**Project Management**
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**Module No # 6**
**Lecture No # 28**
**Important Example of PERT Network Involving Probabilistic time and Variance**

Welcome back my dear students. So this is the twenty eighth, lecture happening for project management and for the twenty seventh one, if you remember I did discuss in the last two slides about the concept of duration. The concept that it is a waited sum of the time based on the fact that I am taking the ratios inside the bracket.

If you remember in the three hundred thirty sixth and three hundred and thirty fifth slide, which I mentioned the ratios of the present values of all the incomes which happen divided by the price of the project and that weights is multiplied with T one, T two, T three correspondingly and I sum them up and then I also showed you that the rate of change of B which is the price divided by B is equal to minus D which is the duration in to the rate of change of Y which is the interest rate.

So that concept of how you do the calculation is not important for our course is more of a learning experience, to use the application. So anybody interested can definitely pick up that concept from good finance book that is the point one, which I wanted to mention. Point two is that you may have seen different formulas coming up for the interested calculation, how interested IRR net present value was all being used.

And I did give you a feel that how they can be used to find out the overall return of the budget and based on the fact that the budget is actual feasible or not. Now our main concern for this trying to discuss this concepts are that they should be in tandem with the concept of project management and how they can be utilized.

So rather than that trying to do any direct problems, though if we do it will be very simple in nature it will be more interesting to see how this concepts are utilized in the project management.

So let me continue and try to finish of the concept of the different type of the unusual concepts based on which the project can be evaluated so the next point which is there in the slide in front of me.

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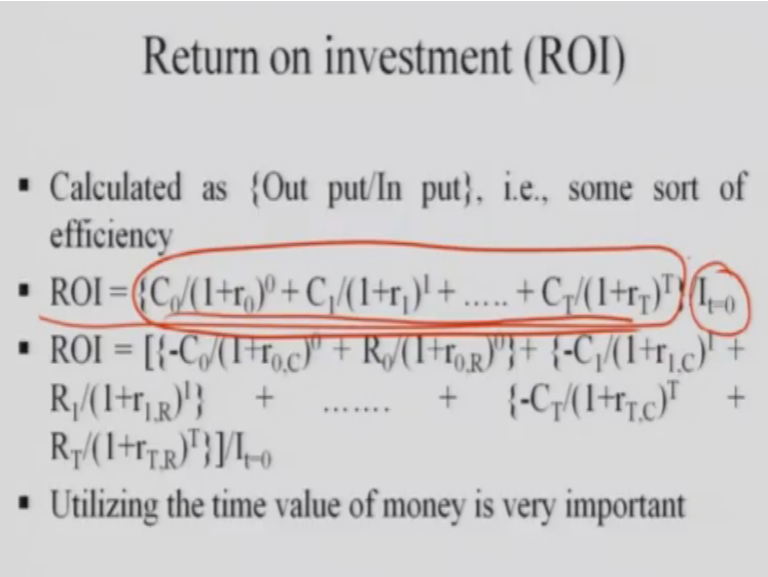


Which is the three hundred and thirty seventh one, is the Return On Investment which is ROI. So calculated technically Return On Investment means, basically some sort of efficiency calculation which given by the ratio of output by input. So what is my output divided by quantum of input would give me the ratios which is the efficiency.

So if I am calculating my return on investment based on the fact that I have only the overall quantum investment, has happening at plus and minus at any point of time t is equal to zero t is equal to one, two, three, four till the last value of t is equal to capital T and the input output cumulatively at these point of time is given by T suffix zero C suffix, one in the last value which is C suffix.

Then the formula which is given in front of you, which is the second bullet point is basically the Return On Investment. So the numerator which is I am circling now is basically the time value of money as of now. So all investment had taken place at different points of time, so those points of times are t zero, t one till t capital T and what are the investments improve also which is happening at those respective points they are C zero till C ten.

So I find out the time value and what is my overall investment happening that have to again calculate and find out which is happening at I zero. So this is the I zero which I have I, T is equal to zero in the suffix. So now you may be thinking that why have I taking only one time. So I am considering very simplistically that all my investments are happening at the bulk amount.

It is happening at I is equal to I time T zero. So if there are any independent payments or investments which are happening, I can find out the time value of those investments also and then add up and use them in the denominator. So I did not do that because I wanted to basically explain that in the qualitative sense.

If you go to the third bullet point which is the Return On Investment based on the fact what are the revenues and what are the costs and again if you remember, I did mention about interest rate being different for the investments and the loans. So the investments loans are different interest rate.

If you remember, I did mention about the concept of demand and supply. So here the interest rates are given for the cost as R the suffix zero one, two, three are for the time and comma C means for the cost similarly, if the suffix is R in place of C they are the revenues. So the first term is minus C zero divided by one plus R zero comma C to the power of zero it is the cost as of now time T is equal to zero.

Similarly minus C one in divided by one plus one comma C to the power, one for the present value of the cost again being calculated at the time T is equal to zero and all the negative values basically signify that if I go to the positive terms which is basically R zero divided by one plus R zero comma R to the power zero.

Similarly the second term is R one divided by one plus R one comma R to the power one, corresponding all this thing so add up all this value divide by investment which I again consider happening at time T is equal to zero which is I, T is equal to suffix and then I find out the Return

On Investment. So Return On Investment in very simple sense if you remember in the first part of the course.

We were doing the decision problems I did mention that you can basically rank them or find out which route to take, if you remember clearly that we were considering the problem from right hand side trying to find out what is the expected value compare the expected value and take the decision, where the expected value was the highest and based on that we proceed one step at a time, till we reach the so called source where the decision was being taken.

