Project Management
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Module No # 08
Lecture No #39
Graphical Evaluation and Review Technique (GERT) V

Welcome back my dear friends students and whoever is taking this course project management and by this time iam sure you know my name by heart and obviously you know the contents of this project management the concepts much better than me considering that we have been discussing many of the issues and you would have done many of the assignments like as per the norm of after first week second week third week fourth week fifth week and all those things and to also remind.

My dear friends this is the thirty ninth class and there would be another fortieth class by which we would wrap up this project management so i will just give a brief finishing touch of this course in the fortieth session but i will continued discussing the GERT concepts which i know in the last two class are little bit theoretical but if you bare with me and read the general book which is pritzker you will understand.

What is the importance and how the and exclusive or inclusive or concepts comment to the picture and how the series parallel or distance can be consumed in general concepts where both let me remind you again the time is probilistic as well as whether path would be taken is probilistic and this is totally different from the concept of PERT where time is probilistic with a certain distribution but the path would be taken or not is not probilostic.

It is fixed and another important point which i keep repeating time and again is basically the looping concept which is there in GERT but is not there in PERT and CPM and obviously the crashing concept which we did in PERT and definitely we did not do in CPM because that would be just an extension of PERT. It can also be applied in GERT and other methods so continuing our GERT or GERT basic network analysis.

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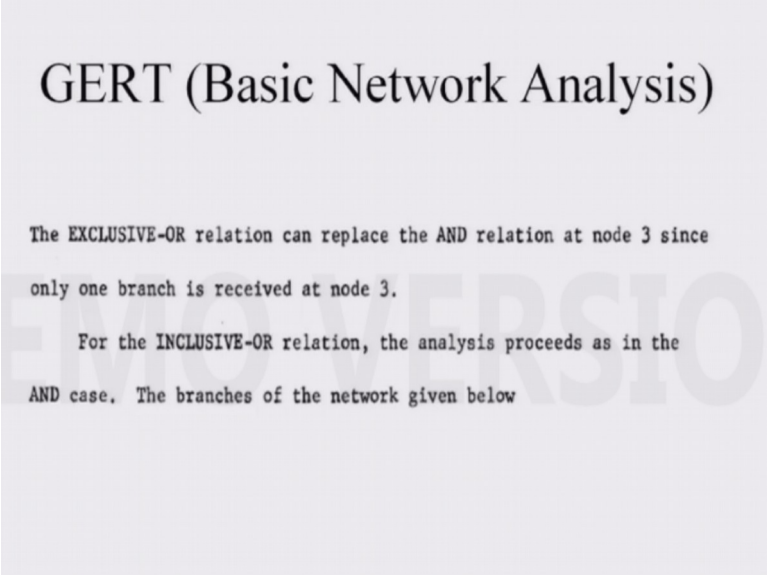

The exclusive or relationship can replace the AND relationship at note three. So if you refer to the slides for the thirtieth session you will understand. What i mean for note three since only one branch is received and at note three. So if you remember very correctly the three combinations of input two combinations output that was left to the GERT and the general detail discussion about that for the inclusive or the relationship the analysis procedure as the AND case.

The branches the network or now given in the next slide for the convenience of the students.

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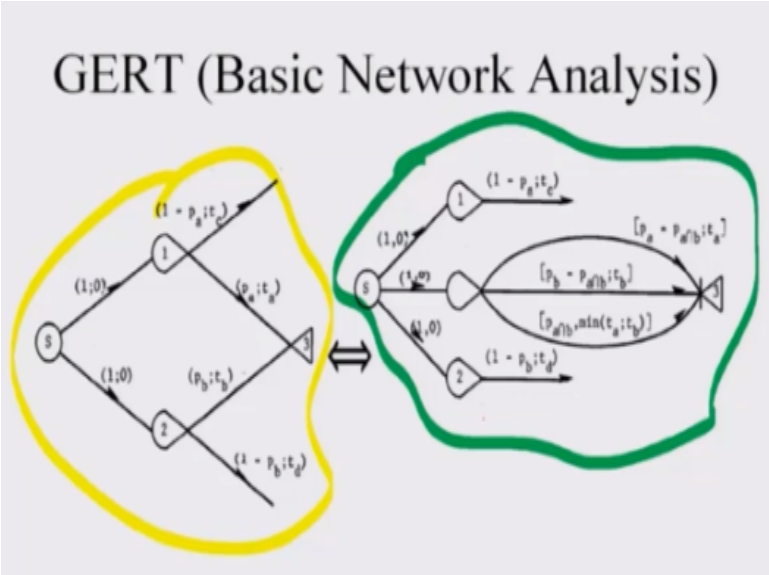

