Project Management
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Module No # 06
Lecture No #29
Scheduling and Crashing Of Jobs

Welcome back my dear students I am Raghunandhan Sengupta from the IME department IIT Kanpur India. So you all of you are taking this project management course and welcome back again to this twenty ninth lecture. So the total number of lectures as you all know is of forty number of duration which is of twenty hours so considering the concept which we have been discussing the last day.

It was basically the concept of the PERT and also we discussed about in details about critical path method and critical path is given what is the overall average time average means expected time which is taken for any activity and then considering the optimist time the pessimist time the most likely time.

We found out how you can find E of T that is the expected value average time we also found out that how the various can be calculated for each and every activity job and how given the concept of the variance. Addition of the variance being possible due to the fact that activities we consider in a very simplistic notion.

They are independent we can add up the variances and then find out the total variance of the path or the critical path or any sub critical path also sub critical in the sense those activities are not critical and then we can use the concept of central limit theorem and find out what percentage of job to be finished? Considering that your average time is say for example T. A average number and your dead line is D.

How you can find out what is the probability of work which is totally finished? What is the probability of the work would be finished between two deadlines? How the concept of cost can be brought into the picture remember i did not mention quite a few number of times that in the PERT CPM and the general in the activity on arc activity on the known concept ever you have we are trying to basically find out the average time and what is the critical path based on which you will complete the job.

So the concept of trying to bring the resources even though i had mentioned that in some details. But, it was not considered as in depth and we will consider that as we go along in the twenty ninth, thirty first and thirty second class and also consider that how the levelling of the resources can be done?

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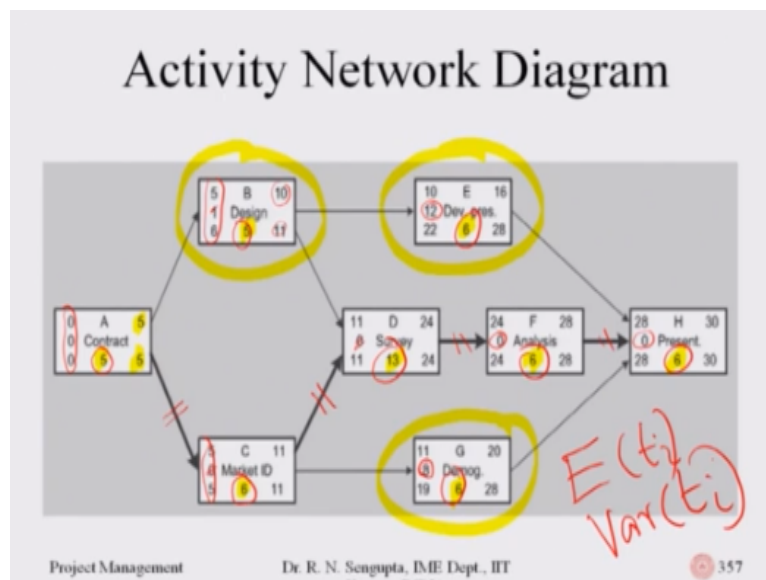
So come back to the slide number three fifty six. It shows the contract is to be done and given based on which you have the design aspect. So A activity leads to B. So it is very simply an activity of node even though the nodes are not circling in nature but it gives you a feel that how the overall project has been implemented.

Then you give some marketing work is to be done and as the design work is being done marketing conception is to be implemented in order to sell that product. So that sellen of the product can be either a moped can be either a fridge whether you are coming out for the new drug it can be that whether it is basically a social project whatever it is there has to be some marketing of that.

Then based on the design and the marketing survey feedback which you get from the customers which you do the survey the actual product which is to be sold get do the test marketing C what are what is the set of feedback it is whether the price is too high whether the reengineering has to be done whether the features based on which you are trying to do a design for the product is absolutely in line or in tandem with the customer ones.

Then as you get the survey obviously there are loops but we are not going to consider the loops as i mentioned for the PERT, CPM and the decision network at least for the bulk portion of the project management course. Then the development process the development face would come then after survey here of the analysis part then the marketing also does a detail survey.

