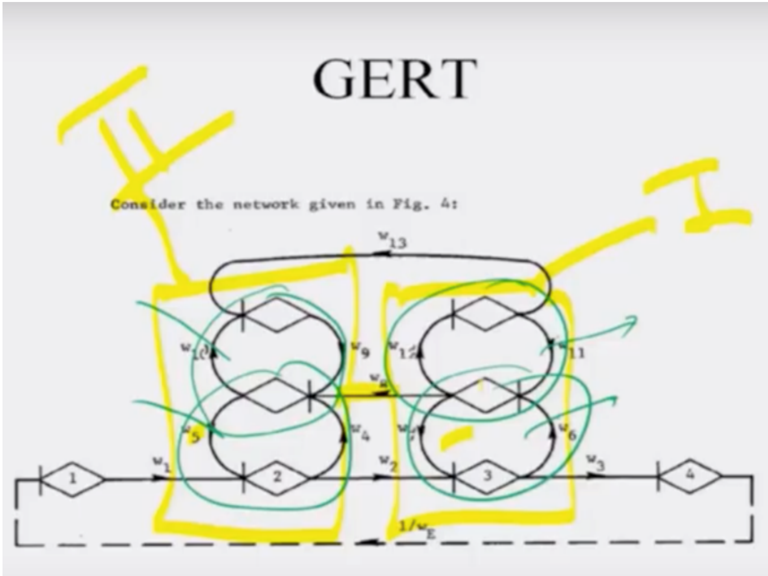

So consider the network which is given here so basically you have the loopings which is happening so let me show you the loopings so there is a loop here. So there is a loop here There is a loop here there is a loop here so all of them would basically be replaced by the looping, looping concept. Once they are done, these to and this one. These two means, this which is one, and which is two they would be combined considering there is series concept.

So this is the series which I bring so once you find out the looping one, combine looping one with one wi with five. So I am basically meaning the one here and the five here combine with the second and the third and proceed accordingly. So obviously, all other circuits would be termed considering the looping, parallel and the series.

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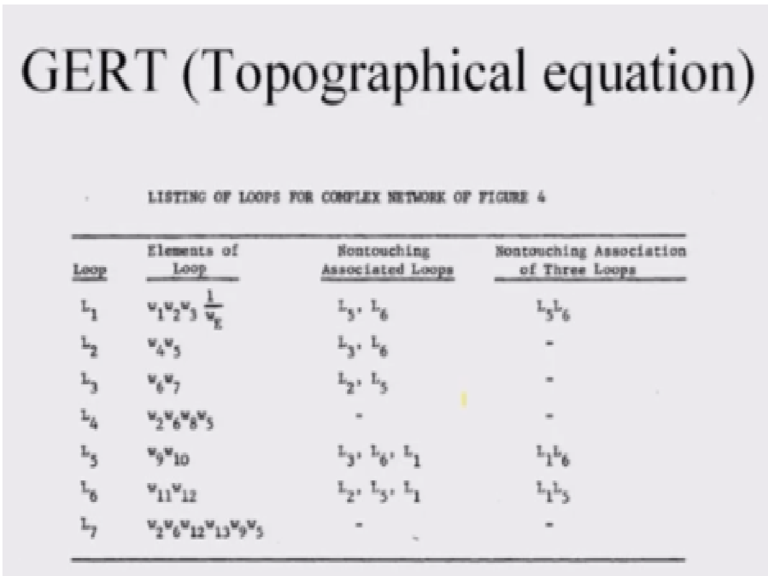

GERT (Topographical equation)

LISTING OF LOOPS FOR COMPLEX NETWORK OF FIGURE 4

Loop	Elements of Loop	Nontouching Associated Loops	Nontouching Association of Three Loops
L_1	$w_1 w_2 w_3 \frac{1}{w_E}$	L_5, L_6	$L_5 L_6$
L_2	$w_4 w_5$	L_3, L_6	-
L_3	$w_6 w_7$	L_2, L_5	-
L_4	$w_2 w_6 w_8 w_5$	-	-
L_5	$w_9 w_{10}$	L_3, L_6, L_1	$L_1 L_6$
L_6	$w_{11} w_{12}$	L_2, L_5, L_1	$L_1 L_5$
L_7	$w_2 w_6 w_{12} w_{13} w_9 w_5$	-	-

So listing of the loopings for the complex network which was just discussed, so there would be loops as I mentioned. If you go back to the slide you will understand loops l1, l2, l3, l4. l5. l6, l7. The elements would loop considering the w-functions would be the first one is w1 into w2 onto w3 divided by 1 by we which is the equivalent value of w, how you found out?

