**Project Management**
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**Module No # 06**
**Lecture No # 27**
**Concept of Forward Rates and Payback Time**

Very good morning, good evening and good afternoon to all my students, so wherever and whichever part of India you are, hope you are enjoying this course and I am Raghunandan Sengupta, faculty member in the IME department at IIT Kanpur, India. So the course as you know is basically on project management and for the last two lectures this is the twenty seventh lecture of half an hour each.

So we are discussing that how to find out the interest rate given the zero rates and corresponding values. So if you refer to the twenty sixth lecture so the interest rate for half a year which is point five for a year, then one year that is one point zero year, one point five year, two years were given correspondingly and based on that we trying to calculate.

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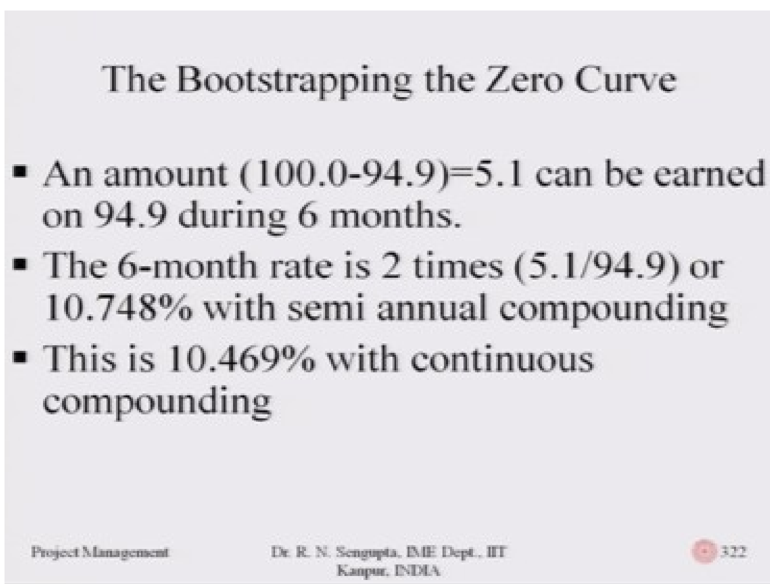



And also it is mentioned that if you see this three twenty second slide, it was mentioned that how the payments were been done. Whether they have been paid in each quarter, whether they have been paid in semiannual basis, annual basis and corresponding values. So going for the second set of calculations, so now if you see the table, so the face value of so called payments is hundred and ninety four point nine was the actual value which you would be getting and earned corresponding to the concept that there would be six time payment happening two times in a year. So if you see the bullet point it goes exactly like this.

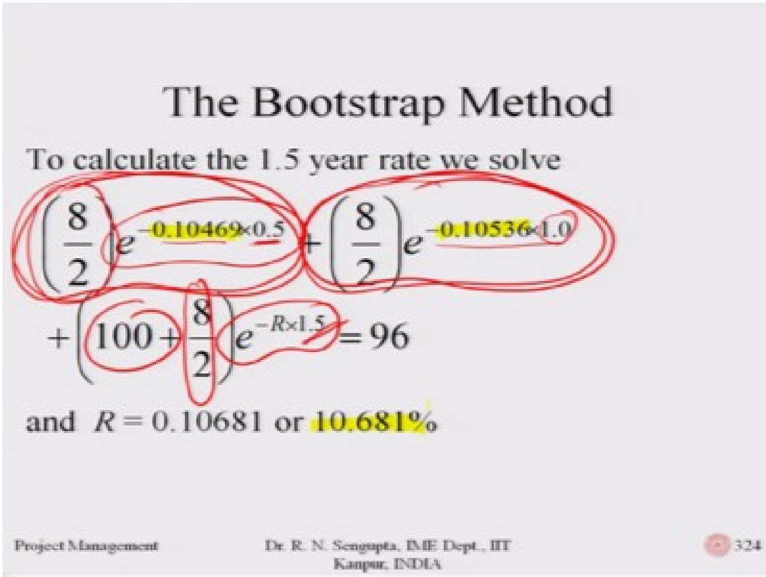
An amount of five point one which is the difference between hundred and ninety four point nine can be earned so that amount can be earned based on what value. So that value is ninety four point four so if I want to find out so called ratio of interest it would be five point divided by ninety four point four. Now the next question would be is it been paid on an annual basis. The answer is no because you will have such two payments happening in one year. So the next bullet point is exactly what it says.

The six month rate is you have to find it out with twice into this amount of 5.1 divided by 94.9 which comes to 10.748. Now this is being semiannual compounding, so if it is semiannual compounding you have to find out the continuous compound rate using the formula which we have discussed in the last class. I have just mentioned that slide so if somebody is interested they can check any basics finance book to understand how this calculations are done.