So now if you see the diagram which is there on the left. So will try to highlight it using yellow colour. So this is the one which is there on the left and if i am able to do the colour specification let me try my level best it is still yellow. So yes so this one so at last is green i

can able to convert that. So if you consider the right one which is so called circled with green outline and the left one which is in the yellow outline they are equivalent.

So let me try to basically highlight that fact so this is basically where my ma this pointed is highlighting is basically equivalent spot now if you see it is a network of only two branches it can be more than two branches also this can be if you come back to the concept of decision trees we have let us say for example for coming up with the this example i keep repeating because this should be a lot of significance of how the decision tree which we did in the initial stage of the project management or so barrier resemblance with the GERT one.

The logical and or all these concepts may not be there in the decision trees but it definitely make sense to repeat that if you consider this path which is leading from S to one S to two and correspondingly. Again going out for one and then one to three two to three two and onwards so this is circular one which iam highlighting with red colour this diagram or the node which is there which you remember there was six of those imported concepts.

So basically they are signifying that if you extend it to the right hand side just understand the node which is connecting one and going forward and who is going forward have the corresponding probabilities as given in the previous diagram which is there one which is outline in yellow. So the probabilities are one minus Pb and the time is T suffix D. So both are the values depending on what the probability situation is and what are the chances and similarly the node.

Which is leading out from one the probabilities? One minus P suffix A comma Tc which is basically the time taken and the equal lent output for this yellow one and the green one if you pay attention i will not go into the details because going into too much of details for the GERT would basically make it much more very technical and mathematical oriented but just try to follow the concept using and or concept.

You will find out is equallance between the structure would make sense that try to place a problistic network with a less intense problisting network is in some way trying to replace the decision trees with their equal lent values of the expected values that is the net present value which we have discussed.

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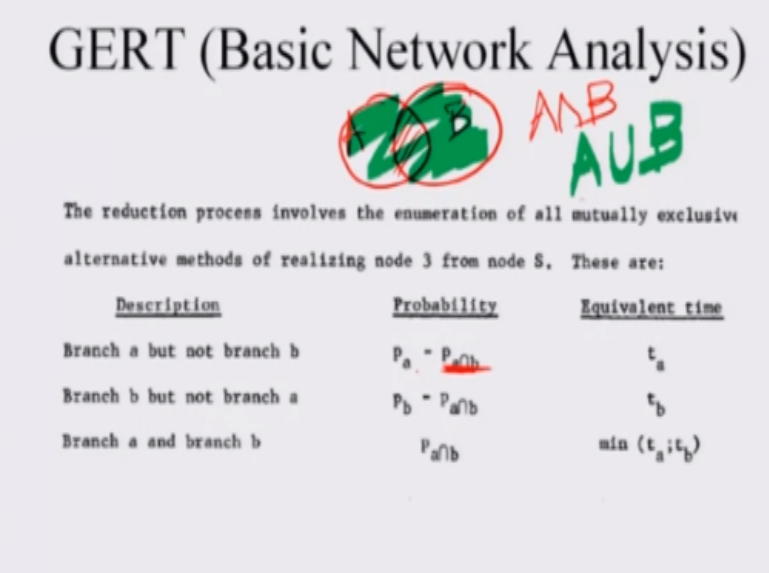

The reduction process which we try to discuss and we try to bring into any of the GERT network involves the enumeration of all the mutually exclusive so called tasks an alter methods of realising three from node S so where is three let me come back to three again i will highlight it in this left diagram the right diagram this is three and you are trying to basically realize that node three from S which is basically considered that source using any different routes which contain

So these mutually exclusive methods enumeration when you do it are like this so first i will read and explain the description which is on the first column then i will go to the probability and then the equallent time so the branch A but not branch B is taken second one is branch B but not branch A is taken and the third one is branch B is taken now when iam speaking this words have a detail look at the second column that will make sense.