And the last value would be, depending on the w values would be w2, ws. In my all these multiplications, w2 into w6 into w12 into w13 into w9 into w5. So if you can find out those values, they can be calculated and the non-touching associated loop and the non-touching associated three loops can be correspondingly found out. So what we have done we have complicated note network loops we have basically divided them into parts and found out the equivalence of the loops which are numbered from L1 to L7.

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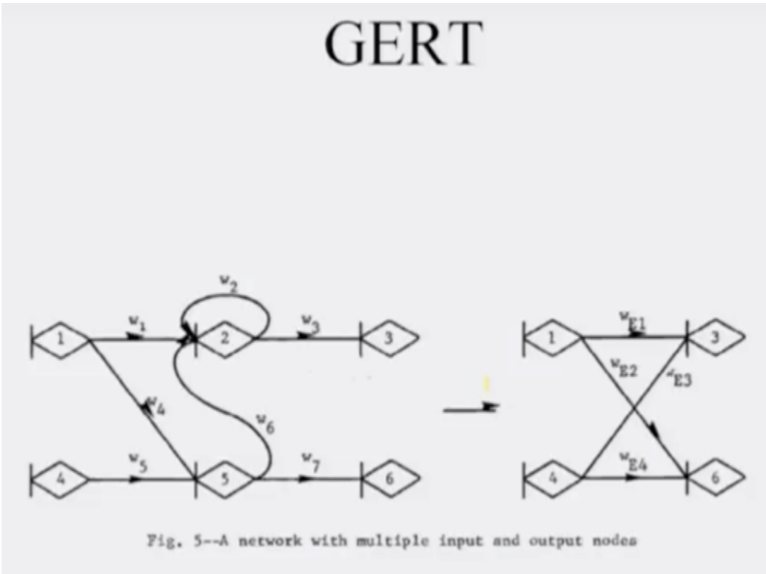

So network with multiple inputs and multiple outputs, so I have just given the diagram such that they give you a very simple concept that how they are equivalent? So the equivalent concept can be brought it into the picture considering the looping for the series, the parallel and the concept being series and parallel whatever combinations which you have.

Remembering that you have the probability you have the time, you have the w-functions. Combine the w-functions in whatever concepts which you have. Then find out the differentiation put the dummy variable zero and find out the equivalent values and then find out the probabilities and the time.

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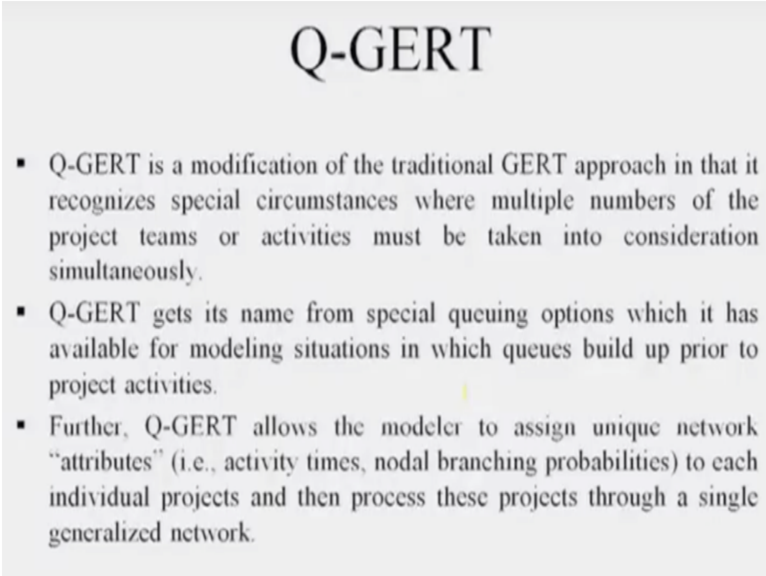