And that was done for the moppet problem that was one for the oil rig problem. Now if you consider the concept of the Return On Investment it is slightly different in those problem of decision analysis. We only considered the expected value and if you remember clearly I did mention in the vague end of the word document where we were solving the problem for the rig problem or the drilling problem.

I did mention that in case if the expected value are the same, what you will do is that you will take the ratio of the expected value to the variance or the dispersion rank them to the highest to the lowest and take the one which is the highest or else you will try to find out that the ratio of the risk to return rank them to the lowest to the highest and take T H one which is the lowest.

So if you see the Return On Investment is exactly the same, so that the numerator is basically the outflow and the denominator is basically the amount of inflow, in the sense the amount the money which you have invested in the negative sense. So obviously loss I am considering as negative of profit or profit I am considering as negative loss. So if you are careful with the sign the confusion would not occur.

So again what you will do is that for the return on investment you will rank them from the highest to the lowest considering the efficiency is where the output is on the numerator input is on the denominator. We will take them in such a way that you get the highest value and if you are trying to basically take the other way around rank then with respect to input to the output you will reverse the ranking take the one which is lowest.

In the other way around like efficiency wise now, why I am mentioning is that again I just try to pause because I thought and I think students apart from doing the course should also think laterally so there are different techniques of non parametric method of ranking and one was that which we did discuss which was the AHP.

One there is a concept of data enveloping analysis which uses simple optimization tool or work tool very simplistic sense and that gives you excellent results, so there we considered the ranking of different decision making units which are the DMU such a way that you find out the efficiencies based on the fact that in the numerator of the output in the numerator, denominator of the input based on the you try to find out what are the efficiency and rank them.

And in case if you are trying to reverse the equation which is input, the outputs then you do the ranking in reverse detection still you get the exact results and they are very heavily utilized in deal. This method is just for the interest of the interested readers utilizing the time value of the money is very important based on the fact you can find out the Return On Investment.

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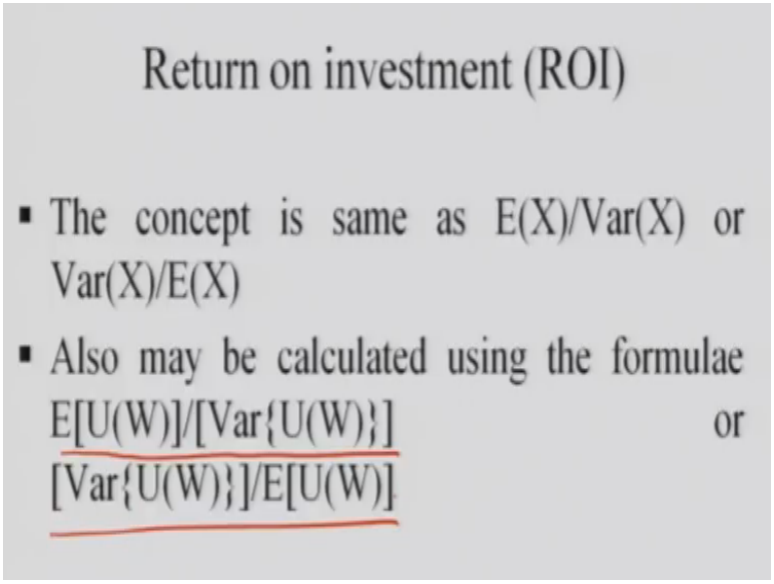


Now this concept which I just mentioned, is that TH concept is exactly same in some sense to the ratio of expected value by variance or the ratio of variance to expected values. So used in the just opposite sense also we may be interested to find out and calculate using the formula of expected

value of the utility and variance of utility because in any decision if you consider even though practically it may not be feasible.

We may be tempted to find out what is utility, what is the net worth of the decision from a project, from a decision, from a gamble, from a certain events, whatever it is so we find out that expected value of the utility divide by the variance of the utility rank them from the highest to the lowest. Take the highest one if you want to rank them from with respect to the variance with respect and divide by the respected value of the utility you rank them from lowest to the highest take the one which is basically the lowest

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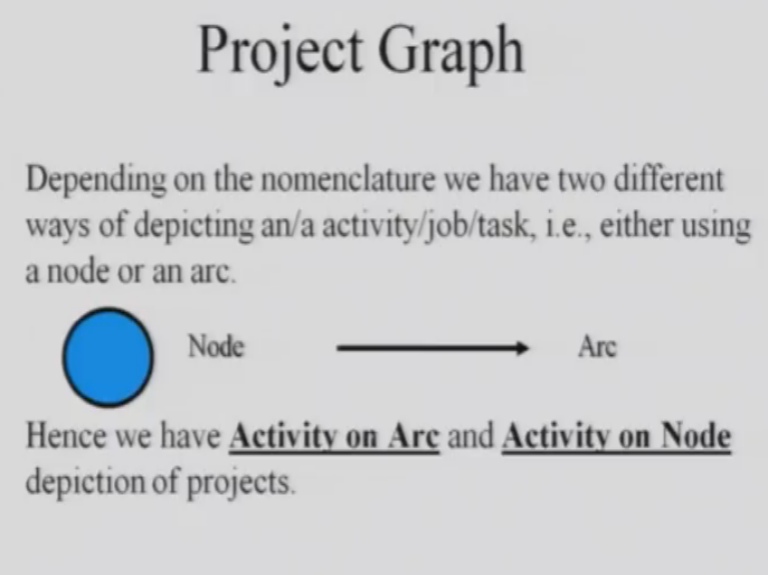


So I will again come back to this project graph and if you remember we did do one of the problems in projects. So it may be a repetition but I strongly urge the students to go through this slides, which I will be discussing because it gives you a different picture that how the concept of variance or the concept of expected value will be utilized to calculate the probability of trying to finish of our work within a certain time frame.

