Project Management
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Module No # 07
Lecture No # 35
Introduction to Graphical Evaluation and Review Technique (GERT) I

Welcome back my dear students and welcome back to this course in project management, as I discussed in the last class which was the thirty fourth class, we said that we will discuss something about GERT or JERT and Q-GERT and try to basically give a very good background and how to stochastic network can be utilized. You may be thinking that what is the point of discussion those networks here.

So if you remember in PERT and CPM the main difference in PERT was there was an optimistic time the pessimistic time the most likely time, and based on that we found out the expected time the variance and then calculated the concept of what was the critical path then we also showed that how the crashing of the jobs could be done keeping in mind leveling has to be done averaging techniques and then critical path was only different from PERT in the sense that the time was deterministic.

Now in the GERT and Q-GERT will just consider the concept where it may be possible that one of the paths or some of the paths are taken or not taken depending on the probability of the moment of either information or the path can be taken when you are trying to build up project which can be launching of satellite or trying to basically come out with a new product whatever it is.

So we will just give very simple examples and go through the basic concept of GERT, or JERT or Q-GERT. And I am sure people will be able to understand the last few sessions we will try to do deal with that in a little bit more detail. So in the thirty fifth and thirty sixth sessions today being thirty fifth one we will try to cover in a little bit slow manner and cover the fundamental.
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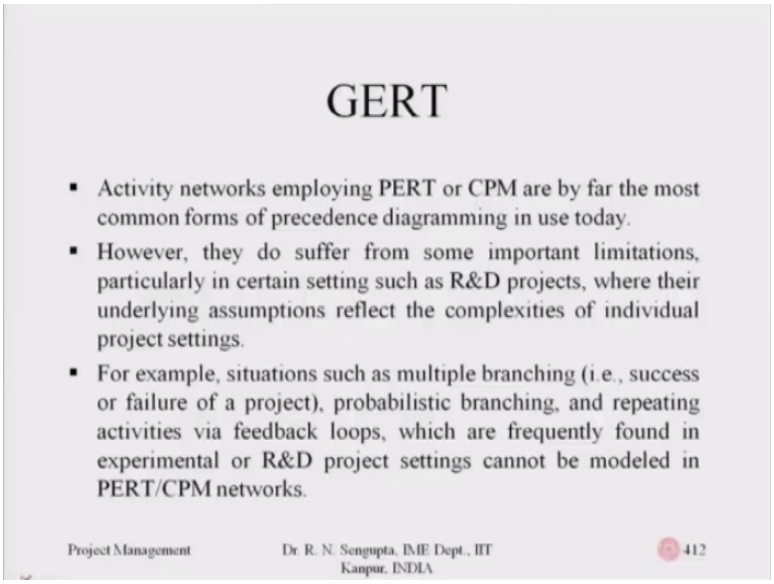

So as mentioned here the activities Network employees PERT which is Project Evaluation Review Technique or CPM, and are far, by far the most common form of precedence diagram. However they do suffer from some important limitations particularly in certain settings such as the R and D projects. Where the probability of trying to invest in some project or trying to basically follow some path.

May not be actually hundred percent certain or the probability may be zero, may be between zero and one so based on that we are trying to basically give the background. So setting such as R and D projects where their underlying assumptions reflect the complexities of the individual project setting based on which we can say whether the path will be taken, not taken or what is the overall probability of the time which is being taken along that part.

For example a situation such as multiple branching, success or failure for a project yes or no or there are more branches whatever it is, probabilistic branching and repeating activities via the feedback loops, so if feedback loops are available if you remember in PERT and CPM, we considered the feedback loops were not there. So now we are considering the feedback loops are there, depending on how the second stage of the work has been over.

So that will give the feedback to the first stage says that the feedback, the first stage can be improved or some other sophistication can be brought into the whole process. So which are

frequently found continuing with the reading, so this feedback loops which are frequently found on experimental or R and D project settings cannot be modeled in the PERT and CPM mode.