So now I will just go into the briefly about Q-GERT and few concept of theory of constraints. So Q-GERT, which is queuing GERT is the modification the traditional and GERT approach. In that it recognizes special circumstances where multiple numbers of the project teams or activities must be taken into consideration in the same time. So in the GERT problem you considered, say for example only one project and the activities had loops. Activities had probabilistic time activities had probably t that a certain loop would be taken.

But to basically make things complicated, now we are trying the queuing GERT. Where the combinations of the projects or the activities are being done in such a way they would be intertwined with each other. Reason being that, in the concept of GERT, we had looping with respect to the jobs which are there in the project itself. But it may be possible that the jobs are at different projects but they are inter-related.

Consider this you have a very sophisticated CNC machine and that sophisticated CNC is very costly you cannot by a second one but that has to be utilized for both two different projects. Consider project one, you are making a very sophisticated machine, say for example for TATA motors and the next one your motor the another machine you are trying to make, is say for example for a company Reliance.

But that CNC machine which you have has to be utilized for both these projects, Reliance and TATA motors. But they have to be done in such a way that apart from the looping which is there both for the Reliance and the TATA motors, there would be intertwined with each other such that the concept of the queuing GERT should be utilized. But the basic concepts of probability time, probability activity happening, the looping would always be there in the Q-GERT.

Q-GERT gets its name from special queuing options which is just available for modeling situation in which queues build up prior to bringing the activities. So queues are can be a part and parcel of the Q-GERT process, which is not there in the GERT. So which means that the jobs are being take taken up as and when they come. Such that the delays of the activities or the time differences which are there are already specified but in queuing we will consider that the arrival and departures are probabilistic.

So it may be possible some of the jobs have already piled up such that they would have a cascading effect on the succession jobs. Which would basically be there in the whole project or in the conglobation of the different projects which you have. Further, Q-GERT allows the modeler to assign unique network attributes such like activity times was 1 important factor in GERT. Probability was there other being nodal branching probabilities.

So if you reach a node, so there would be branching probabilities for going from node one to node two. Which would be there in GERT, but it would be more complicated in Q-GERT considering the concept of queuing is also coming into the picture. So this means that attributes, again I am reading it activities, activity times, nodal branching probabilities to each individual projects. And then process these projects through a single generalized network. Such that you are trying to basically bring an equivalent form of all the complicated networks which are there in front of you.

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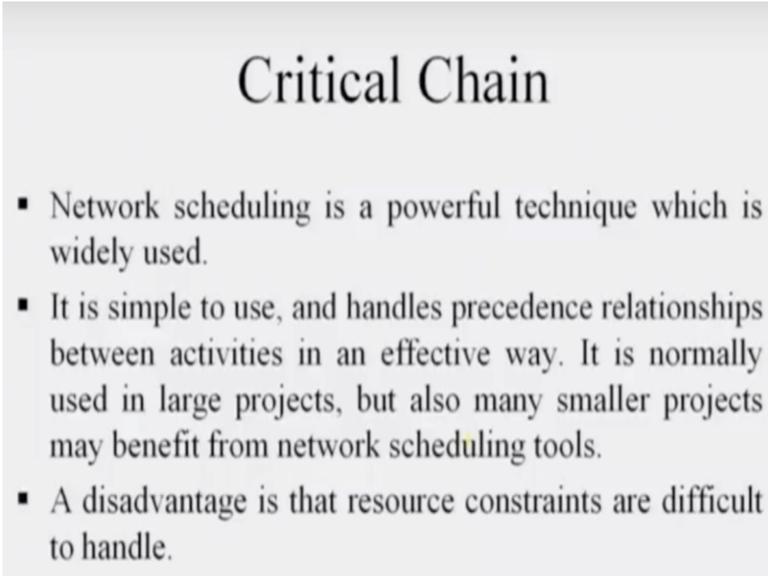

So we will just very briefly go through the concept of theory of constraints and critical chain. So networking schedule is a powerful technique which is widely used we have seen that. It is simply handles precedence relationship in a very nice manner as we saw using the GERT, the Gant chart, the PERT, CPM and so forth on the GERT and just briefly in a few minutes I mentioned about the Q-GERT. We because trying to consider that would become a subject by itself.