What is the demographic concept for whom you are going to sell the product? Whether it is for the teenagers? Whether it is for the middle age people? Whether it is basically for people? Who are in the high income bracket or the low income based on that you combine the phase of the activity E, F, G has given in the slide and then you do the presentation and finish of the project. So this is a very simplistic project which am just going to discuss with the students.

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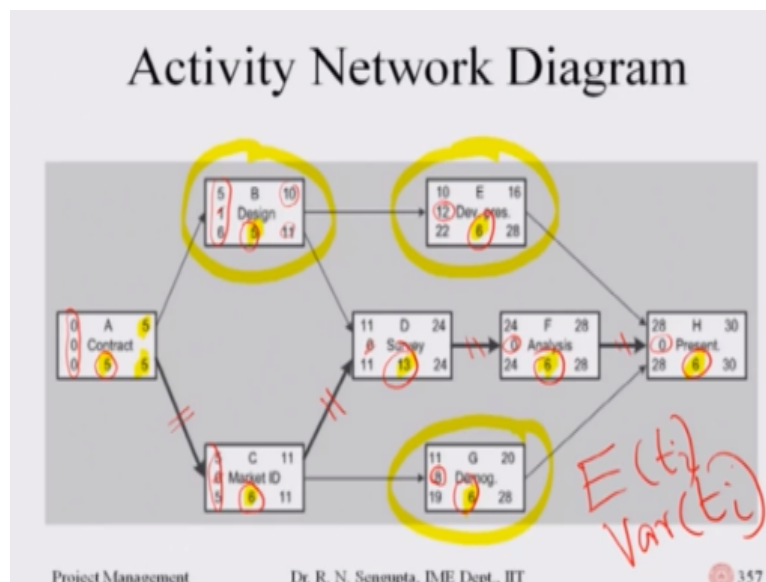
Now considered the same activity network diagram which is again placed in front of you which is slide number three fifty seven but now if you see the numbers which are there.

So, say for example I am considering the numbers in such a way that the left most value. Which is there for A or B. or say for example, C Which is there generally it means the earliest start and the latest start time based on which you can find out what is the overall free slack and the total slack. If you remember i did go into detail that what was the formula for the total slack? What was the formula for the free slack that free slack has to be less than equal to total slack?

If slack total or free slack is totally zero then what does it mean it generally means as you can rightly point out because considering that you have completed about twenty eight lectures of the of this course. And you have been doing a lot of reading and trying to solve the problems. So it means basically the slacks being zero would imply that the activities of job are on the critical path.

So, if you consider A the earlier starting and the later starting are given the total number of days required to finish the work is given here. So, now i will highlight it for each and every activity. So, this is five for A its again five for B, it is six for C ,it is thirteen for D, six for E ,then six for F, then six for G, A, B, C ,D are the activities and lastly it is six for H. So if iam able to calculate or go into the details it means that earlier start you are going to start as T = 0 and if you add the number of days it is 0 + 5.

So, hence the earliest finish and the late finish for this job activity is five years. So it is basically five year and five year. So again iam highlighting this five and five now if i go into Job V the total number of days required is 5 so adding 6+5 = 11 and again adding 5+5 = 10 which means that we consider the early start early finish late start late finish.

Considering the concept of the forward past that means you are going to the left to the right from where you start which is the source to the sink and the backward past very start from the sink to the source there it will give you the total slack of the free slack. So the slacks which are given for the job would be for B at least is one.

Then for C it is zero. Iam just highlighting those for D it is zero then E it is basically given 28 – 12, 22- 10 it is basically 28 – 60. So it is basically 12 then for G it is 8 for F it is 0 and for H it is zero . So if i want to concentrate and find out the critical path it becomes very obvious to all of us considering that we have already done this concepts and the concept of free slack from the total slack is clear to us