Based on 10.748 which is the semiannual compounding rate we find out the continuous compounding rate is 10.469 percentage. For the third set of values which is given in that table so you earned ten rupees or ten dollars or whatever it is, based on the fact that for a face value of 100, 90 was basically the value which was earned.

So ten can be earned on a value of ninety the time duration is one year which means the twelve month interest rate would be because there is only one payment happening in one year so it will be one multiplied by the ratio of what is the amount you are getting divided by so called input, price or what is the actual value you are investing, so that is 10.0 by 90.0 so the ratio comes out to be 11.111 interest rate is being calculated on an annual compounding basis so if you covert it to a continuous compounding basis the value comes out to be 10.536.

So we have found out the continuous compounding interest rate for one fourth of a year, for half a year, for one year. So these values have been calculated, based on that now we will proceed. **(Refer Slide Time: 04:28)**




So now if you see need to calculate the one point five year that is one and half year interest rate, now the question if you check the table, I am not going to go back to the table you can refer back to the table in your last class which was the twenty sixth class.

So it says that eight rupees, eight dollars, eight euros whatever was there was been paid on six month basis which means that I will get or the person who is going into the investment would get 8 divided 2 per sex months but this six months has to be calculated based on the fact that what is the present value of that amount which accrues to me as of now.

So if you see the first value on this slide which is the three twenty fourth slide so as I mentioned eight by two is the amount. So when it's been paid you should ask your question so it is being paid after six months as of today. Now the next question would be, what is the amount of that value as of now so you have to basically find out the time value of money which means it is continuous compound which you have just found out which was 10.469.

You basically multiplies with e to the power minus because as you come more nearer to time T nearer to zero the value will decrease so it is e to the power minus 0.10469 which is the interest rate multiplied by half which is zero point five is the time period. So based on that you will find the first term which is the actual value for the so called four rupees which you would have got or which you get after six months.

So the next calculation would be based on the fact that you will get another eight rupees, eight dollars whatever it is divided by two obviously sorry my mistake because it is being paid two times a year. So this four rupees would be obtained after another one year but in this one year time period this continuous compounding rate is different what is that, that is 10.536. So again you multiply e to the power minus 0.10536 into one because now the time period is one year.

So this value which you get is actually the value which you would have got now provided no four rupees was paid after one year. Now the last term which you have to find out on the equality side is given the fact that the value was hundred rupees so that you will get back plus the interest rate. So what is the interest rate again the answer is eight by two because it being paid again after six months.

So for the one point five years' time period first six months this value you have got next six months this is the value which you have got. So out of after one point five years one and half years the interest rate this plus the principal amount. Both are being calculated based on the interest rate which is continuous compounding for a one point five year period.

So hundred is known to you, eight by two is known to you, if you see the e to the power terms so minus R, why R so this is the continuous compounding interest rate based on the fact that is true for a time period one point five years because that one point five years is coming here. So based on the fact that the actual value which you have got would have obtained is ninety six as per the values in the tables so when you equate that find out R value it comes out to be 10.681.

So now what you have got is this, this value which is 10.469 is the continuous compounding interest rate for half a year, 10.536 is the continuous compounding interest rate for one year and 10.681 is the continuous compounding rate for one point five years.

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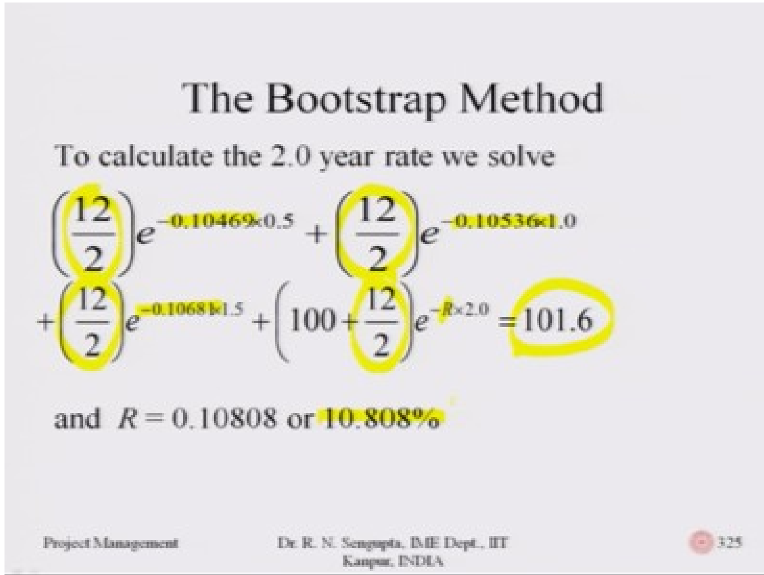


Now I need to find out the continuous compounding rate for two year period. So what I do is that, I again go back to the calculation check what is the interest rate being paid, now it is twelve rupees, twelve euros whatever it is on a half a yearly basis.