So it is normally used in large projects, but also many smaller projects may benefit from network scheduling problem. A disadvantage is that resource constraints are difficult to handle if you remember that I did mention time and again that initially for PERT and CPM time is the concept. Then we saw that if resources are unlimited, then well and good. If they are limited, then obviously the problem would come up that how you will try to handle the concept of resource constraint in the PERT and CPM and then it can be extended to the GERT and the Q-GERT GERT also.

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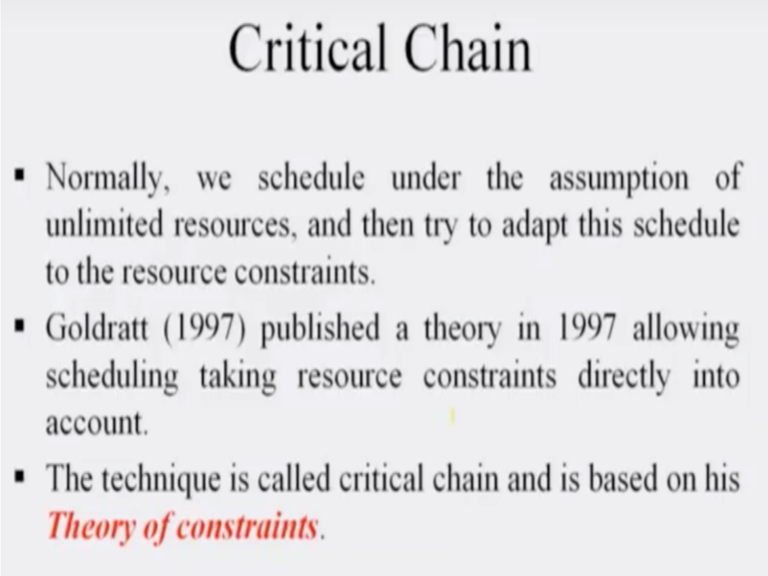

So Goldratt in nineteen ninety-seven, came with the concept of critical chain. So normally we schedule under the assumption that unlimited resources and then try to adapt the schedule to the resource constraints. So initially we considered resources are absolutely not a problem. Then we scheduled it and then tried to tackle the problem considering and each and every level there are resource constraints.

But Goldratt in nineteen ninety-seven said it was false, Goldratt published a theory in nineteen ninety-seven allowing scheduling taking resource constraints directly into account such that they would be made. This technique is called the concept of critical chain and is based on the theory of constraints the work which is a part and parcel of operation research and operations management.

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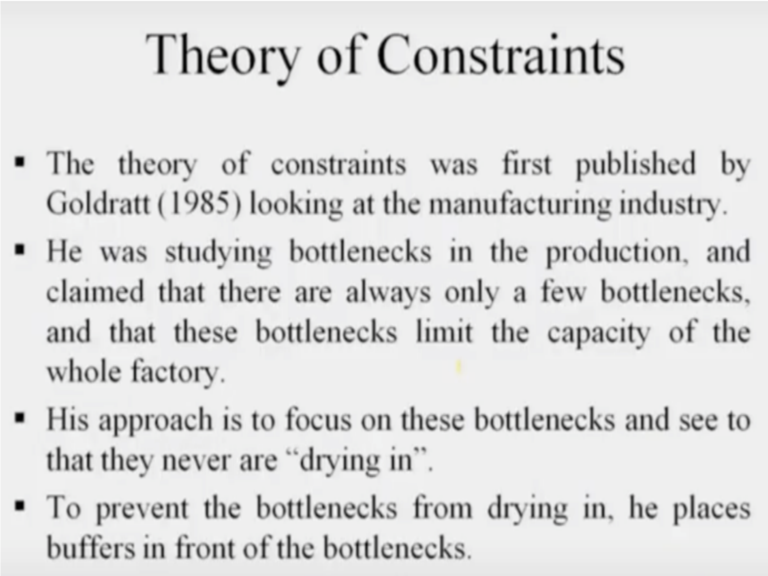