So depending on the nomenclature we have two ways of depicting, if you know definitely have gone through the other slides and I have been mentioning the time and again which is the activity on arc and activity on node which is AOA and AON concept. So here is the node which is showing as the blue circle and arc is the black arrow which we have.

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PERT Example (AON Network Diagrams)

| Immediate Activity | Description | Predecessor |
|---|---|---|
| A | build internal components | ___ |
| B | modify roof and floor | ___ |
| C | construct collection stack | A |
| D | pour concrete and install frame | B |
| E | build hi-temp burner | C |
| F | install control system | C |
| G | install air-pollution control device | D,E |
| H | inspect and test | F,G |

Now consider the pert concept a simple another example based on activity on node network diagram the fist column on the immediate activity which you have in front of you which is A B C D E F G H till the last one and the description are given. So it is basically trying to build up the system where you build the internal component modify the floor and the roofs then build up any air instruction.

So you are trying to basically implement a system for air pollution and corresponding structure. So description are given in detail in the second column and the third column basically has a predecessor which means that before C, A should come or before E, D should come, if you see the last one before H F and G should come. So based on that we will first draw the diagram

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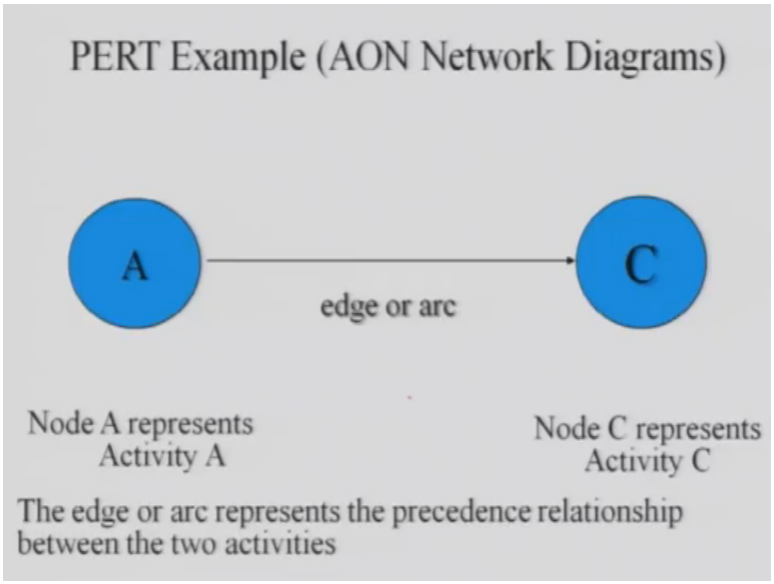


So node a represent activity A node C represent activity C. So this is edge on arc and the edge or arc represents the precedence relationship happening between two task or activities or job

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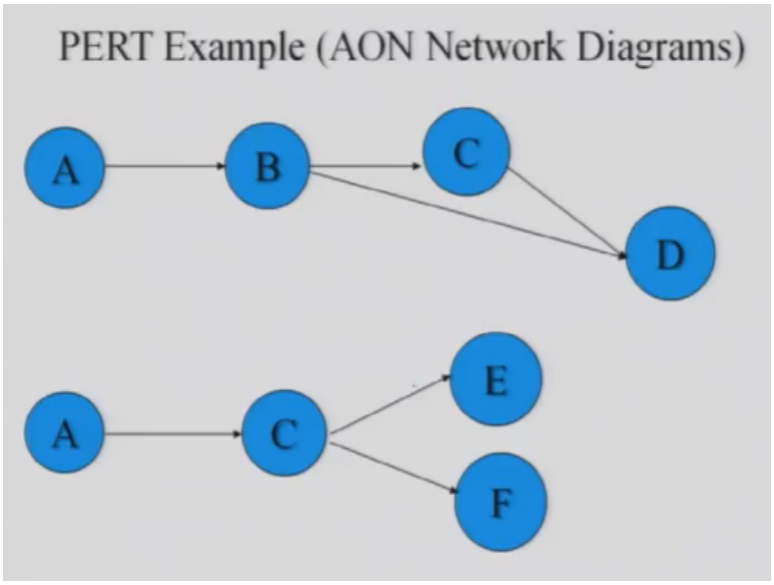


Now in this diagram I have not started the problem in this diagram. So these are the different sequences of jobs you may have of A leads to B, B to C, C to D also, but it cannot be completed until unless B and C is finished in the second set diagram which is there in slide, three hundred and forty two. It means that A leads to C and then C leads to E and F in some sequences. But it has it gives us no relationship between E and F. So E and F can end at any certain point of time provided they are being followed up after C