Now if branch A but not branch B is taken so if you consider the actual route the probability was already P suffix A now if they are usually exclusive and we are trying to find out the concept of probability it will basically be from the point of you of the win diagram would be the intersection of A and B so i will just draw the win diagram in a very simple sense which would not at all be required.

Basically to understand project management as a concept but it will give you a clear picture that how the AND or and this exclusive or inclusive or the AND concept are been brought into the picture considering the mutual exclusive concepts of the alternative methods so if we

have a set A and set B. So this would man basically it is A. So it is A this is B this is A intersection B and obviously if i go to the highlighting one using the green one.

So if i highlight this green one which is the whole area. So this would become A U and B now if you consider this one which i have just noted down this become means A intersection means that branch A and but not branch B is taken. So obviously it will be the initial condition or probability which is been suffixed if you come to the next one but branch B but not branch A is taken which you mean again.

It will be intersection P intersection B but it now would be subtracted from the concept that branch B is taken and not branch the intersection of A and B .So in the initial case it was PA minus PA intersection B and in this case it was Pb minus P A intersection B like now i want to extend that like there are branches ABC so obviously would mean the intersection concept and the minus concept just be extended in a similar manner.

Say for example if means that the branches are due to the concept of mutual exclusive alternative method properties we may have PA plus PB minus the concept of PA intersection B intersection C i would like to basically finish this slide and go back to one of the decision trees concepts related the problems of the oil rig and drilling example similarly in the last row you have branch A and branch B.

It has to be both of them are not taken hence it is probability of A intersection B and the corresponding **co** time values if you take if it was branch A and not branch B obviously it will be T suffix A basically becomes a very simple logic. Similarly when i come to branch B and not branch A. It becomes again a simple logic that a time would be corresponding to T suffix B and the last one which you see the last row which is branch A and branch B.

Technically it means is an intersection hence it would be as per the concept for the logics of exclusive or inclusive or an AND concept it would basically be simple minimum of the two values of TA comma TB now if you remember just few seconds back i said that i will i will bring some simile or some similarity with the problem which we did under the oil drilling problem which is decision tree there you had the condition probabilities.

So condition probability that the whole well or the geological structure was wet or dry depending on that you had a huge amount of probability of finding out the oil in the similar way if you consider this concepts minimum or the intersection one it means that the condition that concept A or task A or the activity would be taken up would depend on the fact that whether B is realized or not realized.

So hence the concept of condition probability would come which would mean that let me write it down the Pa given B was already occurred now this adding for the first time but people with simple basic concepts of probability would understand that .So it would mean that P A intersection B divided by PB that means provided B has occurred what is probability that A would also occur. So you can extended it .Say for example probability of A given B and C has occurred and you can proceed accordingly to whatever level you want to make.

So which means that the number of so called branches number of sub edges and the nodes would be much more on number in the GERT process.

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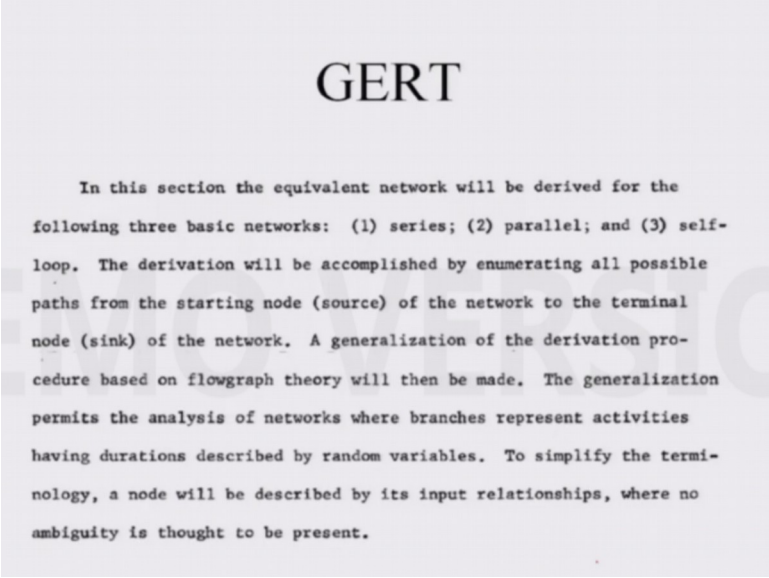

So continuing the discussion in the selection the equivalent network would be derived from following three basic networks which is the serious one which is the parallel one and the self loop one where you ever loop and the series one. It would have some implication with the max and the minimum time and the max circuit and the minimum circuit so series L very simply consider from the point table of simple serious circle which can be drawn in electrical engineering.