It will be basically will take the path which is A to C .So iam just marking it with ash. Its A to C then it will go to D because i will not be taking a C to G because that is not critical and along the critical path then i will basically go to E to F and then i will complete overall project going from F to H so these jobs which iam going to highlight now are circled now and
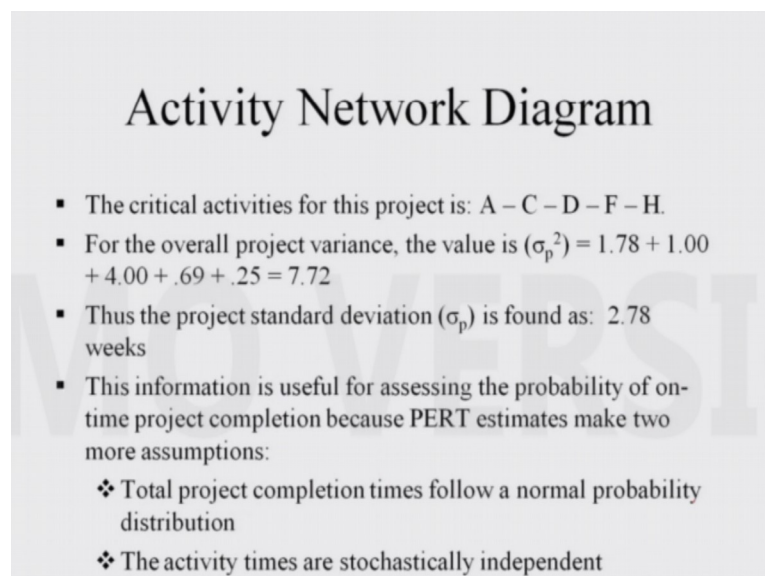
not along the critical point .So B is not there E is not there and G is not there now having said that the data which is given in front of you gives the early start and early finish.

So we assume that the optimal the pessimist time the optimist time and the and the most expected time are already provided based on which you can find out the average time .so, the average time which we find out for all the jobs are what i had already mentioned. So basically for job A it is five .So obviously they would be a variance so iam not going to discuss about the variance but come into the concepts.

As such five is the expected time of the average time for B similarly for C it is six, for D it is thirteen, for E it is six, for F it is six, for G it is six and for lastly for H it is six. So, you can find out the value of E t for E job and can i can find out the variance of E job so it is basically E t suffixed for the job number. So i would basically with a job number and adding up if you know and if you have gone through in depth the slides which i have discussed.

My lectures which have been repeating time and again that addition of all the time is taken along the critical activities gives you the time for the critical path and addition of the variances along the critical activities of the critical path will give you the variance of the total critical path and then we can find out the standard deviation .So again coming back to the same diagram.

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Activity Network Diagram

- The critical activities for this project is: A – C – D – F – H.
- For the overall project variance, the value is $(\sigma_p^2) = 1.78 + 1.00 + 4.00 + .69 + .25 = 7.72$
- Thus the project standard deviation (σ_p) is found as: 2.78 weeks
- This information is useful for assessing the probability of on-time project completion because PERT estimates make two more assumptions:
  - ❖ Total project completion times follow a normal probability distribution
  - ❖ The activity times are stochastically independent

The critical activities for the project is as rightly pointed out in the last slide which was three fifty seven is basically A,C,D,F,H for the overall project.

Even though the data is not given but we considering the data is there and we do the calculations the calculation concept remains the same there is no change but I am just skipping that part and going into the discussion directly. So add up all the variances which is 1.78 + 1. So this is for A, C, D, F, H in that sequence then a 4 then a 0.69 and 0.25 so to total variance is 7.72.

If i want to find out the standard deviation it will be very simply the square root of 7.72 which is the standard deviation for the path which is critical? Which is A, C, D, F, H thus the standard deviation has given in the third bullet point is found out to be 2.78 approximately.

So, this information is useful for accessing the probability of the on time project completion because PERT estimates makes two assumptions which again iam repeating total time project completion follows a normal distribution considering the central nearby theorem may be it is true. Even though the distribution of for each and every activity is totally different which we have discussed and the activity times are stochastically independent because if they are not independent then adding up the variances would not be possible.

So as a result please excuse me even though I am repeating that but i strongly feel the concepts which I am lately repeating time and again are very important to understand the general methodology of the concept based on which you will try to solve any problem in PERT CPM and got later on.