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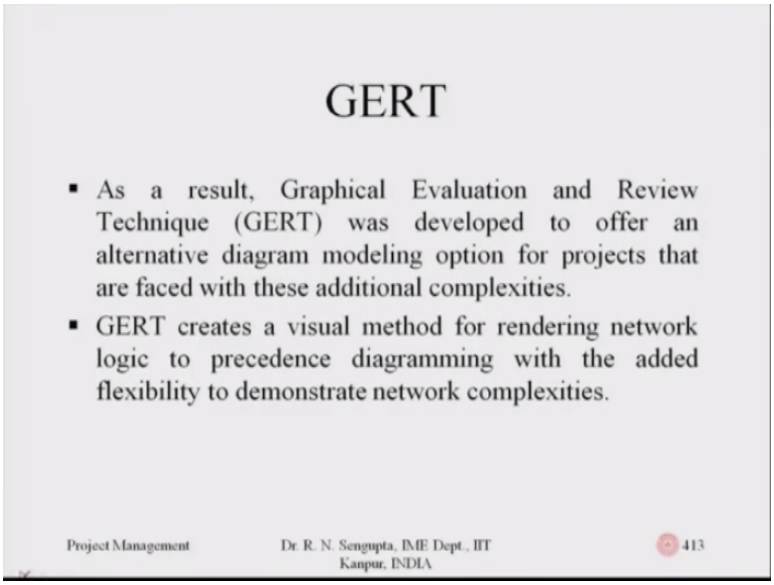

As a result, we come into realm of Graphical Evaluation and Review Technique which is GERT or JERT which was developed to offer an alternative diagram modeling option for projects that are faced with these additional complexities. Like go, no go, yes, no, if, or else whatever be the concept, those paths are not certain, and you will take that depending on whether those parts can be travers not be travers.

Feedback loops are available not available, based on that you take the decision. GERT creates a visual method for rendering network logic to precedence diagram with the added flexibility to demonstrate network complexities to a larger extent. Like in PERT and CPM, obviously we saw, that looping was not there, then the concept of end to start was the main concept which you worked and found out the times.

But obviously the concept of start to start, start to end, end to end all these other concept criteria can be included, how they can be modeled. We have already discussed, even though we solved the problem for the case where it was end to start. But other three concepts can be incorporated depending upon the formulas which we implement. Now in GERT, these concepts number one

they would subsume automatically and number two as it has been mentioned just in the previous slide, that the concept of looping would also be considered in the GERT process.

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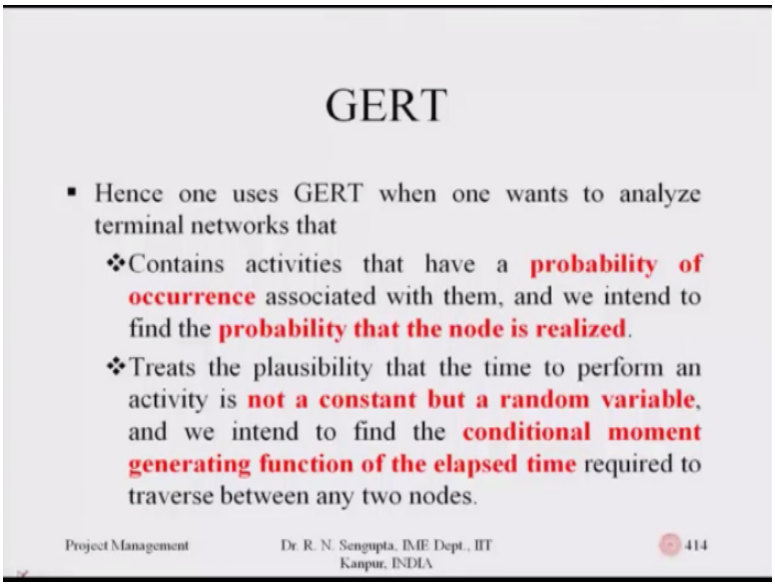

Hence one can use GERT when one wants to analyze terminal networks that so what are those conditions and what are the main criteria based on which we will work, they contains activities that have a probability of occurrence associated with them and we intend to find the probability that the node is realized. So we will be basically focusing our analysis on the precedence diagram considering the nodes or the activities. So it can be done for the arcs also but we will stick to the nodes because it will be easier for us to explain.

So we will consider that, what is the probability of occurrence of that note occurring considering the fact that the precedence diagrams, the precedence relationships are actually met. Depending on the probability that has been stated so, obviously there is a probability based on which we want to find out that what is the probability of that note occurring in the network. So in the GERT, also we want to analyze terminal networks that treats the plausibility that the time to perform an activity is not a constant but a random variable.

So, provided a node is yes or no, possible or not possible, we also have the time factor also coming into the picture which means if node two, is possible it may also be possible that from node two to node three there would be a probabilistic time which we have to take into

consideration in order to solve the overall network. so and we intend to find out the conditional moment generating function for the elapsed time required to traverse between two nodes.