So the interest rates are being paid if you note down careful it is after six months, after one year, after one point five year plus after two years also but at the end of two years apart from interest rate you will get the so called principal amount which was hundred. So let us do all the same set of calculations as we have done in the previous slide which was three twenty fourth slide.

So the three twenty fifth slide are exactly the same based on the fact that the concept is what we are trying to follow is based on the same set of formulas. So this twelve by two which is appearing four number of times the first one is after six months I am repeating it please bear with me, the second twelve by two is after one year, third twelve by two is after one point five years and fourth twelve by two is after two years.

So now if I check the interest rate they are basically being obtained from the table as well as from the calculation which we have just finished in the three twenty fifth slide. So 10.469 is the continuous compounding interest rate for six months, 10.536 is continuous compounding rate for one year and 10.681 is the continuous compounding interest rate for one point five years.

So now we do not know the continuous compounding interest rate for two years, we consider it as R, equated to the value of 101.6 as given in the table. Solve this equation get the value of R the R value comes out to be 10.808 continuous compounding interest rate for a two year period.

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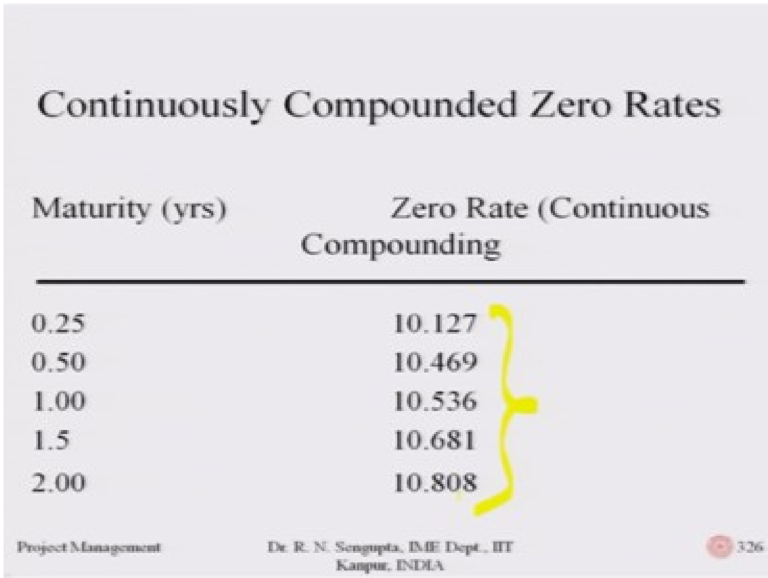


Now if you note down the values in the table so the first column is the maturity in years so they are the zero interest so if you want to find zero interest continuous compounding for the time period. So again I am repeating so the time period are such that you start your clock ticking at T is equal to zero stop it at T is equal to 0.25 and there the continuous compounding interest rate is given as the first value in the second column which is 10.127 so in between there are no payments.

Then if I go the second set of values, second row it is for half a year it is 10.469, for one year it is 10.536 it is the third row and corresponding to the values in the fourth and fifth row, they are for one and half year it is 10.681, for two years as we just calculated on the three twenty fifth slide it is 10.808, so we have got the zero rate continuous compounding.

If you want to find out the compounding of the simple interest whatever it is for those time periods you can do that using the simple formula which we have shown that how to convert any compounding interest rate to the continuous compounding interest rate and vice versa.

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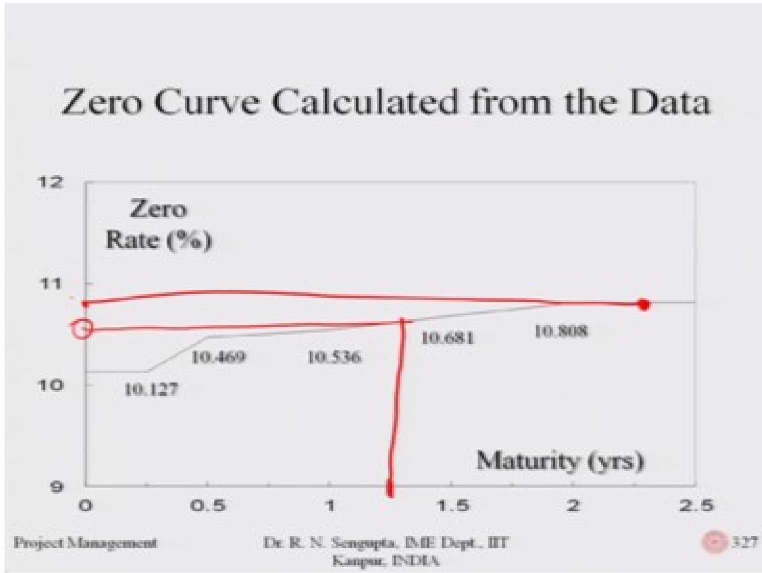


So I plot this values with maturity along the x-axis with the interest rate being plotted on the y-axis, so remember this is the interest rate based on the fact is the zero interest rate intermittent payments for the projects are not there. So of I plot the value and in a very simplistic notion what I can do and why it is important is that say for example I want to find out what is the continuous compounding interest for the project and I want to have some feel that should be the actual value.