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So as a result the normal bell curve can be used to present the project completion time. So the normal distribution here implies there is fifty percent likelihood that the project completion would be will be less than thirty weeks. So if you add up all the times for along the critical path you will get the sum of the average time of all the paths which are critical .So what i do is that if you see the calculation i add up the expected value.

So these I all those set of jobs and activities which are critical so here it is fifty percent chance of completing that job before thirty weeks or whatever the values which i find found out and fifty percent of chance is there that the job would not be competed. Fifty percent would not completed within that actual time so this is not the due date.

Due date is basically decided between the person who is doing or the group which is doing the project and the set of customer who will be served after the project is completed. So then again if I want to find out the delays the overall run cost again I am just repeating for the sake of repetition the things really get in very fine tune for all the students I need to use the probability concept using normal distribution or the normal deviate so what I do is that i need to find out the probability so this x is the random variable which denotes the time.

So if I want to find out what is the probability that x will be less then some D this is the D is the due date which I want to find out or which is given So it can be both given as well as considering what I want to find out why I am mentioning in point one and point two point one is considered they are the same way of looking the picture into two different angles point A if D is given.

Then I know the distribution X which has got as per the concept of central limit theorem some expected value where the expected value is here which I am circling now this is the sum of all the expected time for all the critical activities and the variances as you remember it is basically the sum of all the time.

So DS VT1 is for the first activity dot dot till the last one so considering there are N number of such jobs along the critical activity. So if i find out the square root it gives the standard deviation of the whole set of activities or the critical path then i again use the probability of X minus E to the power t.

This E to the power t is basically the sum of them all of the sums of the critical paths corresponding to thus square root under the square root you will have the sum of all the variances that should be less than equal to D minus E t divided by again square root. So again so i know E t i know variance i know D. So this becomes a standard normal deviates the as well as Z1. So i check my standard table and find out the probability.

So this value alpha i find out and i say to the team members who want an answer based on what percentage of the job would be finished within the due date. I say for example, seventy percent of the work is finished well i keep that as the set of information which i will be able to answer considering the point which i mention where D was known now consider the picture from the other point of view which is point B consider D is not given and iam told that i have to finish about seventy percent of the job well in time.

Considering that there are some deadlines consider this dead line i need to build up this whole complex which is a house and consider the people if they want to move as fast as possible considering the overall electrical system of the whole flat or the apartment is ready, the sewage system is ready, the water supply is ready, the security setup the garage parking space everything is ready.

But due to some reason they are willing to move into the flat because it is entails a huge amount of cost for residence that they are willing to move in spite of the fact that the overall boundary wall is still to be done. So i know that that boundary wall would is basically ten percent of the job level boundary wall then cementing of the overall area or trying to basically have the roads outside the apartment building that is not completed. So consider that is ten percent of the job left.

So now if i try to answer the question as per the concept which i mentioned point B is that ninety percent of the job is finished based on which i can tell the residence to come in so if ninety percent of the job is finished i need to find out what is the dead line by which i need to find out so what i do is that i again take the same formula probability of X less than equal to D where D is now unknown to me but what is known to me is that probability of X less than equal to D that probability is ninety percent.

So what i do is that i again us the concept of central limit theorem i use the concept of standard normal deviant i write the formula as it is which is probability of inside the bracket the formula is same iam just going to repeat that X minus Et which where Et is the total sum of the average time of the critical paths divided by the square root of the variance of sum of all the variances of the critical paths.

So, square root of that is standard deviation less than equal to D where D is now unknown D - Et which is again Et value. I have repeated that time and again divided by the sum the square root of the sum of the variance. So that probability is given as ninety percent. So given ninety percent is that using very simplistic concept.

Here i understand we are using a very simplicity concept but it will give you some idea that the calculation can be done i check the standard normal table from there i find out that what is the realize value of Z small z for which ninety percent is true from there. I find out z value consider z value is given as say for example 2.5. I am this value iam just mentioning to you it may not be 2.5. So what i do is that i have this value of 2.5 this is equal to D - E of t divided by the square root of sum of the variance.

So 2.5 would now be multiplied by the square root of the sum of the variance which is the noun term and i take E to the word t on to the left hand side becomes plus. So only unknown in this whole equation which iam going to highlight is D.