Because that will give us the overall functional form, between the probability and what is the actual time taken, such that trying to find out the overall critical path so overall average time would be possible for us. So moment generating function would be needed that if you want to find out that what is the overall relapse time such that adding up the times for all the paths, or all the activities which actually make the critical path would be important.

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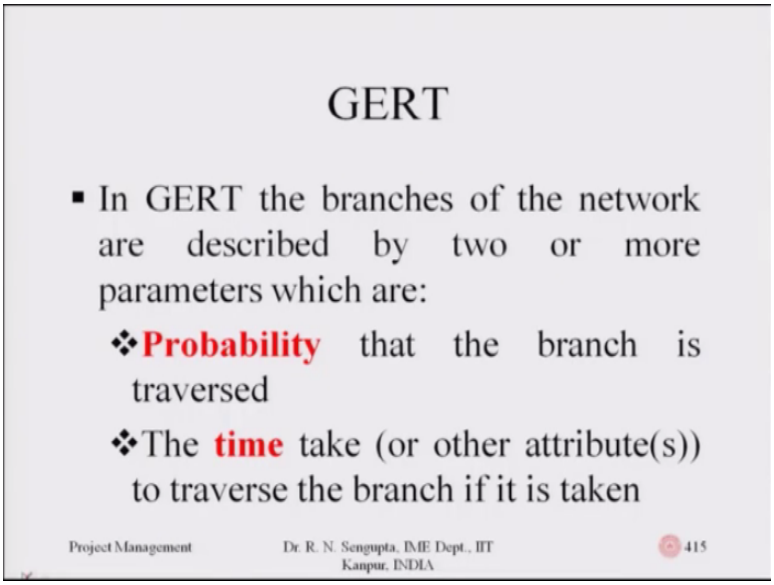

In GERT the branches of the network are described by two or more parameters, so we will consider parameter one and parameter two are the essential ones which as I mentioned in the last class as a probability. Probability of the occurrence, so it says that the probability the branch or the node is realized so that is the first part which will node by probability with the number suffix where that suffix will denote what is the activity of the job taken.

And what is also the activity of the job which is being traverse. Say for example from, node A to node B, so either we use the suffix A or we use the suffix A, B to denote so it is travelling from node A to node B. The next parameter is the time, which is time taken or other attributes, this other attributes can be anything apart from time. So, but generally we will consider for the initial network, that the probability and time are the two important things.

So it will mention that the time taken or other attributes to traverse the branch if it is taken at all. So obviously, if it is not taking the time factor it would not come. Hence we would not be utilizing the concept of moment generating function to find out what is the average time, but if the node is traverse or if the node is realized obviously you will have to find out what is the actual time based on which we will do our calculation.

So, technically we will consider two parameters which are important for us because based on the parameters we can find out what is the average time, what is the set of past which we will follow in order to complete the overall projects. So one is time, one is the probability.

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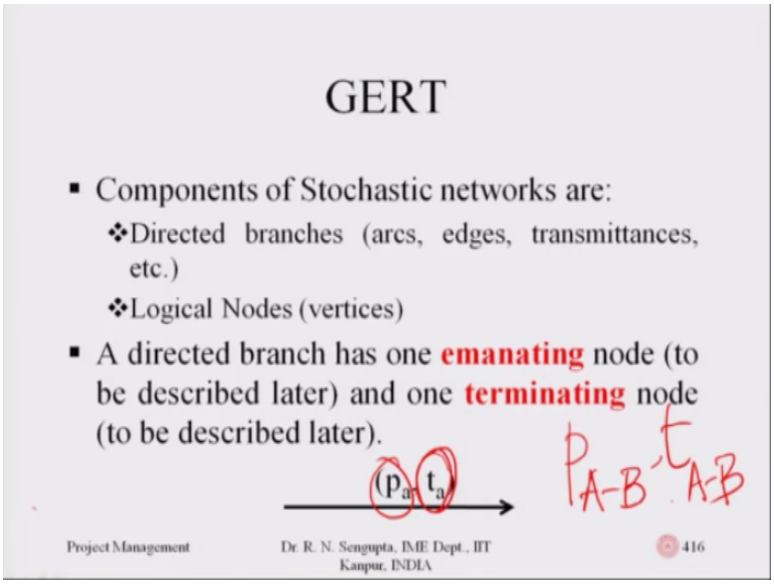

So the components of the stochastic network are the GERT processes are two things one is the directed braches which is the arcs, edges or transmittances, so similarly exactly what we did in PERT, CPM, we had the arcs and we had the nodes. So directed branches are basically the arcs. Now what is important to note that in GERT, the as import also in CPM also, you will basically have a precedence diagram but the precedence diagram in the case for the GERT may not be sacrosanct due to the fact that they may be looping.