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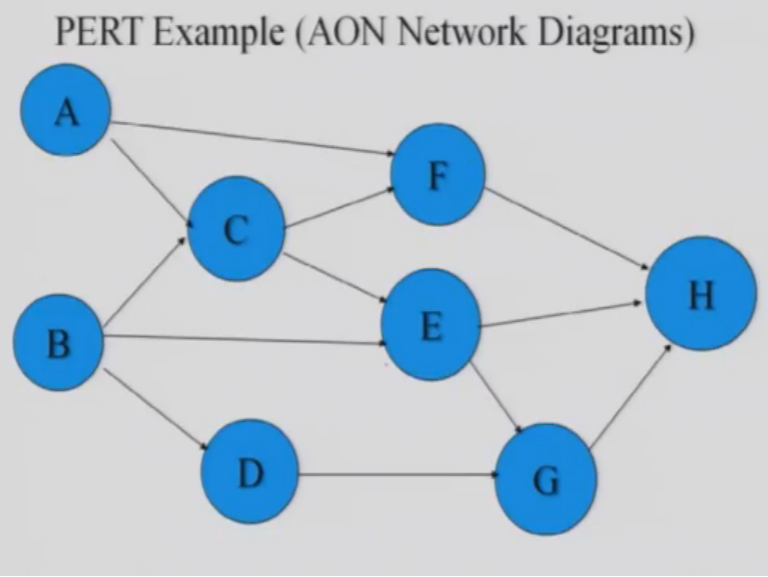


So in this simple diagram we have basically given that A and B is basically the so called starting nodes. So this I want to point out very clearly. So if you remember technically its always best to have one source and one sync because based on that we can build able to time the overall project in the best possible manner means.

We are able to find out the time duration of the project, time duration of the activity, time duration of the critical path, time duration of the non-critical path in the sense that they give us the exact answer rather than having two different nodes of starting or two different nodes of ending. So in this problem we have two different nodes for starting. So A leads to C and F, B leads to C and D and E. And if you follow that set of nodes and arcs which is there C goes to E F and D goes to G and after F E and G finishes.

You basically end the overall project which is H if you note the right side of diagram which is on three forty third slide means that I am trying to follow the norms as per the nomenclature, where there is only one node which is the sync which basically ends the task if we consider the left part of the other diagram which is this slide which is the three forty third slide it means that I have basically two different sources of start.

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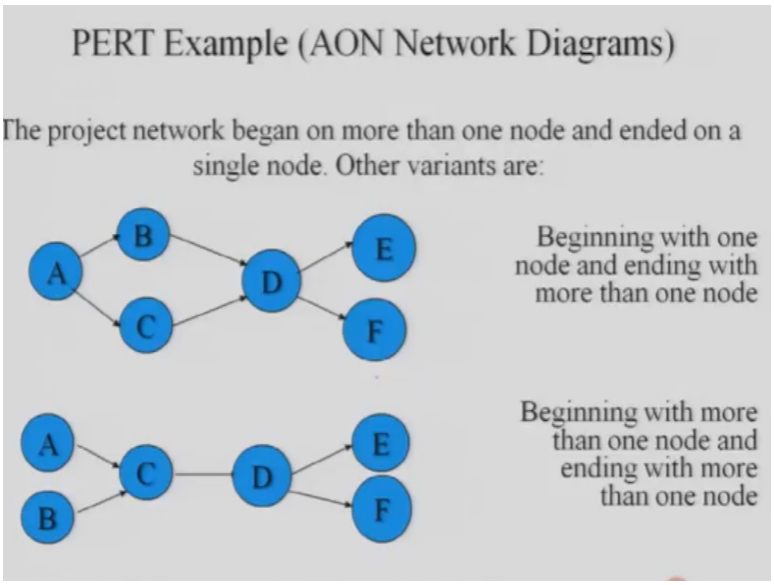


The project network began on more than one node and ended with single node, other variants which if was in three forty third, in this one if I have a project is happening in this three forty fourth slide project is starting at A but ending at two different nodes which is E and F. So it says beginning with one node and ending with more than one node.

And in the in the second diagram in three forty fourth it was someway related to three forty third. We have two different node which is starting and two different nodes E and F which is ending so hence it is written that beginning with more than one node and ending with more than one node.

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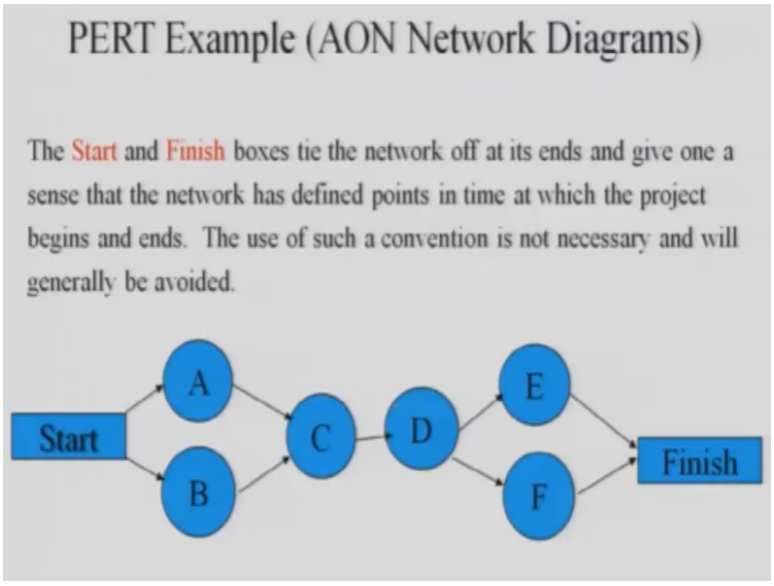

The start and finish boxes, so in this may be repetition but I still request the candidates and the participants of this course to please go through what has been discussed because we are trying to cover lot of different concept in a very simple way such that at least kick starts the concept in the candidates and they get interest and read the books. If you remember I given a lot of good reference books which are generally used.