So D is basically two point five or whatever the value is multiplied by the square root of the sum of the variance plus the expected value of the sum of all the sums of the expected value of the critical path i find out D. So i can say to my project management team to the set of engineers see if i need to finish of ninety percent of the work for this apartment i need to finish it by this due date. So based on that i recalculate how the work can be done.

You are trying to see the picture from two different angles but this two viewpoints gives you a lot of information that what is the state of the affairs for the job whether the D which is the due date is known or whether the due date is not known now here i would like to mention two points point one is something related to the way of calculations we are doing if you remember in one of the last problems.

I did not make a decision based on the fact that D1 and D2 which are the two deadlines are known to me and based on that i tried to find out what is the percentage of the work which is to be finished between D1 and D2? i could have also taken a view point from the other point of view that i need to find out that what is the percentage of the work which would be finished for the same example, which i just stated that apartment building one that ninety percent of the job is to be needed to be finished by that dead line. So i find out D.

So consider that is D1 that value comes out to be known to me and write it down now when i present it to my team they say that no let us also try to find out that rather than ninety percent what is the due date based on which i can say that ninety two percent of the work of the project is finished then again re redo the calculations considering that now alpha is zero point nine two and i find out a new dead line D2 so these D1 and D2 are known to me are considering the fact.

That alpha in one case is zero point nine zero and the case zero point nine two. Then i place it to my team. And say like look here D1 D2 are known to me. So is it feasible. So based on that i do whole set of recalculation whether resource consistence are there whether i need to basically use resources in some other activity than try to reduce the other activities to some extent and then try to find out which is feasible now whatever i had been talking about for the last three or four minutes may not make much sense.

To the who may be the students? Saying that well you are trying to do the calculation in a very simplistic sense. Does it serve the purpose? So i will give you a very simple example say for example we all know the famous laws of newton and in one of the formula it is force is equal to mass into acceleration. So if somebody is from the commerce background is from the science background whoever is taking this course iam sure you would have done the Newton's law of motion in class ten or twelve or eleven.

So on that case we do not consider in friction. But still the results which we get using force is equal to mass into acceleration gives us very good results based on which we can do our calculations or say for example in physics we have the period simple motion of a simple pendulum and we try to find out that what is the time period depending on what is the length of the string even though friction is not considered gravitational force is not considered any in damping noise is not considered and all these things.

Still we get very good results considering that the formula which is true in theoretical sense in the same way the mythology we try to utilize or we have been discussing for the last five six minutes based on the fact that we are trying to utilize the central limit theorem and we are trying to find fit the normal distribution for this calculation the overall answer gives us a very good picture how the work is going on. So as that based on that we can go into depth of trying to analyze the project in much detail.

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So continuing in the three sixtieth slide so therefore, if you wanted to determine the probability of that the project would finish no later than thirty two weeks we would need to determine the appropriate area under the normal curve upto the completion date on or before thirty two. So if you remember average time was thirty now the actual probability.

We want to find out what percentage of the work would be finished is still that due date of thirty two. So D is thirty two. So we can use the standard normal equation to determine this probability as i mention and if you go into the concept so what you have is that you find out the probability so the probability the formula looks little bit different is not is exactly the same. So i need to find out the probability and one the left hand side it was basically X - E t by variance of t variance.