So say for example, if you are following the loop, from A to B and then coming again back to A then the precedence relationship between A and B is not clear as what it should be in the PERT

and CPM. So incase for a PERT and CPM we are saying that job or activity or task A has to be finished before job B, then it obviously mean the arrow is basically ready from job A to job A but in GERT if there is a looping that means after B is completed.

The feedback again comes back to A then obviously this so called precedence concept would not hold true so, but obviously it will make things much more complicated but it will give you a much better, practical picture how things are going on. The next component in the stochastic network or the logical nodes. So logical nodes are exactly like the one which you have nodes in the PERT and CPM, the logical nodes are only coming into the fact because in PERT and CPM the types of jobs.

The activities you have mentioned, were actually realized such that the overall project should be finished. But in the GERT it may be possible that if the probability of the actually reaching that node is zero, that node would not be realized but, considering that node is a part and parcel of the overall project which would basically have, give a much more clear picture that the how, the project should be done.

Now say for example, that you have two sets of activities, let me try to explain, obviously I will come out with an example. So you have activity A and B now it may be possible that the project would be completed either if A is done or B is done or both of them are done and if both of them are not done which means the project is not realized. So in that case, the path stating through some source till some sync goes through A but does not go through B.

Which means the job is realized as mentioned in my last statement. Case two, starting from the source to the sync the overall job passes through B, but not through A still the project is realized. Third case, the overall sequence of job started starts from the source to the sync, so whatever the source and whatever the sync is and completes A and B in some sequence which means still the project is realized and the fourth notion is that it starts at source.

But both A and B are not realized which means the project is not finished so hence, the probability of trying to realize A or B or both of them would come into the picture in order to

basically calculate the problem, probabilities occurrences and also the time. The second point states in this slide is a directed branch has one emanating node that means from where it is starting to be described later I will come to that later on.

And one terminating node which will again I will describe later such that particular R or that particular directed branch, or edge would basically have two sets of information one is probability which you see is this one which is Pa which means probability suffix a meaning it is basically following the arc A, or it feels basically starting from activity A to activity B, so obviously corresponding probability will be given by A to B, that is probability.

And this time T suffix A which you now see is basically that the time taken to traverse the arc A but if it is basically from activity A to activity B or job A to job B hence it will be T, which is time with a suffix A dash B,

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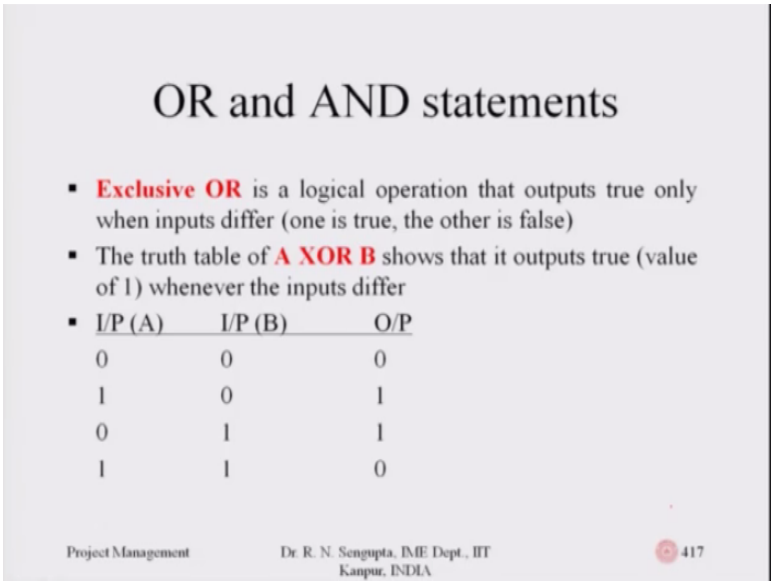

Now I will basically discuss some of the very simple logical concept, which is exclusive R and the GATE, GATE means basically the very simple concept of logical operator, so I will go with a very simple example and explain that, I am so sure that make things much clear when we consider the examples, in a very simple matter, so let us consider the first the exclusive R.