So if you go along the x-axis mark a value one point two five years go vertically up where it cuts that graph zero curve go again on a left to the horizontal direction and the value which you find out can be used as a theoretical value based on which you can do some calculations and find out that what would be the projective turn and so and henceforth.

If I want to find it out say for example for two point two five years again I would go vertically up stop at the point where it basically marks along this curve and then go horizontally on to the left I will come to one value along the y-axis which is the zero interest rate use that value on my calculations.

So if my values when I do my calculations are very close and if I have more number of values trying to basically fit a better curve in place of this set of straight lines which I have for this curve for the zero interest rate would be much more useful in trying to do our calculation. So one word which I did miss that even though I thought it is not important but I will just try to mention that even that is nothing to do with your concept which you are trying to cover for the project management.

PERT is that if you remember in the three twenty sixth, three twenty fifth and those previous slides the word which was there on the top of the slides was bootstrapping, bootstrapping means basically means that you are wearing high heeled boots till you your knew and as you basically tie the knot you come up from the lowest level and then keep tying it up till you are able to secure your boot in your legs.

So it means that as you go up and do the calculations some eternity methods have to be used in order to find out the concept that how you can find out the interest rate and bootstrapping is just for the interest of the candidates and students is very heavily used in the statistics and stimulations different concepts of bootstrapping which can be used on a cuticle frame work to find out any good results, that is from the statistical point of view just I want to mention that.

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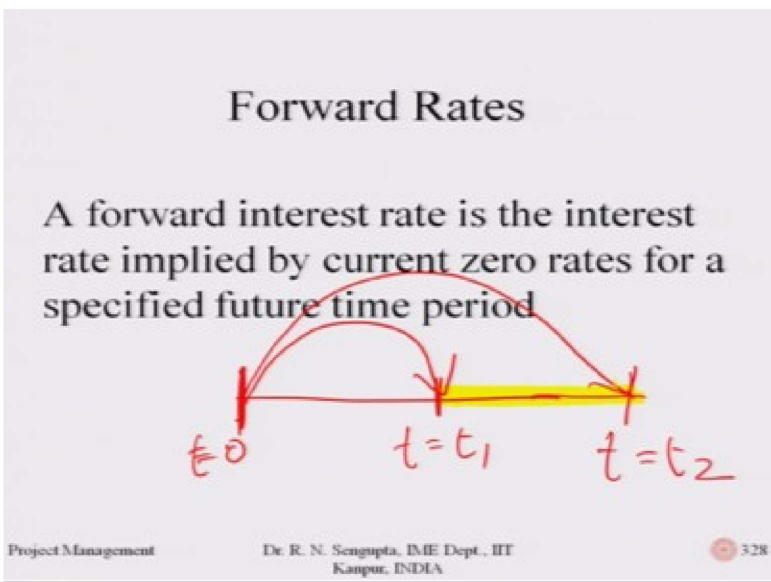


Now I want to find out the forward rate so given the all the calculations which we have on the zero eth rate my main intention is to find out the forward rate so this is the interest rate implied by the current zero rate for a specific future time between two zero interest rate. So let me draw a simple diagram and I am sure it will be clear to all the candidates.

Consider my t to the power zero is this point which I will mark consider this is t = t1 and this is t = t2 so I have the zero interest rate from zero to t1 also I have the zero interest rate from zero to t2 and correspondingly all the zero rates are there. So what is of major interest rate interest to me and I want to find out is that what is the interest rate, which is applicable if I start my clock at t is equal to t1 till the time t is equal to t2.

So if I have some interest rate happening for the project between the time of t1 to t2 it would be essential on my part to have the note that what can be the actual or probable values of the interest rate happening between the time so that I can do my calculation and understand what is the overall cost implication for the project in a much practical sense so, this is what we want to do given the interest I want to find out the forward rates.