And if you search the market you will get the hardcopies of those books at a very cheap price and if you search the net I am sure you will get the soft copy of the PDF format of this book. The start and finish boxes tie the network off at its end and give us a sense that the project has a starting point, has a ending point. So if you remember the last diagram which we consider in the three forty fourth slide, it had start A, B, so they were being combined a start and ending of F is combined for finish.

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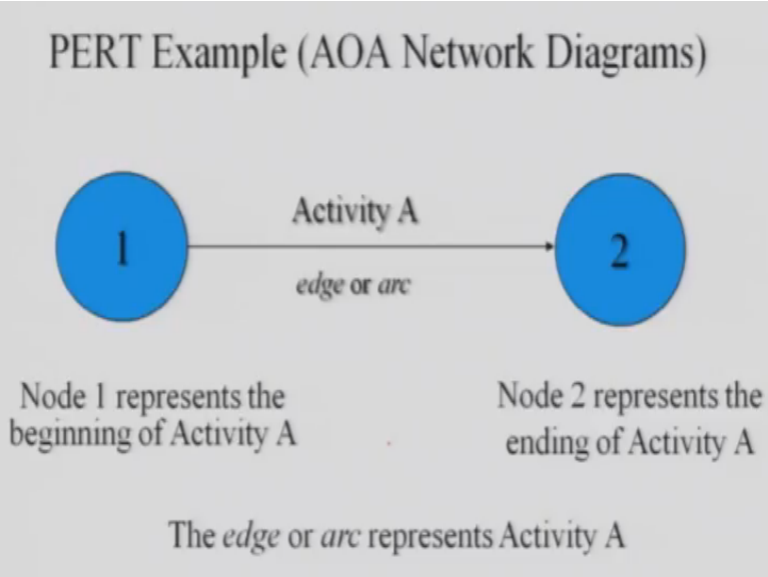


So now node one represent activity A node two represent activity A. So now the activity is basically being convert, if you see the two diagram it is AOA which is an activity arc, the other one was basically activity of nodes. So whichever you do you be very careful that once you start following the concept use that throughout the diagram.

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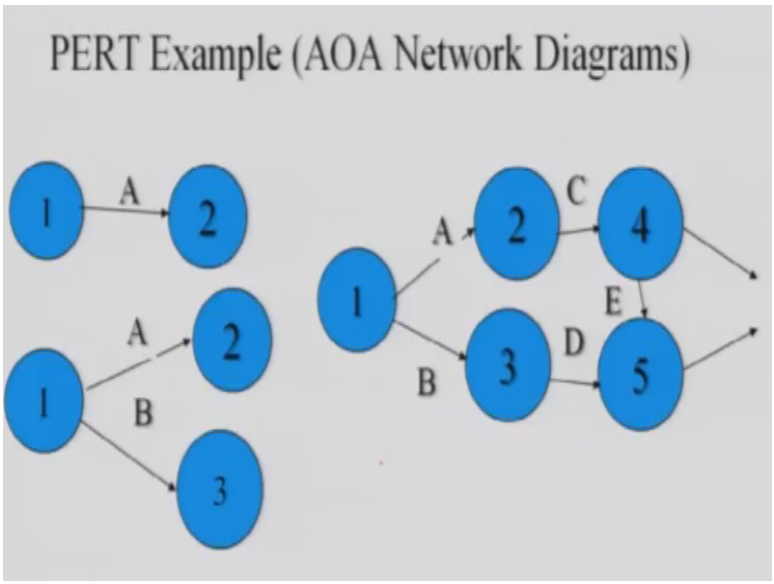


So these are the examples of activity on arc one leads to two. So that is activity A in between in the second one it is one to two or one to three basically needs A and B, then in third diagram one to two, one to three, two to four, three to five, are basically a b c d and between four and five it is four and then you can continue.

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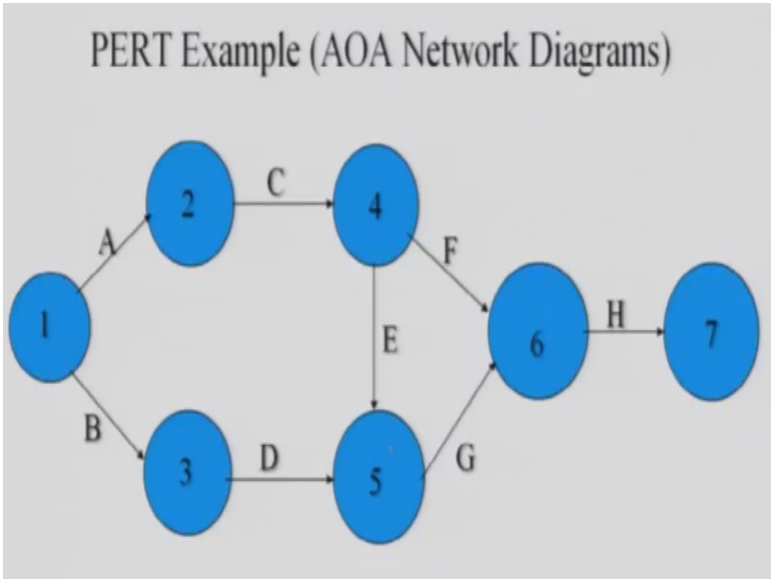


So this is another way of triangle to show the diagram. So here basically one source and one sync. So now you basically implement that accordingly. So this is activity of arc so the activities are subsumed and being shown in the arc it is not on the nodes.