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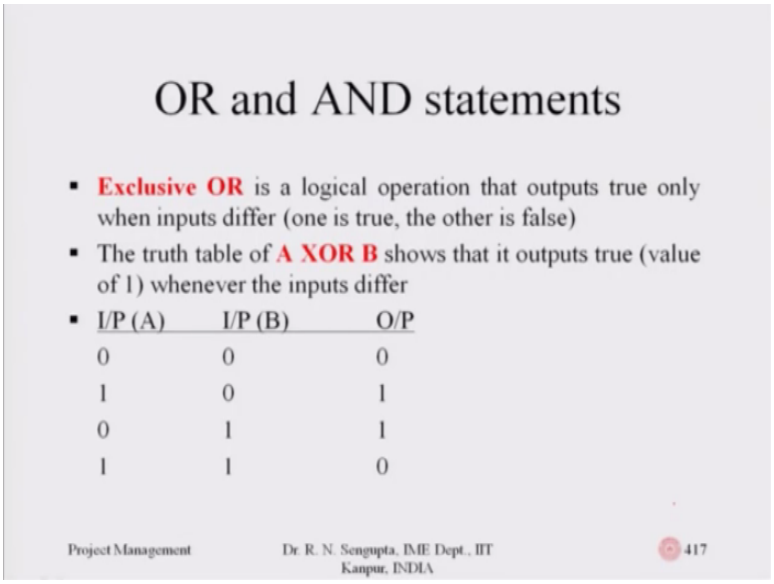

So an exclusive R is a logical operation that outputs are true only when inputs differ, so say for example you have two operators a GATE, on and off switch sort of thing. So consider the gates are A and B, and they can be expanded to A, B, C, D to whatever the combinations are now, the first statement says the exclusive R, or is a logical outputs true statement if only A is on, B is off, or B is off and A is on.

So which ever so consider sorry, let me rephrase my statement, A is on, B is off, and the second one is B is on, a is off so which means that one of them is false and another is true or vice versa so let us see how it is basically notified in the table. So the truth table, second point says that the truth table of A XOR which is exclusive R of B shows that its outputs are true values are one, so we will consider the outputs to be true that means the output is one.

I realize one and if it is not true, which is a false it would be zero manorial operation sort of thing whenever the outputs and inputs are different so let us consider how they are different and what are the actual outputs. So input A has an information which is zero which is false input B has information which is again zero which is false, so false and false basically means both of them are of the same category hence the overall result is false.

Which is the output, which is zero, which you see that first row, the second one is that input A is one which is true, output B which is off, which is zero, hence as both of them are different as per

the simple definition of exclusive or the combined effort is basically one. So in the next one if you have the input A is off and the input B is on which is one then the combined one is what one is off another is on but now in the third row it is just opposite to what we saw in the second row.

So in the second row A was on, B was off, now A is off B is on, they combine overall output is basically one which is true and in the last instance if both them are on or both of them are true which is they are of the same category hence the overall result is false which is zero. So this was the exclusive R, now consider the inclusive R.

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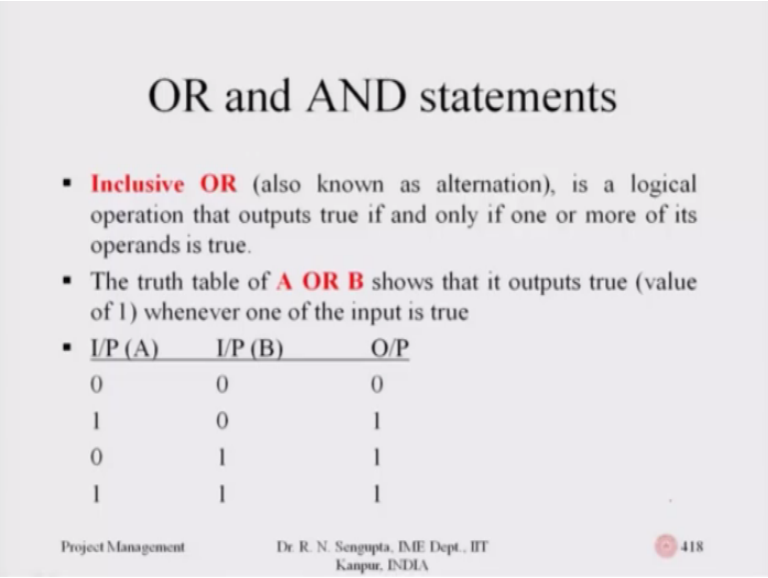

Also known as alternation is basically a logical operation that outputs true if and only if one or more of its operands are true. So this inclusive R is basically denoted by A or B in that case in the last slide it was A X or B which is exclusive. So the truth table here for A or B shows that the outputs are true whenever one of the input is true. So if other is false, if both of them are true obviously the answer would change accordingly for the last statement which I make.