Similarly you will the parallel circuit similarly on the same line from electrical engineering and the looping one which were considering that fact that looping is allowed in GERT process. The derivation would be accomplished by in enumerating all the possible paths or all different type of occurrences which you can occur from the starting node which is the source if you remember the source S which has been mentioning time and again like in the last diagram two S was reading to three.

So S is basically the source S in the end can be sink also so that is the nomenclature we have to change so though sink is denoted so starting from the starting node source to the network to the terminal node as i mentioned is the sink a generalisation of the derivation is the procedure based on flow graph theory graph theory which we have will then be made and we would try to understand how they can be utilised in the GERT process.

The generalisation permits the analysis of the networks where branches represents activities having duration described as a random numbers variable which is T suffix A T suffix B what i have been mentioned to simplify the terminology and the node would be described by it is input relationship so what are the different type of inputs from where they are coming where no ambiguity is thought to be present we would be basically try to generalize them through using the input or else we will specify them how they can be described.

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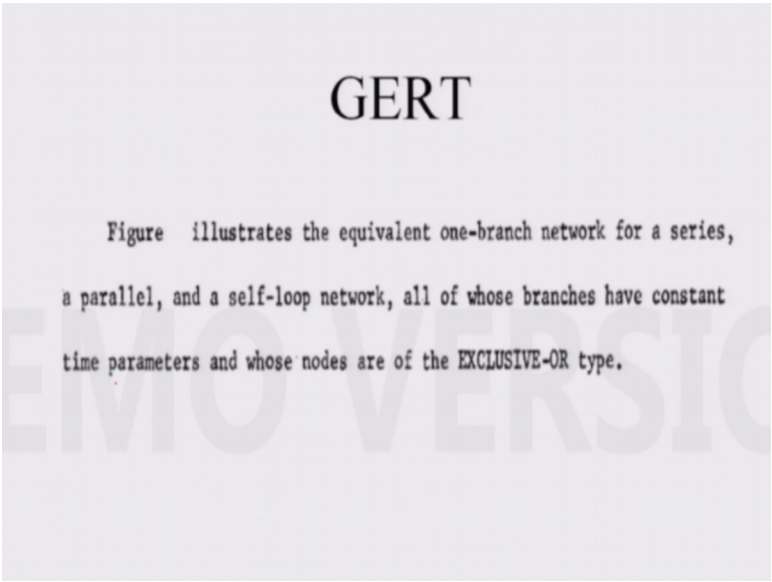

So the figure we are going to consider illustrate the equal lent one branch network for a series a parallel and a self loop network all of whose branches which have constant time parameters so we are considering time as constant thing and whose nodes are the EXCLUSIVE OR so it

can be done for inclusive OR also it can be done for the AND network also but we will concentrate on the ex or which is EXCLUSIVE OR.

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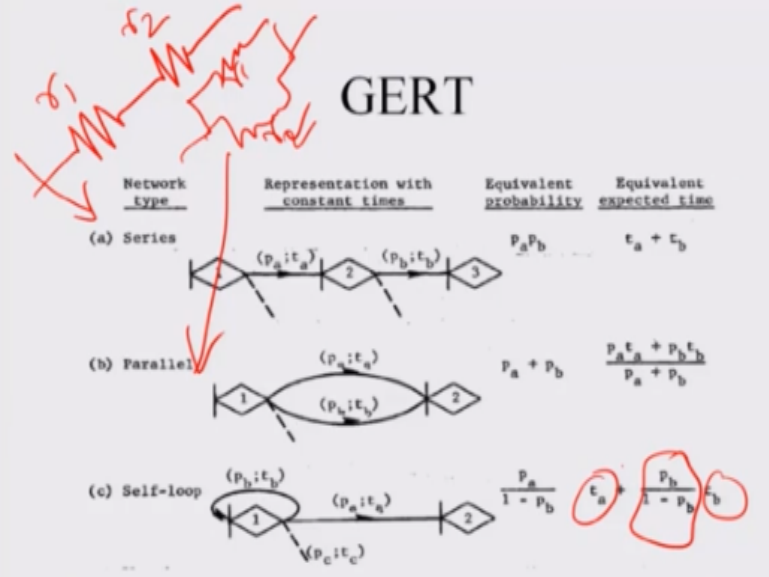