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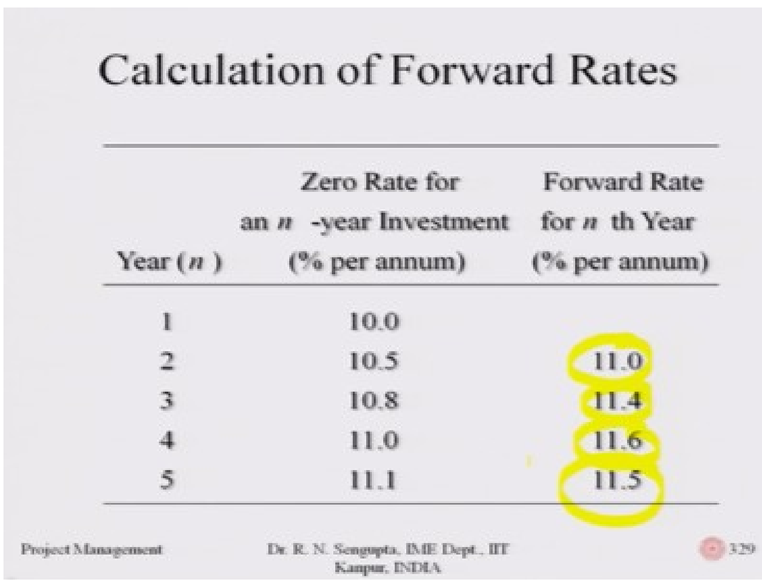


Now in this calculation I am again showing the table so this table basically the first columns are the years, years are again discrete they can mean decimal also that does not matter. The zero interest rates are given starting from which is second column starting from 10.0 to 11.1 which is for year one till year five respectively and the forward interest rates are given here and we will like try to basically discuss the concept of calculation of how they are found out.

So the forward rate for, if you see the second row which is for year two the last value is eleven so this is I am marking here is eleven, if you see the third row the last value in the third row is eleven point four then if you go down along the third column the next value is eleven point six, the next value is eleven point seven, this is of interest of how we find it out.

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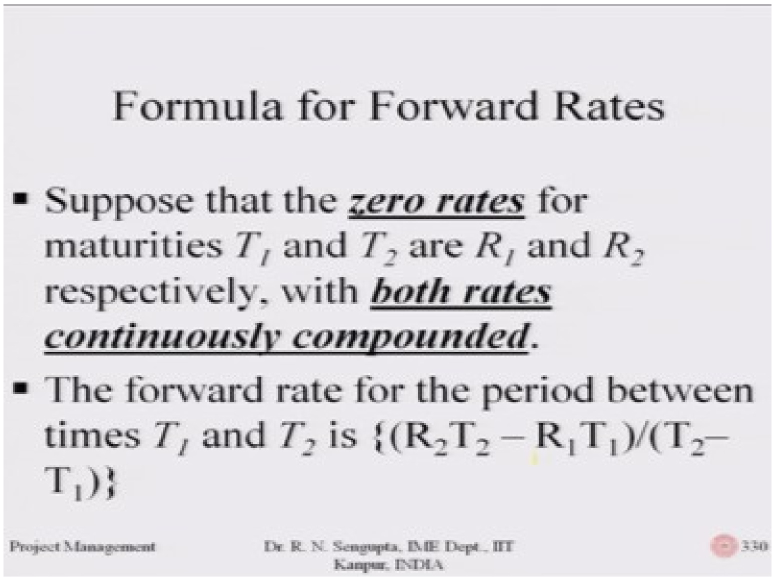


So suppose the zero interest rates for maturities for T1 and T2 if you remember in the three twenty eighth slide I mentioned T1 and T2 so there T1 and T2 were T with small ts so that hardly matters whether you are using capital T or small t the actual concept should be made clear to the students in the best possible manner. So suppose the zero interest rate or maturities T1 and T2 are R1 and R2 respectively and both rates are continuously compounded.

So this is, this has been highlighted because I want to mention that if you are trying to calculate the forward rate based on the fact that it should be continuously compounded you use the zero rate and based on the fact that they have been continuously compounded if you are using the simple interest rate use it for both the zero rate and calculate the forward rate based on the fact that you want find out the simple interest.

So try to do the calculation on the same datum and changing it from simple interest to compound interest or compounding to continuous compounding interest then it hardly matters. So the forward interest rate would be found out using the fact it is equal to R2 into T2 minus R1 T1 divided by in the bracket T2 minus T1 I that can be derived very easily I am going into the derivations or simple concept how they are actually used to derive.