So let us basically see the table and you will understand input A is off, false, input B is off which is false so combination is always off. Now let us concentrate on second row, third row and fourth row, one at a time, the second row says input A is on, input B is off, hence the combination shows that one is on which is A that is off which is B then the combined one is on which is true which is one.

The third row is exactly opposite to opposite means input A, B are output are opposite with respect to what was there in the second row, so in this case A is off, B is on which means A has a value zero, B has a value one different with respect to what we had for exclusive R, or A or B, here you see both input A is on which is true which is one input B which is on which is true which is one, hence the combination of them is a combination of true.

True which is one as mentioned in the slide inclusive R so obviously you should remember the exclusive and the inclusive R, are different very subtly in the first case, they are of the opposite concept but here one of them has to be true such that to give you the result as one. Now we come to the AND statement.

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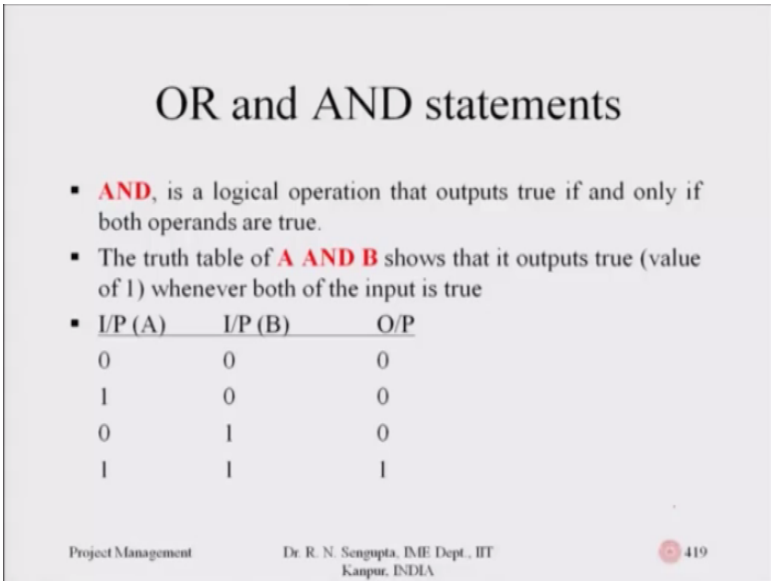

So in AND is a logical operation that outputs true, if and only if both of the operations, operands are true. So the truth statement is basically written as A and B it shows that its outputs is true value, true value is basically one here what we are considering, yes, no, or whichever combinations you want to take is basically if you switch on, off, on will consider true, off as false. One and two, one will consider true, zero as false, consider their colors black and white, black as blue and white as false whatever the combinations you have.

Go, no go and all this combinations. So as A and B should shows that its output value is a value of one when a both of this input is true, so let us consider them, input A which is a name concentrating on the first row, input A is false, input B is false both of them have value zero, hence the combination of both zero. Second row, input A is one which is true, input B is false which is zero, combination of one and zero is zero.

Third one is exactly opposite that means input A and input B is exactly opposite to what we saw in the second row, so input A is zero which is false, input B is true which is one, combination is zero and as you find in the last element of the third row. And the final on which you have seen is basically the AND one if both of them are open or closed depending upon how you are basically trying to analyze the problem. Both of them are true.

A is true which is one, B is true which is one, combination is basically one, so I will try to basically highlight this three exclusive, inclusive AND, once again going through the slides so, I am going back to the last to last slide in order to make things much clear. So in this case you see zero gives you an output zero. One, zero gives you an output one zero, one gives you an output one, and one gives you an output zero.

Since basically you can use plus and minus operations also, in this one zero which is inclusive R, zero gives you an an output zero, one, zero gives you an output one, zero one gives you an output one and one gives you an output one so this was inclusive or the previous one was exclusive OR, now we come to the AND one so this is the AND one, zero gives you an output zero, one zero gives you an output zero.