So now the diagram or the slide which you have in front of you basically describes the concepts which are just mentioned two seconds back. The time parameter and it is basically for the exclusive or network and there were three concept which you are trying to bring one is the series one one is the parallel one one is the looping one so the first column basically states the characteristic the second column basically with the diagram on represents with the concept time.

So if it is flexible time it can be turned accordingly that flexible means the problistic time the equal lent probability is given in the thirds column and equal lent expected time is given in the fourth column so now we will proceed correspondingly. So to make life simple so let us first consider the diagram and simultaneously try to explain the probabilities and the time which is basically we will follow the concept.

Each row now if it is a series circuit let us go consider this problem in the other way so if it is series one let me draw a very simple series circle electrical engineering and let me mention this R one and R two R one R two very simply consider them they are to the resistance .So if i want to find out the resistance technically it would be done as per the concept of electrical engineering where find the additional values and proceed and if it is probability one they would just be multiplied in the sense that provided R.

One has happened what are the R two has happened what are the probability that the emotion flow or the path would be taken up would just be the multiplication of the probabilities of R one and R two which is in this given as P suffix A and P suffix B and the corresponding time would be because you have to travel both the paths and if they are series so obviously it will be some of the times will be taken two traverse path A and B.

Now if i go to the concept of the parallel circuit so the concept of trying to find the parallel relationship is just the reverse where in this case we just because they are parallel to each other so R one R two are like this. So this is R ne so this was series in the concept this are the parallel concept .So in this case rather than find multiplying the probability. We will just add the probabilities because that the probabilities is consider in such a way.

That the sum the probabilities will give you the probability of trying to traverse. The parallel path because you have the information has to go through both the parallel paths and if you use simple electrical engineering concept very simply which anybody does in class in the first years of engineering the formulation of the equal lent time would be given as it is given here in the second row.

Which is probability Pa into Ta which is probability Pb into Tb divided by probabilities of A plus B.? Now if i have three one three circuits and all of them are parallel the concept would just be extended where it would be Pa into Ta plus Pb into Tb plus Pc into Tc where Pc and Tc are the corresponding probability and time or the C path or the arc the circuit and that would come in the numerator and the division would be done by the denominator.

Where the denominator would be Pa plus Pb plus Pc and it can be extended accordingly .So if you go to series circuit it would be again accordingly so if they are three it will be Pa into Pb into Pc time would be Ta plus Tb plus Tc and the self loop one will basically have to see the loop is coming inside depending on what is the start and what is the finish. So if i have the probabilities the probability would have traverse Pa and then it would be coming back.

So it will be divide by one minus Pb and the corresponding time would be calculated as given which is Ta plus Pb in divided by one minus Pb which is the actual equal lent probability multiplied by Tb though this is Ta this Tb and the waiting one or the waited or the waits which is happening with the looping one would be Pb into one minus Pb and if there are three

corresponding such loops which can be made complicated as possible. We can find out the equal lent structure and there equal lent probability and equal lent time.