The theory of constraints was first published by Goldratt in nineteen eighty-five and basically became into the force in the nineteen nineties. And it was quiet useful and helpful in trying to solve many of the complex problems. But the problem was that, in general, the conceptual framework of theory of constraints, critical chain were excellent but trying to basically bring into the practical sense, it was a little bit more complex but it was very difficult for people to understand.

So he was studying bottlenecks in the production and claimed that there is always only a few bottlenecks. And these bottlenecks limit the total capacity of the overall network. So say for example, you are trying to do, whatever the sequence of the job is, whether there are queues, or whether they are not queues, whether time delays are there, leave that. Very simply consider, there are three main jobs.

One is the working on the late machine. One is the work on the grinding machine. And one is say for example working on the shaper. Consider very simple machines in the shop floor. Now consider due to so some reason that the time taken in these three machines are t1, t2, t3. All of them are probabilities, probabilistic, irrespective of the fact that the jobs are be are being done right on time, but consider they are probabilistic.

To make life things simple, even though probability is a concept for the practical sense, we make the time as equivalent deterministic time. But now when we try to basically implement that in a work schedule for the project, it may be found out say for example that the grinding work which is a precision work takes a lot of time. So that would basically be a bottleneck because all the scheduling things which you are going to do for the subsequent work and all the inventory control which you are going to do.

Inventory what I am trying to bring it for the first time for this whole project, because resources are an issue. And inventory being very high very low would have a consequence on whether resources are constraint or not. Consider the grinding one is one of the bottlenecks, because it takes time. So we will be more interested in trying to basically schedule the work for the grinding machine in such a way that the work can be done to the maximum possible extent.

Such that the resource allocation for all the three machines in our case we are considering only three machines can be done to the maximum possible extent, with minimum resources. Such that the resource constraints along with the bottlenecks are considered at one go, point one. Point number two you can think, that there would be such resource constraints at every level. There would be different types of resources there would be different type of bottlenecks.

So trying to consider the bottle necks, or trying to basically pinpoint the main bottlenecks would be the best possible way how you tackle it. His approach is to focus on these bottlenecks and see to that they are never drying up that means bottlenecks do get the jobs each and every time. To prevent the bottlenecks from drying, he places buffers in the front of the bottlenecks and tried to basically do the work such the buffers are resource constraints in some way. Such that the addition of the resources on other work is basically planned accordingly.

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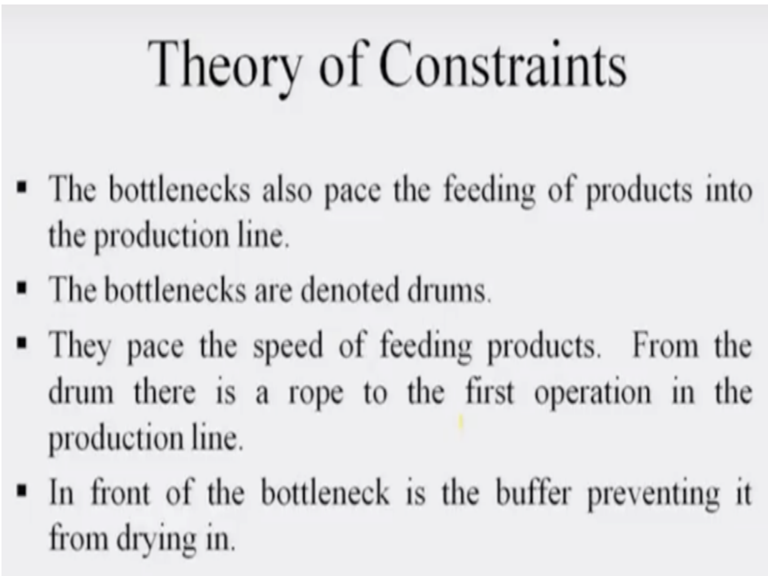

The bottlenecks also pace the feeding of the products. The bottlenecks are denoted by drums and based on that you proceed. They pace the speed of feeding products. From the drum, there is a rope to the first operation in the production line. So that rope would basically mean that what is the linkage which is happening between the bottlenecks and the resource which are put there. In front of the bottlenecks is the buffer preventing it from drying up. So technically it means that the rope is some sort of linkage which is happening between the buffers such that you can take corrective actions for the bottlenecks.