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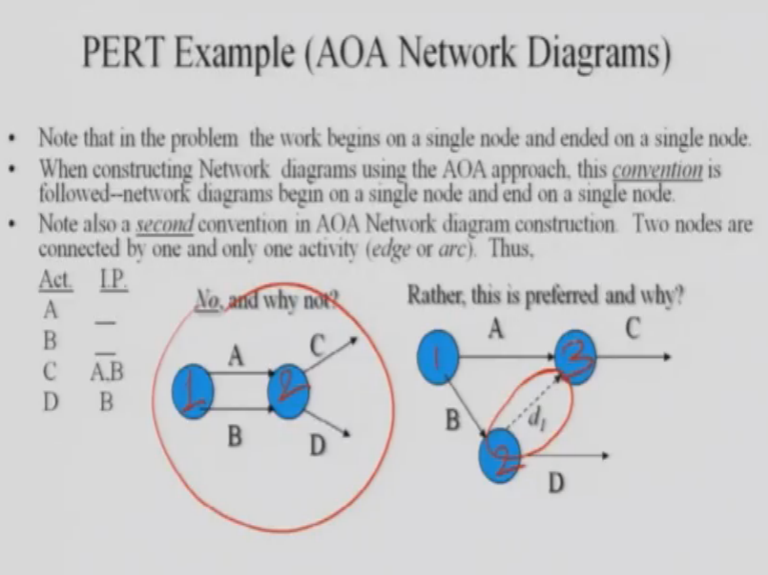


So known that in the problem begins on the single node ended in single node. So when constructing the node and the AOA and AON, you should be careful. So if you see the first diagram which I am trying to basically highlight. Now it basically has two different arcs A and B going from so called the node one and two which is not marked. I am trying to mark for your own convenience.

So this is not allowed because here it means that two activities are basically same, started the same thing which is not allowed. If you remember one of the earlier assumption based on which we started the factor C point. So rather than we try to basically bring a dummy activity in between the jobs which I am trying to basically or the node I am trying to mark as I am A, B and one, two and three it can be done accordingly.

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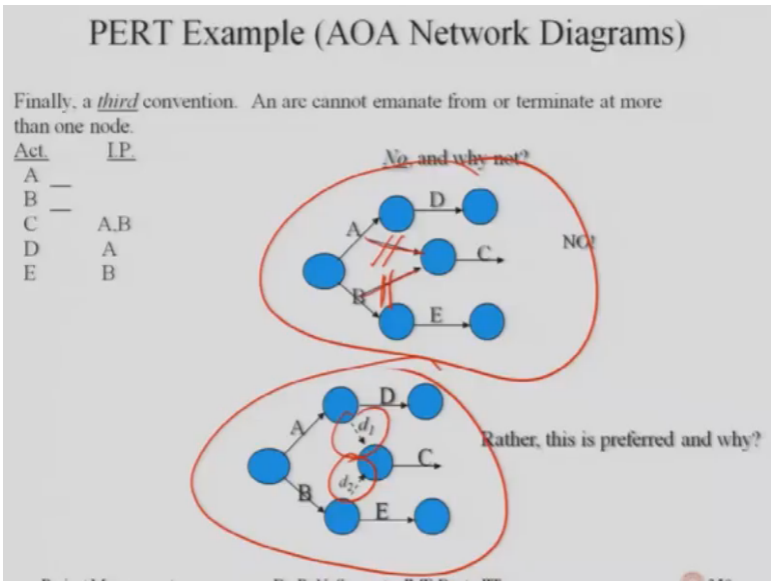


Here in this case two different dummy activities are being considered which is D one and D two. In this figure in second set of figure and if you compare with the first set it means that the activities which are node I am marking with the lines, they are basically starting somewhere within D and somewhere within E which would be very confusing implement.

So all the precedence diagram are giving on the lift as I will leave that to the reader or the class people or people who are doing this class to see and make a comparison what I was discussing.

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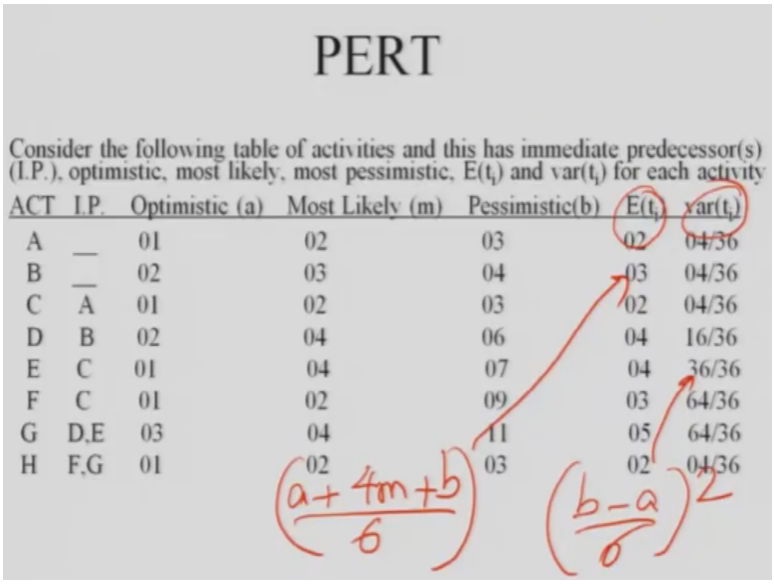


Now let us go consider the following table activities which are given so the activities are given activities starting from A to H and the precedence are given in the second column the optimistic

time which you remember the optimistic, the pessimistic, the most likely time, based on which we will do the calculations is given in the third column which is the optimistic time A the fourth column which is the most likely time M and the pessimistic time basically the B which is the fifth column.