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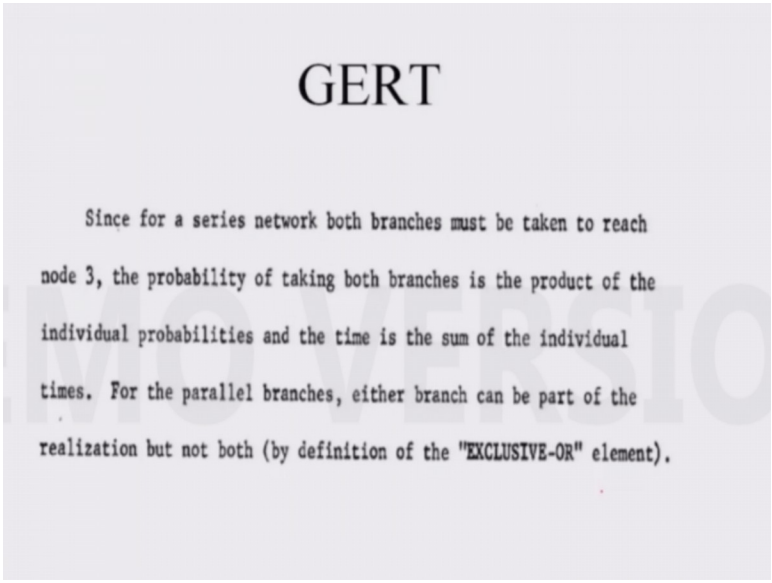

Since for a series network for both branches must be taken by node three the probability of taken both branches is the product as i mentioned of the individual probabilities and the time in the sum and the individual times as i both mentioned and which is there in the first row in the last slide for the parallel branch is either branch can be part of the realization but not both a definition of the EXCLUSIVE OR concept and is based on that way calculate or give the concept.

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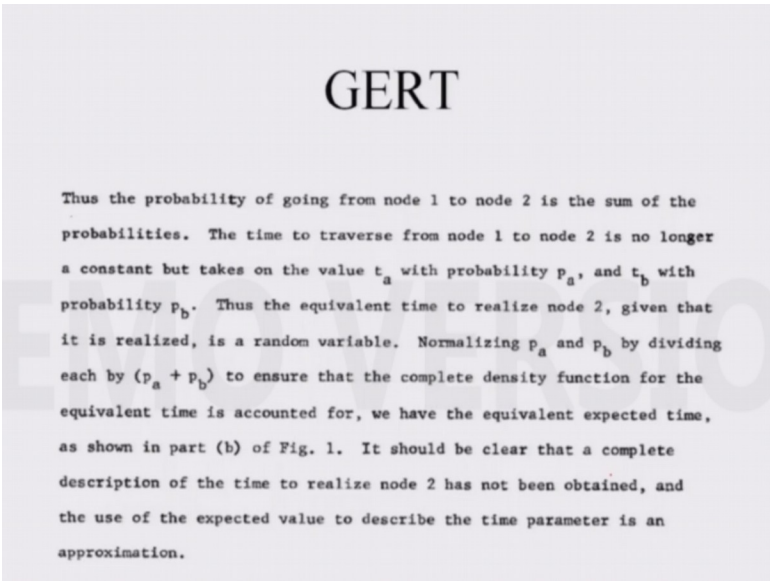

Thus the probability of going from node one to node two is the sum of the probabilities so if basically going for one to two in depending on whether the parallel basically. So it will be

calculated accordingly the time to traverse from one to two nodes is no longer constamatic on the value Ta with probability Pa and tb with probability Tb. So Pa is the probability and is the time and corresponding for B Pb and Tb are the corresponding probability and the time now if i want to normalize.

Normalising so that means i want to basically being a similarity in the comparison normalizing Pa and Pb by dividing by the sum the probabilities so it will ensure complete density function for the equal lent time as i counted and then we will have the equal lent expected time and the expected probability as calculated and again iam mentioning it can be extended for more than two of these connections which is possible in any complicated GERT process.

It should be clear that the complete description of the time to realize note two has not been obtained and the us of expected value to describe the time parameter is approximation based on which we are trying to proceed. Now why the approximation if i remember that these realizations are random in the similar way in one of the classes last two classes and discussed that if you have box and chits are marked one to five and then you pick up chits only three in number.

So the average would itself be random in the long run as you keep doing the experiment that the average of the average is would be actually equal to the expected value of the population. Population means the whole set that means chits one two three four five there cannot be any other set of observations apart from that set of chits. So even the population should be infinite but iam trying to basically give an example how these things can be understood.