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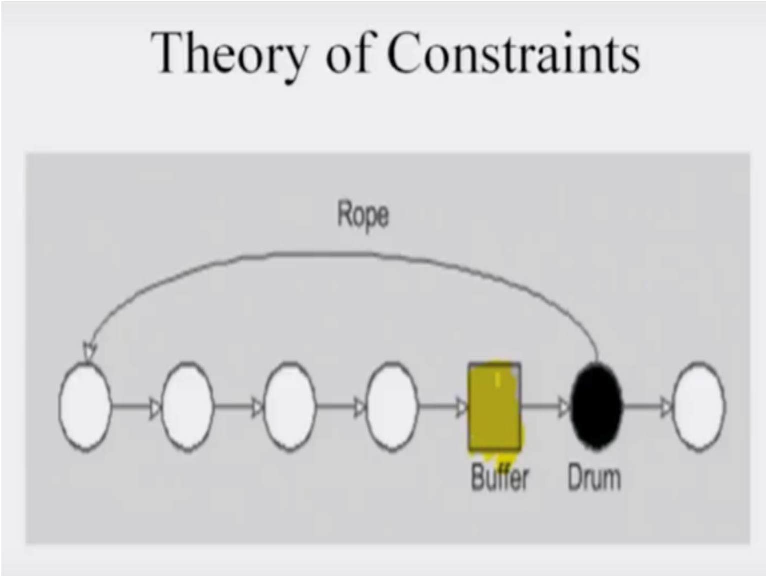

So this is what I mean so there is rope from the drum. And there is this light colored one is basically where I am putting the color yellow is the buffer. And this buffer basically has a consequence on the drum and how the feedback happens.

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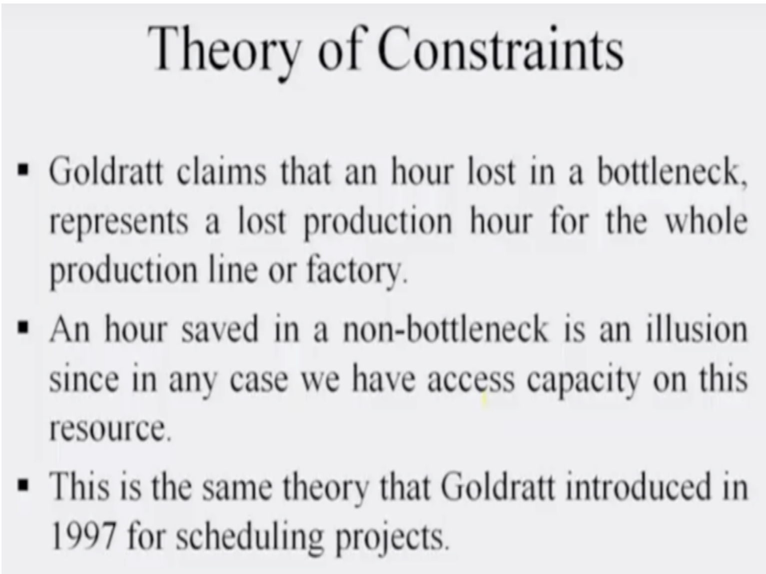

Goldratt claims that an hour lost in the bottleneck represents a lost production. An hour saved in a non-bottleneck is an illusion since in any case it will be stopped at the bottleneck. This is the same theory that Goldratt introduces in nineteen ninety-seven for the scheduling projects.