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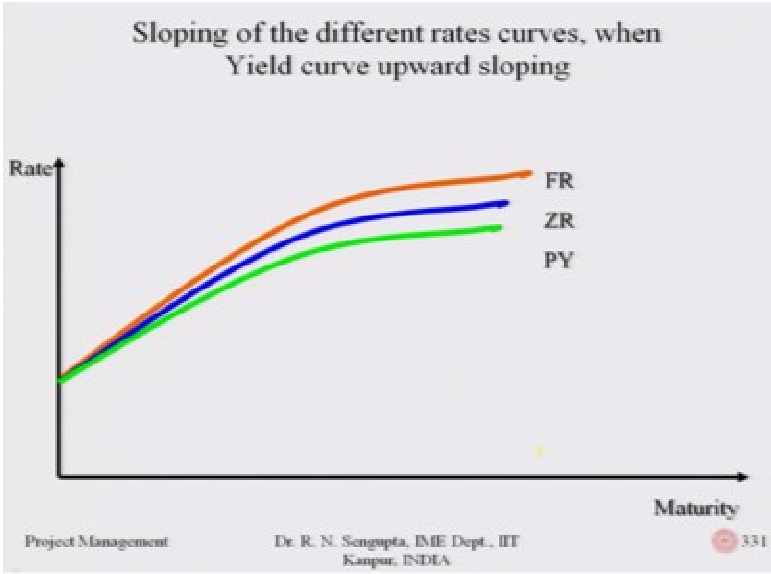


So based on the fact that the forward rates are R2 into T2 minus R1 into T1 and divided by T2 minus T1 you can find out all the values in the third column from the table just shown. So the table was basically the three twenty ninth slide where in the first column the years, the second column you have the zero interest rate and the third column you have the forward rate forward rates can be calculated considerably formula and the year and the zero interest rate.

Now I just discussed in the qualitative sense the sloping of the different curves when the yield curve is upward sloping or the yield curve is downward sloping. So if you remember we did discuss and find out the projectile and all the pile then we found out the zero interest rate for the projects and we considered the forward interest rate.

So if the power yield is increasing or if the forward rate is increasing and the dy dx any of these value is increasing then the sequence of the again without going to theoretical proves I would just want to mention the sequence of the forward rates, zero rate and the power yield are as shown here. Here the orange line is for the forward rate, blue line is basically for the zero rate and the green one is for the power yield and for the project if it is downward sloping that if the dy dx are negative

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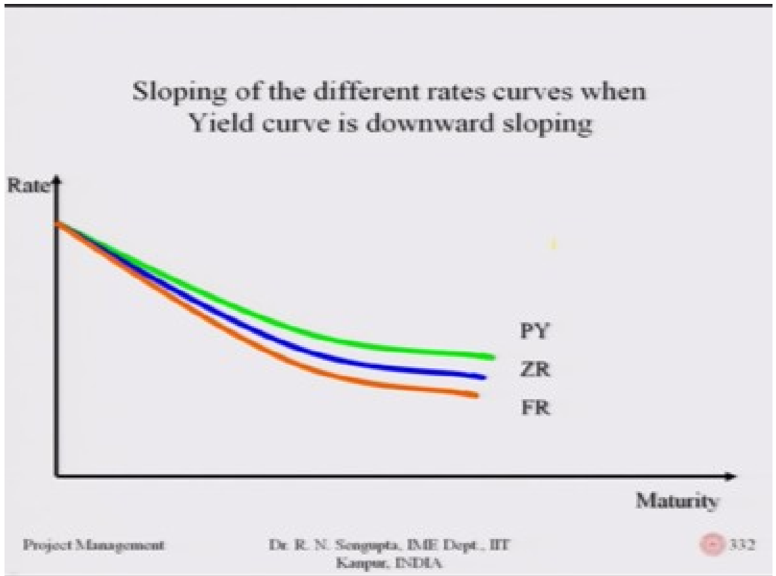


Then the graph will look exactly opposite where the forward rate which is the red one would go bottom then you have the zero rate and then you have to power yield. So these just for the concept that as you are using this different type of concepts of interest rate or the IRRs or the net present value or the fluctuation interest rate or fixed interest rate, the zero rate the forward rate.

So those can be used in a very intentional way to find out what is the net present value of a project or whether its positive or negative depending on the out flows or inflows that you can make a judicious decision whether that project can be taken forward to its next type of logical conclusion.

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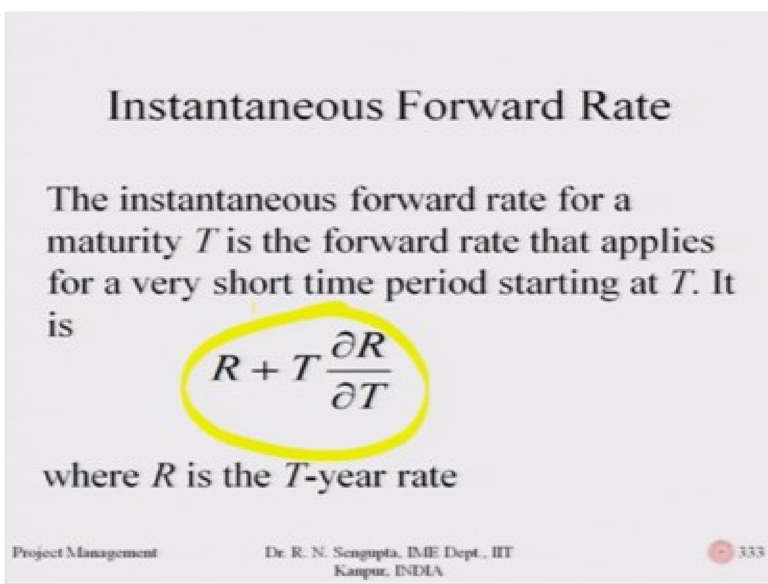