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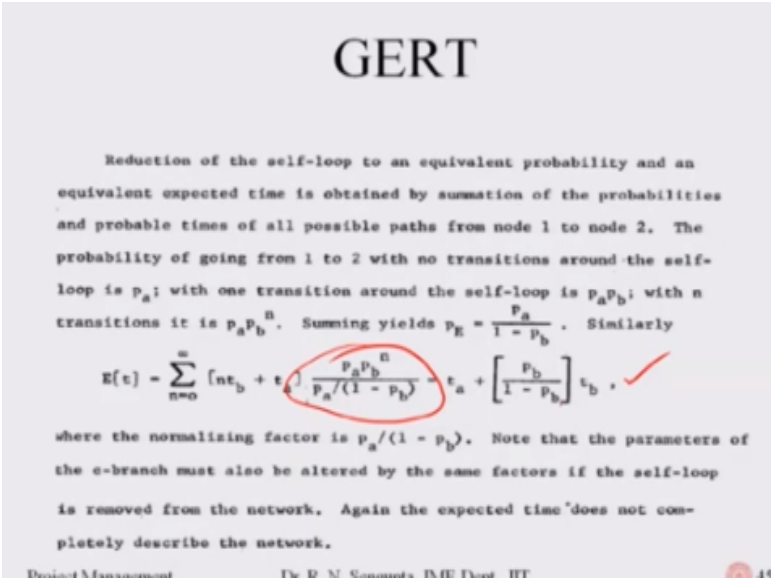

So the reduction of the self loop the loop which is was the third diagram which was there to an equal lent probability and an equal lent expected time is obtained by summation of the probabilities and the probable time to all possible paths node one to two one to three. It will be done accordingly the probability of going from one to two with no transition around self loop is Pa with one transition self loop the time would be as they are in series.

It will be Pa into Pb with transitions being N in number obviously it will be in P .So first time it is Pa into b Pb time and again it happens it would be again Pa into Pb. So if it is happening two times it will be Pa into Pb square. So both of them square three times it is happening it will be Pa into Pb both of them cube and if i do it N number of times depending on how many loops are looping is there.

It will be calculated Pa into Pb to the power N summing would deal basically the the equal lent values probability being Pa one minus Pb similarly the expected value can be found out using the simple summation. So what we are doing is that time to find out the equal lent summation for number of such inter action being from N is equal to zero from N is equal to infinite and you can find out the probabilities and this values were given in the diagram or last table.

So what we did it was trying to basically summit up from zero to infinity and then find out the equal lent probability and the equal lent time so now whether it says the prop normalation factor is Pa divided by one minus Pb node that the parameter or the C branch must be altered by the same factor where self loot is removed from the network again the

expected time does not completely describe the network because expected values and the time would be taken to consideration to study the net work GERT process.

From the analysis of the basic network presented above it is seen that two branches in series the probabilities with the branches are multiplied to obtained the equivalent probability for the two branches for parallel branches the probabilities probabilities add up so this rule adhere the basic law of nodes presenter previously for flow graphs .So whatever concepts or the flow graphs are just being welcomed in this case which means that the probability associated with the node.

Can be computed at the some of the probalities of each incoming branch. So times the probabilities of the node from which the branch has emanated. So if you see it they the multiplication and the normalation done in such a way that it basically adheres to the flow graph concepts alternatively by setting all time stochastic network zero and allowing the other parameters to assume a wider values of range of values reduces a stochastic network to a flow graph.

And that can be done but we will study is points from the point of view from the stochastic network considering in the initial signs in the general way.
That both time and the path which has been followed are both probilistic and they are given by P suffix the path from T suffix the path number or name whatever it is A B C 1234 as is mandated by the concept which you are trying to use it is now possible to stay the