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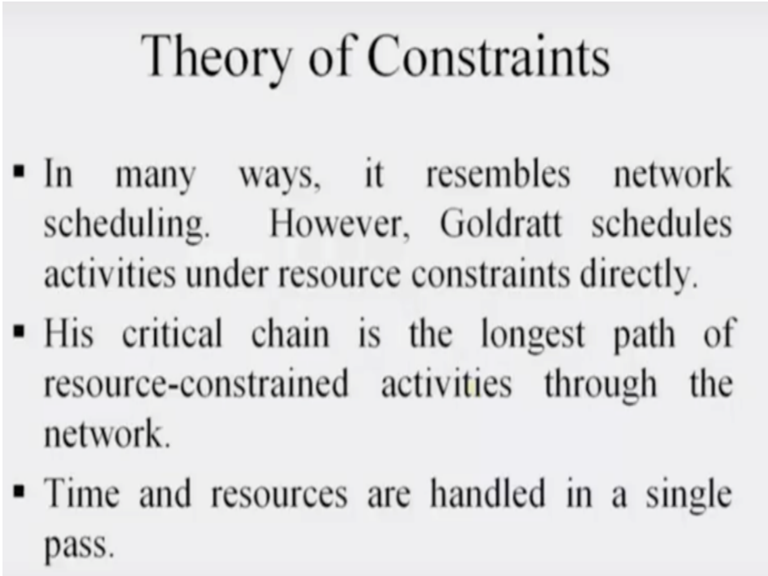

So in many ways, I will I am going a little bit fast in order to basically to inculcate you with the interest of what is theory of constraints and then basically try to wrap up this course. In many ways, it replace resembles network scheduling. His critical chain is the longest theory of resource constraint activities throughout the network, which is very widely utilized. Time and resources are handled in a single pass in any of the concept. So with this I will try to close this project management course.

So it has been a very good experience from my part and I am sure the students have learned the concept of what we mean by network flows and network diagrams. Then how the different concepts of PERT and CPM can be utilized? How the concept of say for example different variables, societal concepts, how it basically makes into the project sense? Those are considered.

How the different type of decision making tools can be utilized? How the expected value can be utilized. How the variance is an important factor it can be utilized to at least rank a project? How the different type of financial issues, the IRR, the fixed rate of interest, floating rate of interest those could be utilized? We found out how the duration concept can be utilized?

Then we go went into the concept of utility analysis. How the different concept of utility, different four different types of utility functions could be utilized? Then how the concept of normality and the concept of utility being quadratic; they make a sense one-to-one correspondence and what was the reason for that. Then we use simple concept of inequality, Chebyshev inequalities, though briefly and how can be utilized considering the central limit theorem to be true.

Then we went to the resource constraint scheduling and how jobs could be scheduled considering resource is an issue. Obviously if thinking of the fact that in PERT time was probabilistic. And then we solved a very simple problem in the resource constraint networks, PERT concept and CPM was basically a simple follow-up of that. We also considered the four different precedence concept which was there, end to start, start to end, start to start and the other fourth one.

Then we went to the area of GERT and considered how the six combinations on any of three inputs and two outputs could be accomplished the concept of series, parallel and the looping concept. How the probability of time was important. How the probability of a network or the arc being taken was important. And then so I gave you a very brief though I we did discuss in some details and I did mention that the best book would be Pritsker that how the looping concept can be considered. How the w-function can be used for any combination of series, parallel and the looping one.

And then we went to the moment generating concept and then tried to wrap up giving a very brief discussion of the theory of constraint, the concept of this critical chain which has been proposed by Goldratt. With this, I will close this course. And you as I mentioned that we will have 8 different assignments for eight different weeks. There would be an N term.

And whatever quires are there as the course progresses please get in touch with the office MOOC office NPTEL and the TAs and myself will try our level best to immediately answer to all the quires. And the readings which I have given, I will strongly suggest my dear friends my dear students to have a look at the readings. And follow these notes or the slides which are given not as the end in all concept but that is just conglobation of the different concepts which I have used.

And I am sure they will benefit in the long run trying to utilize the concept of project management. With this I will close this course. And thank you very much for all the attention. And I am sure everybody will do very well in life all the best thank you.