So now I need to find out the expected value and the variance if you know the expected value it is basically A plus four M plus B divide by six. So this is the expected value if I want to find out the variance it is B minus A by six whole square. So this is that variance, so variance and the expected values have already been calculated and they are given in front of you.

So I am not going to do the calculation please pause, see the three thirty fifth first slide and giving A column which means the third column B which is the fifth column and end column which is the fourth column. Do the calculations to find out the expected value and the variance.

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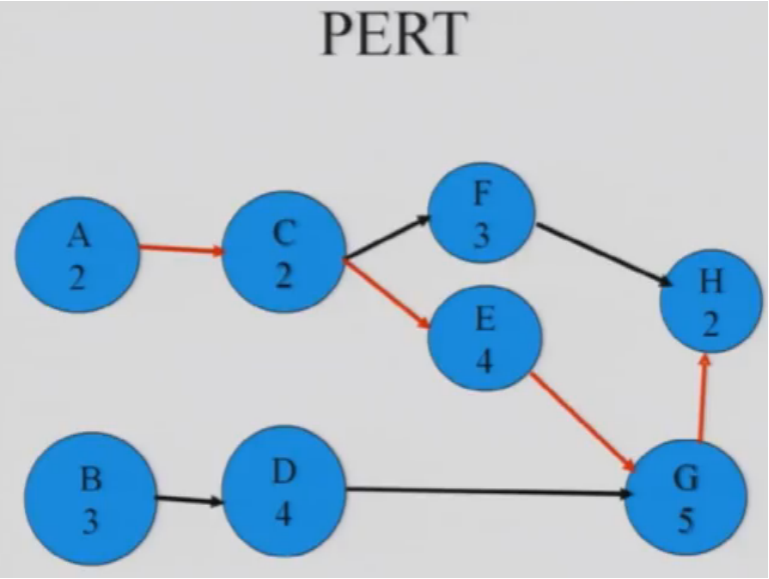


So I have drawn this graph based on the precedence diagram do it yourself you will find out but only one thing which I want to point out is that there are two nodes starting, so that should be avoided. So in order to avoid that and also keeping in mind that the red line which you have in front of you which is joining A to C, C to E, E to G, and G to H is basically the critical path you can do the calculations then find out the critical path. So that I will leave that to the candidates.

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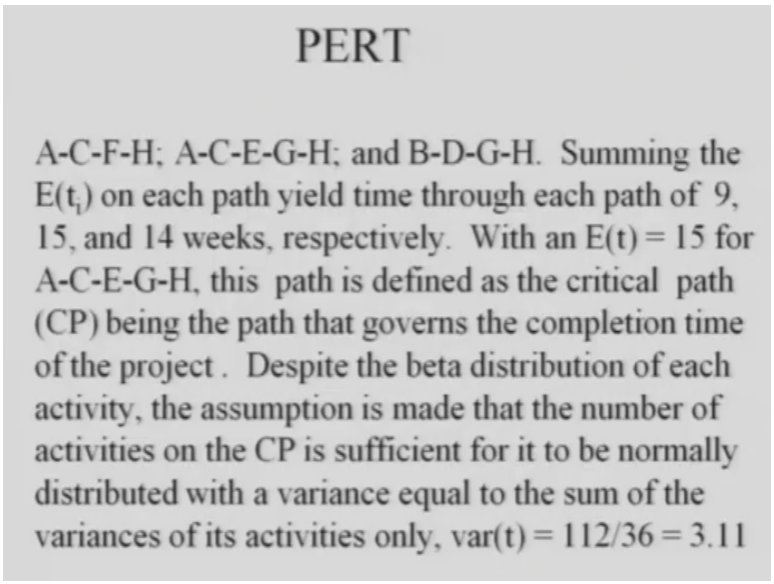


So A, C, F, H, A, A, C, E, G, H, B, D, G, H so if you sum up the expected value on each path yields that so each path the time durations are given as nine fifteen and fourteen weeks the expected value of E,T is fifteen for A, C, E, G, H path. So this path is defined as critical path as I mention.

So despite the beta distribution which you have of each activity assumption is being made the number of activities in critical pathway is high and variable will be the central variable theorem. So for which the variance we need to find out, so how we will find out the variance. So let me repeat it you add up the variance or all the path which are there on the critical path.

And that is sum the variance if I want to find out the standard deviation on the critical path add up all the variance. Let us find out the square root, so this I have already said few times but I am just repeating for the benefit of the candidates so now I basically converted if you remember in the three thirty fifth slide the activities were there on the nodes now I have converted the activity.

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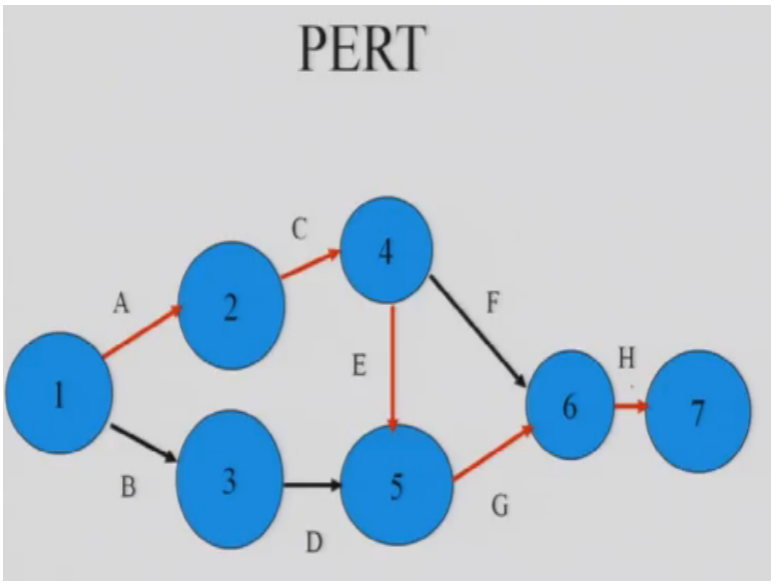


It is on the arcs, so now there is only one starting node which is one so it is not A, it is one so again if I follow the critical path is exact with the same A, C, E, G, H exactly as similar to the diagram which I showed in the three fifty two slide, so these are is basically PERT concept based on the fact there the activity on the arc and that one was activity on the nodes.