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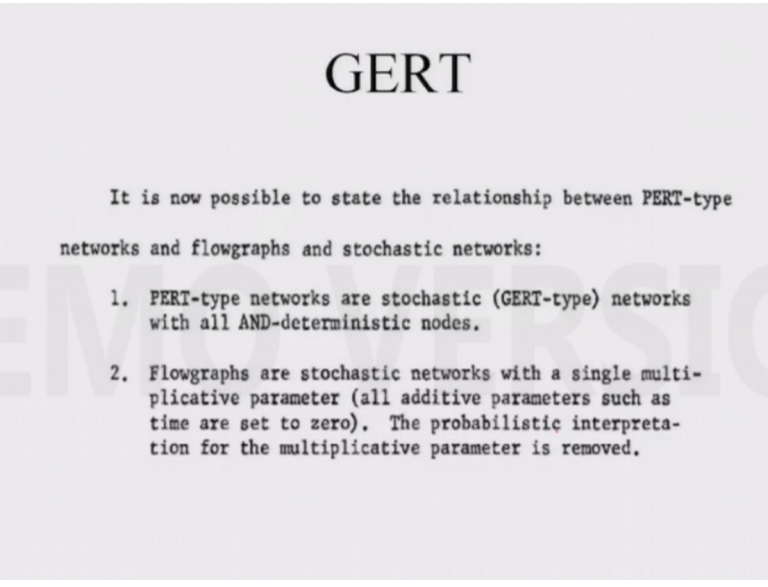

Relationship between the PERT type network and the flow graph and the stochastic network so let me go to the important relationship slowly and you will understand that point one is PERT type networks and stochastic networks that means they are GERT variety networks with all AND deterministic node. So it is basically AND the deterministic combination so we will go back to the combination which we have three into two that means three inputs and two outputs.

If you combine them one of the work basically the AND the deterministic one flow graphs are stochastic networks with a single multiplicative parameter all additive parameters such as are set to zero depending on how you assume the problistic interpretation on the multiplicative parameter is removed in order to basically make sense how they can be done.

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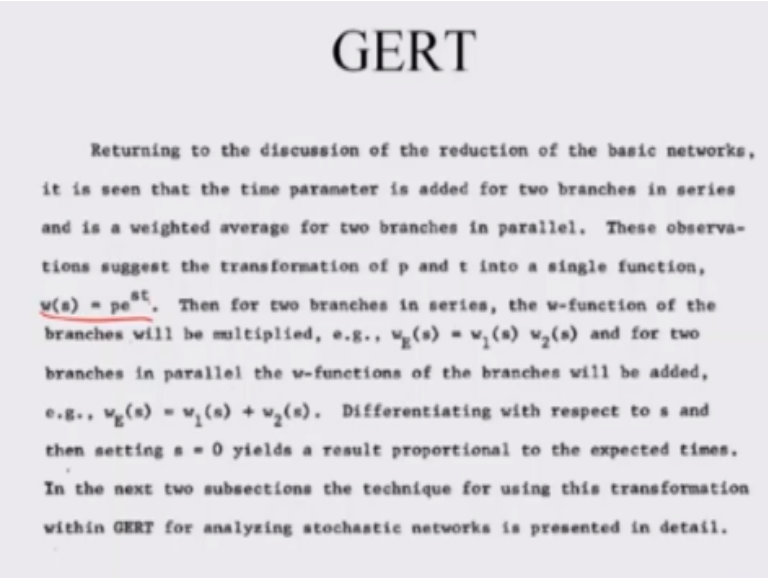

So let us return to the discussion of the flow processes returning and with that baa we will go into details in the fortieth lecture. So let me read the slide which i will try to explain to the students returning to discussion of the education of the basic networks it is seen that the time parameters added for two branches in series and is a weighted average for two branches in parallel these observations suggest the transformation of P and T into the single function based on which you will try to basically find out what is the probability?

And what is the time for the overall network? If you remember in one of the slides initially when you started the GERT process we say that we are interested to find out the moment generating function because that what is give you the actual characteristic of the probability and the time. So we will take the single function which is which will mention as W functions

here. So then for two branches the W function would just be the multiplication. So the WE which is expected the average value for S or the source of the sink.

Whichever you are trying to realize or S can be just be utilised as it is. It will be if there are two it will be W one into W two and W one would be given by P one into the power St one and W two would be given by p two into the power Se two and for two branches in parallel they would it just be the sum. So series would be multiplication and the parallel would be sum differentiating with respect to S will try to basically out. What is characteristic functions or the properties and based on that we will proceed.

So with this i will close the thirty ninth lecture and with a node. That i will try to finish wrap up GERT in the fortieth lecture and also try to basically go very briefly about what we have covered we do agree that considering the forty lectures we have to cover a lot of things and we tried our level best in going through that but iam sure that the students would have really learned a lot of new concepts once we end the course and i will try to basically highlight that in the last class thank you very much for your attention have a nice day.