So now I want to find out the instantaneous rate so if you remember the formula for trying to find out the forward rate was what it was R2 into T2 minus R1 into T1 divided by in the bracket it was T2 minus T1 so if you do the limiting concept there and try to utilize the value comes out to be as I am circling which means the instantaneous forward rate for the maturity T is the forward rate that applies for a very short duration time starting at time T.

So this R which you have is basically the year rate or the T rate which is given here and this delR delT rate of change of the interest rate function with time and that multiplied by the time duration of the time which you have to take.

If you utilize this formula will be able to find out using simple concept of bootstrapping to find out each time period forward rate given the value of the zero rate is known to you based on that you can find out at any instant what is the project yield, what is the project value, what is the project's IRR and so and so forth values based on which you can take judicious decision about the project.

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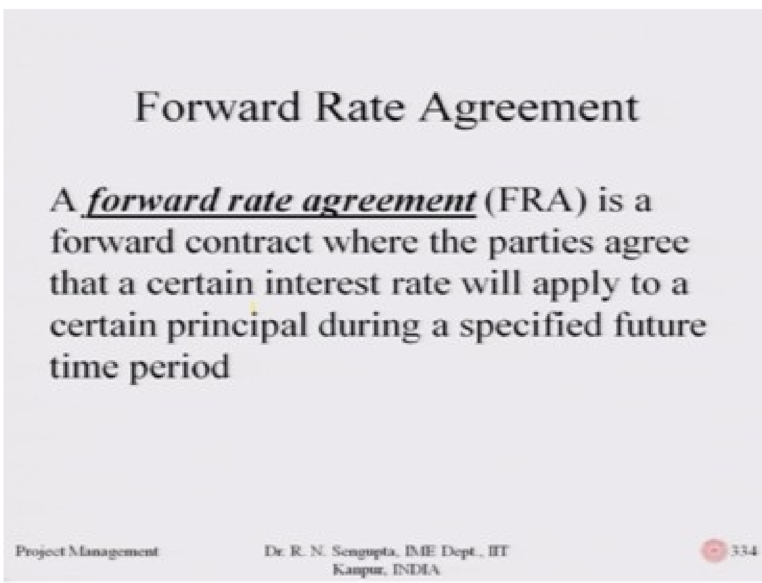


Now I will just come to the concept of forward rate agreement even though it is not immediately applicable for the projects but forward rates agreement would give you a picture that how they can be utilized to at least judge whether the project rate is actually feasible or not from the financial point of view so forward rate agreement is an forward rate contract where the parties agree that a certain interest rate will apply to a certain principal during a specified future period. So if your forward rate is very high or very low it will have some consequence on based on the fact that what is amount of money you can borrow and what is the amount of money you can invest based on which you can take decision and compare projects such that the expected value of the project is high the or the various of the project is low or any other values.

Such that the project implementation can be done thinking about the overall scope of the project which is there which is the deliverables of the social impact would be the financial implications of the project, what is the overall effect of that project on the organizational variables which you want to consider and all these values.

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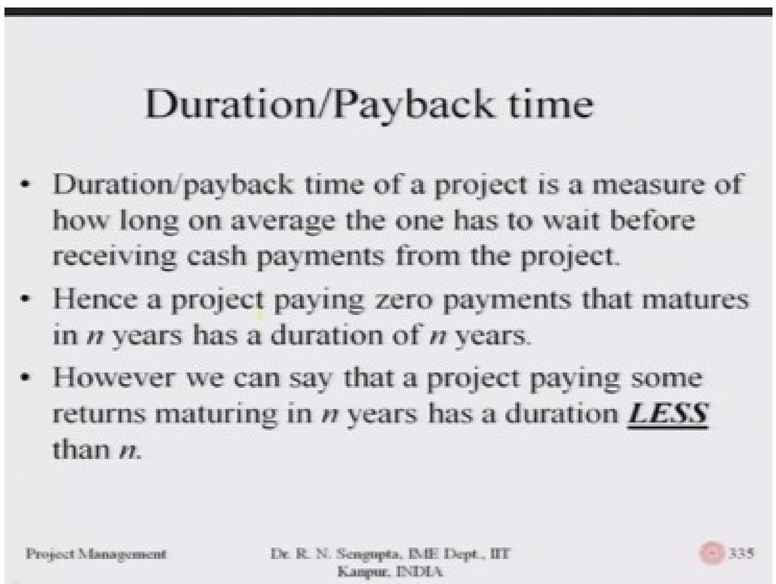


Now we will come to the duration on the payback time so very simply duration and payback time means the time starting from today which is T is equal to zero till some time such that I am able to get back my overall investment and make the money in such a way that whatever investment I have done is returned back to me. So consider the IRR, IRR if you remember is the internal rate of return so that rate such that the net present value rate basically becomes zero.