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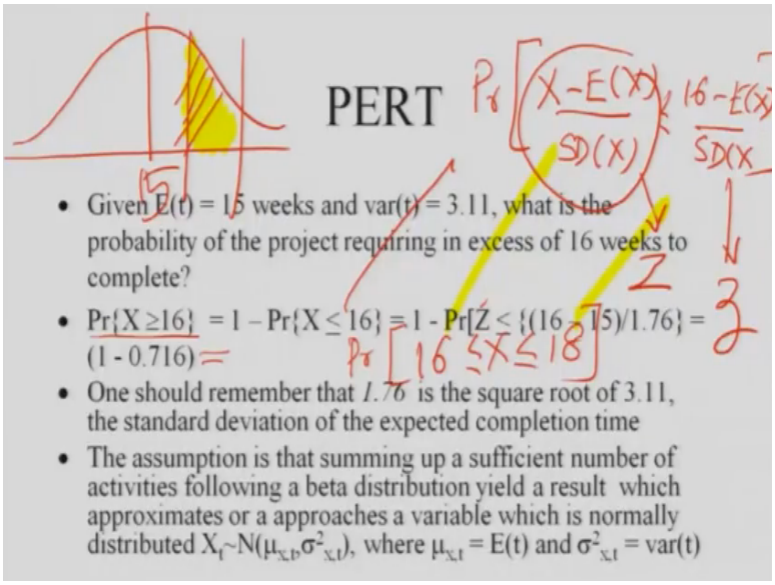


So given that the given expected value is fifteen weeks I find out the variance which is I just did in the three fifty third slide. So variance comes out to be three point one one which is one should remember. So before I go into the first bullet points let me go to the third one should remember that one point seven six is the square root of three point eleven the standard deviation of the expected completion time.

So now we have this average value given as fifteen and the project duration on some dead line based on the fact of what has been mentioned by the customer or what you think is feasible what is the actual timeline based on which you are doing your word depending on your different constrains A, H if you think that any time excess of sixteen which is problem for you. So you try to find out that if start your project you have proceeded sometime in the project.

You want to find out that what percentage of the overall project is to be completed within that sixteenth week, provided that the average time to complete the word depending on the PERT concept is fifteen weeks. So again you will simply go to the simple calculation remembering the facts that even it is beta distribution for each activity cumulatively when you take them using central limit theorem it becomes a normal distribution.

This I am trying mentioning time and again because this is a very important concept which comes in project management and it comes in every sphere of our life in different type of calculations. So I want to find out that what percentage of the job is to be finished. If it is greater than time period is greater than sixteen. So this is a probability will be one minus probability x less than sixteenth.

So what I do is that I have this calculations x minus the expected value of x divided by the standard deviation of x that should be less than equal to sixteen which is the duration minus expected value of x divided by standard deviation x which is a very known equation which we have been solving. So this gets converted into standard normal deviate Z which is here and this gets converted into a small Z which is a realest value which we know.

So what is the realize value sixteen minus fifteen which is one divided by the standard deviation what is the standard deviation if you remember it is one point seven six which is the square root of three point one. So that value we can find out of depending on what minus zero point seven six it will be giving very rough value it will be something less than thirty percent. So about twenty eight point some value of the work is still pending.

In case the work project has exceeded the sixteen weeks, so I want find out what is the positive implication. You have to go through the contract see what is the overall cause value which is incurring so what is that twenty five point eight percent I am taking a value of twenty eight percent roughly, so we will try to find out. So what activities are left what is the overall cost complication? For that are they very high value item? In the sense is do those activity take a lot of time.

So always it means the cost complication is very high added to that you will all always have the fact that if you have exceeded the duration of sixteen, obviously there will be penalty. So is the penalty on linear case is the penalty on a non-linear case. So all this implication will come into the picture.

So now if you remember also that if I want to find out I am again repeating a small set of problem which we have done earlier in one of the lectures. So consider that I want to find on what is the probability over? What is the portion of work? Which is possible to do in a time frame of say for example sixteen to eighteen week? So what I will try to find out is that the probability that x which is the time duration x is greater than sixteen but less than eighteen.

So we will do the calculation problem, so if I go to the normal distribution even though I have not shown that I am trying to show it here so this is the fifteen value average this is sixteen this is eighteen I need to find out what is the overall area inside this graph so this you can do using easier integration but for this always standard normal value has a table doing it so again convert the sixteen into the standard normal eighteen into standard normal.

You can find out the probability so assumption is that the summing up of sufficient number activities follow beta distribution yields a result which approximates or approaches the variable which is normally distributed and as XT is given in normal distributed sum in on standard deviation or of variance. So this is again being relatitive for the benefit of the candidates who are doing this class thank you very much and have a nice day.