One zero gives you an output zero, zero gives you an output zero and one gives you an output one, so with this information we will try to basically come off with the clear strategy or the GERT,

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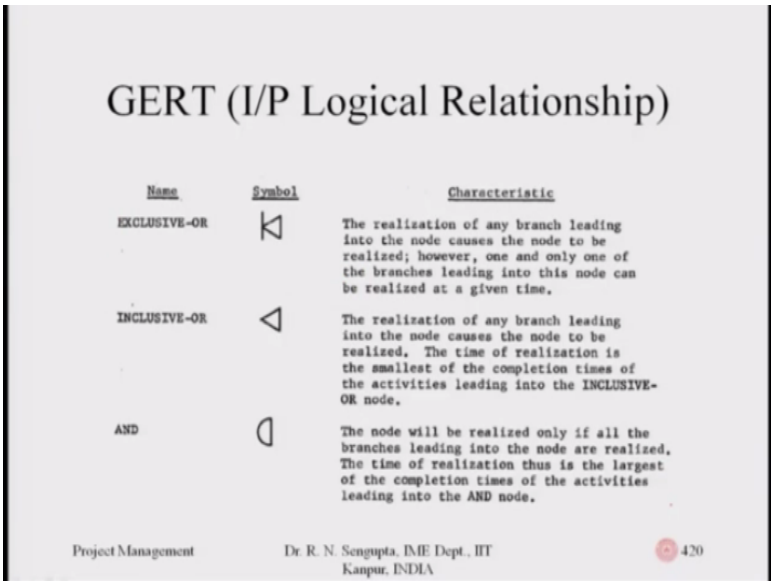

So in the GERT, they are basically six different combinations, three in the input and two in basically the output end so three into two is six, so what are the input information, let us go one by one. So if you see the first, second and the third column in this slide, the first gives you what is the consequence whether it is an exclusive OR or inclusive OR or an AND, what is the second column is basically the diagrammatic representation of the code.

Or the node and the fourth column which basically is the characteristics which we will try to describe. So this when I go through the three type of inputs and the two type of outputs you will find out that it may mix quite a good logical sense considering that we have already considered the exclusive OR, the inclusive OR and the AND logics. So the exclusive OR is basically denoted by a triangle with a vertical line on to the left.

So it means that realization of any plan. So this is important please pay attention here, the realization of any branch leading into the node causes the node to be realized, however one and only one of the branch is leading into this node can be realized at a given time so that means only one of them can be realized. So hence it is an exclusive OR, so if you basically follow the overall concept which we had for the exclusive OR concept.

It will make sense like how it makes a clear similar concept or the logical operators and trying to bring that into the GERT network. The inclusive OR has only that triangle, as in the exclusive

OR without the vertical line on the left. So it means the realization of any branch leading into the node causes the node to be realized as rightly pointed out, now here is what is the important fact, the time of the realization is the smallest.

Which is the minimum of the time of the completion time of all the activities or all set of activities leading into the inclusive OR node so if you have basically different times, you will basically consider the minimum of them and proceed accordingly, so in the first one which was the exclusive OR it was one and only one so you will only take one of them and the second point what I am trying to highlight is the importance of the statement.

You will take the time and realization based on the minimum fact and now come to the AND statement of the analogy which is basically a half circle and with the circular part on the left hand side that is as it is there the node will be realized so let me read it carefully, the node will be realized only if all the branches leading to the node are realized so all of them have to be realized that the time of realization does is the largest.

So if all of them has to be realized obviously it would be realized after all of them have been basically met, the condition have been made, so obviously logic says you have to take the maximum time. But if you consider the inclusive OR it would be realized the moment one of them is true which is basically a simple concept, minimum time is the important factor which has to be taken into consideration for inclusive OR.

So again coming back to the AND concept the time of realization does the largest of the completion time of the activities leading into the AND mode.

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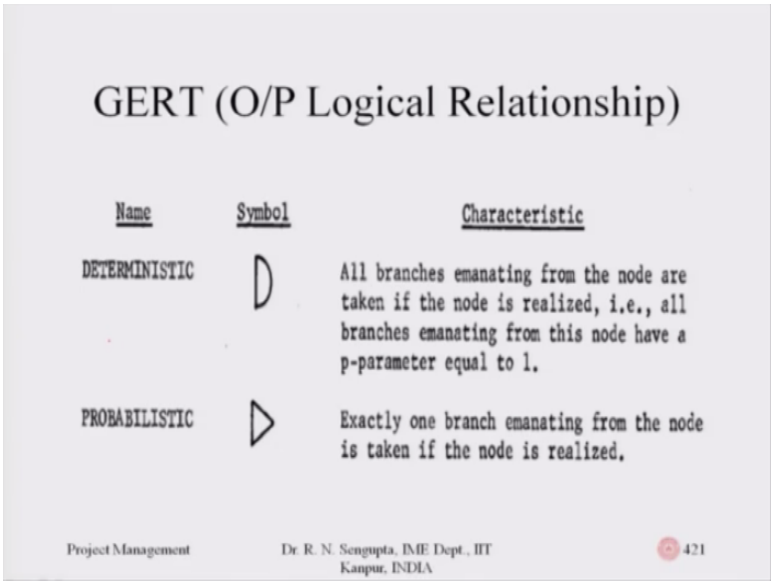