I am just writing V so that is equal to less than equal to D minus E t divided by variance t so loo look here what we have if the due date which is D here just concentrate on the right hand
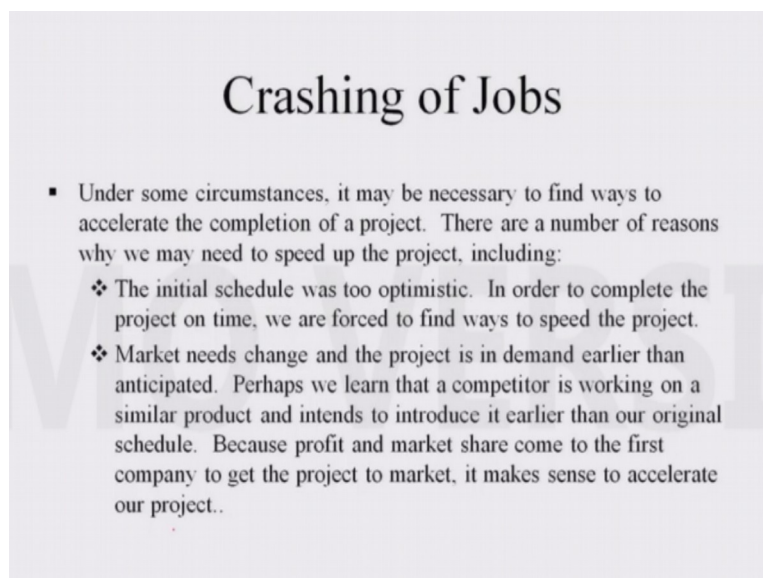
side of the probability equation .So i need to find out alpha. So D is given is due date which is thirty two which is here i have written here this expected value data completion is given as per the con and the problem which we just discussed was thirty. So thirty two minus thirty comes in the numerator which is D minus E t.

And in the denominator you have the square root of the variants. So thirty two minus thirty comes in the numerator which is D minus E t and in the denominator you have the square root of the variants which i just discussed in the last two slides. If u see the problem it was two point seven eight. So it will be divided by two seventy eight. So the value comes out to be seventy two percent which means that within the due date of thirty two number of weeks considering the average value is thirty for completing that whole project

Considering is the critical path we would be able to finish about seventy two percent of their job and then again taking the view point of B which i dint mention. You can also find out that what is the due date considering eighty percent of the job is finished or what is the overall probability of the work would be finished between the due date of say for example thirty two to thirty three and all these calculations can be done.

So now we will come to the concept of crashing the jobs first i will give you a feel in the qualitative sense and then going to try try into basically solve a problem. So under some circumstances.

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Crashing of Jobs

- Under some circumstances, it may be necessary to find ways to accelerate the completion of a project. There are a number of reasons why we may need to speed up the project, including:
    - ❖ The initial schedule was too optimistic. In order to complete the project on time, we are forced to find ways to speed the project.
    - ❖ Market needs change and the project is in demand earlier than anticipated. Perhaps we learn that a competitor is working on a similar product and intends to introduce it earlier than our original schedule. Because profit and market share come to the first company to get the project to market, it makes sense to accelerate our project..

It may be necessary to find ways to accelerate the completion of a project there are number of reasons. why we need to speed up the project and included some of the reasons are if u see the bullet points the initial schedule was too optimistic we basically plan in such a way we thought everything would be on time the resources would be utilized just arriving at the optimal time man power would be available or the resources is based on which we will to use a raw materials would be arriving on time.

So these were some optimistic thought process in order to complete the project on time. we force to find ways to speed up the project so now market needs have changed consider suddenly we find out that the prices of one of the raw materials is going to increase in by drastically in two weeks which means that i have to basically plan my work accordingly or the customer demand that the project should be delivered not on the thirtieth week but say for example it has to be delivered on the twentieth week.

So they can different consequences of that or say for example we suddenly face a technology constrain like you a digging the tandle in Switzerland and suddenly you face a huge amount of technology barrier where you need to do special type of excavating un under the abs and there the technology need is huge quantum so those changes if you are not aware they come into the picture. So perhaps we learn that a competitor other case can be competitor is working on a similar projects.

Suddenly comes soviet technology breakthrough and he or she a person who is doing the other project is progressing very fast on those lines. So it is there what iam reading perhaps we learn that a competitor is working on a similar product and intense to introduce its earlier than the original due date based on which you were planning because you wanted to flood the market with a similar type of product with respect to the competitor coming a little bit ahead of time but suddenly the overall game has changed.

So there you have to basically take a very ration and realistic thought process another can be because profit and the market share comes to the first company .so in case it is coming we have to take the decisions accordingly. So to get the project to the market it make sense to accelerate the project in time and considering all these things you have to re-evaluate the process.

So i will go in the thirtieth slide to do crash a job and then try to give a view point and how it can be done in a very simplistic sense for a simple problem at hand. Thank you very much.