Now for the output logical criteria or the relationship for the GERT one, these are the one, we have the deterministic one which is the hemisphere but just opposite consider in the AND, and input one for the GERT and the probabilistic one is a triangle, but the triangle is just the mirror image, what we just saw in the other two exclusive OR, or the inclusive OR concept. So in the exclusive OR you had a, the vertical line on the left or an inclusive OR which was a triangle on the left which was now just opposite in the probabilistic one.

So obviously all this three to two combination would be done, I am going to come to that so let me first go through the brief explanation of the output logical relationship so the deterministic concept which is the word written in the first column then the symbol as you see and then the characteristics, so characteristics leads as it is all branches emanating from the node are taken if the node is realized that is all branches emanating from this node have a p parameter equal to one.

Which means the probabilities are there so you will have node to the probability such that the sum is one, obviously that should be true because one of them have to be taken. The probability okay, so let me clear this because I think they would be a, some confusion later on when we try to solve the problem. So from p, parameter I basically the probability which is the overall time duration taken for the deterministic one is basically one.

The second point is basically in the first column is for the output logical relationship is the probabilistic one it states exactly one branch emanating from the node is taken and the node is realized. So one of them have to be taken the probability would be given, but if you, if I go back, what I mentioned just few minutes back or few seconds back, if I take all the probabilities of all the possible realizations then the sum would technically be one.

And likely so, so we have three inputs two outputs let them come, let us come remind them. So now you have the GERT, the overall symbolic operators which are there,

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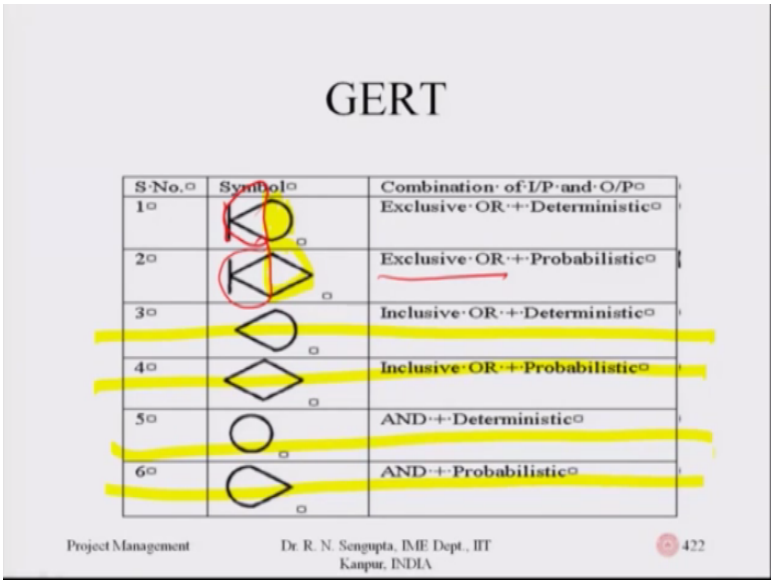

The first one you see in the first column, you have the serial number, second column you have the symbol, the combinations of the input and outputs is given in the third column, so let us go one by one the symbol which has an exclusive OR and deterministic one is give, like this because in the first part you have the input concept, and in the second part you have the output concept, so I am basically trying to highlight the input one with the red one and the output as the yellow highlighted.

Similarly when I go to exclusive OR, or the probabilistic which is the second row then it is basically a triangle rumbas with the straight line onto the left which is this part so this is basically the exclusive one, or any this triangle which you have on the right hand side which is basically

the probabilistic part similarly when you come to the third incisive OR plus deterministic the symbol is given in the third row, for intrusive or the probability you have the fourth row.

So I am sure you will understand if you wants you can go through the three input and the two output the fifth one is basically AND, and deterministic and the sixth one is the AND, and probabilistic. So with this logical operators I will end this thirty fifth class and I will strongly heard the student to please go through the concept once again and with this we will start with how the diagrams can be drawn for the GERT network and try to explain that in the thirty sixth and the later on class. Have a nice day and thank you very much.