So what I want to find out is the duration such that depending on whatever the inflows and outflows are I am able to get back within the time frame the overall amount of money which has been invested by me considering all the outflows and inflows in a accumulative sense. So it means the duration payback time of the project is a some sort of measure that how long on an average that one has to wait before receiving the cash payment from the project.

So shorten the duration means faster I am able to get back the money longer the duration is that it will take much longer time as that project may not at all feasible because it takes a long time to get back the money which have been spent. It means that it is not worthwhile to take a decision to invest in that project hence a project paying zero payments that matures in n years has a duration of n years. So I will discuss that within the formula within one minute which would be coming in the three thirty sixth slide

And the last point which is there in the three thirty fifth slide basically means that however we can say the project some returns maturing in n years has a duration less than n which means that if I am getting some money before the two years for which the duration is there then means that I am able to retrieve my money from that project in a much faster duration so the duration would be less than two years.

So obviously that would depend on what is the interest rate what is the payment period happening and what is the total amount of payment happening at what duration of time. Duration is basically the so called average value but that average value is waited depending on the inflow and outflow of the time duration which you have. So how it is done let us see.

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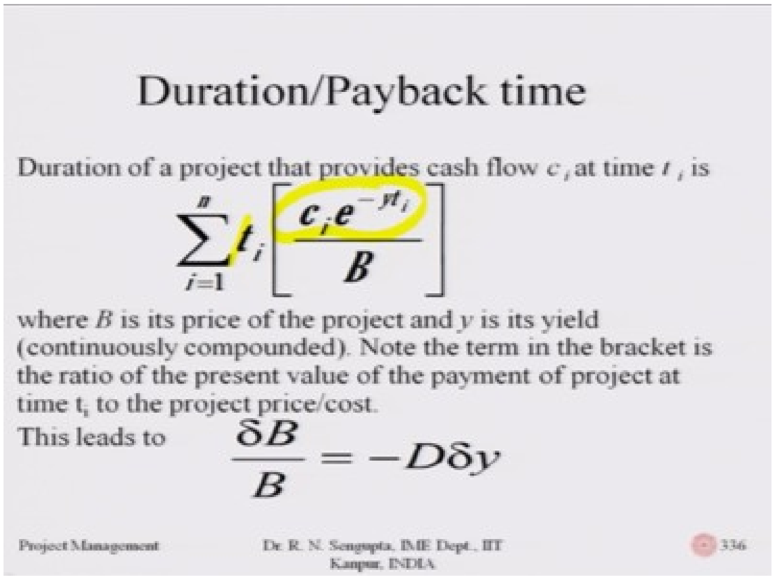


Now let us consider this formula which is given in the three thirty sixth slide as I mentioned duration of project can be provided that you have the cash flow are given positive or negative that material that Ci is positive it will be taken as a positive sense, it is negative it will be taken as a negative sense. So the cash flows are given as Cis at time period Ti so this Ci may be any random fluctuation of returns or random fluctuation of payments whatever it is and this time period which you have need not be of equal intervals.

So it can be the quarter of a year then it can be the three fourth of the year then it can be one point five of a year then it can be five years whatever it is it does not matter. So let us concentrate on the terms which is outside the bracket which is ti and the term which is in the numerator of the terms which is inside the bracket. So the inside the bracket if you see it is ci multiplied by e to the power minus yti so this y value which the yield which is the continuous compounding yield which we have already discussed in the last class.

So now when I am basically sum up c1 into e to the power y minus obviously minus y into t1 plus c2 into e to the power minus yt2 if you add them up this is the total value of the inputs or outputs which is happening and I am trying to find out what is the net present value of that amount. Now what I want to do is that I want to find out the duration on the payback periods such that in each value of b which I have b is the time duration which I need to find out.

So if b is the price of the project and y is the yield as I mentioned if you multiply the overall amount which is coming at time to zero multiple by t it basically means some sort of averaging which you are doing based on which you want to find out the average time that the value would have so called zero net worth to me at that point of time. So if you see the terms inside the bracket.

Inside the bracket whatever your terms is basically those are rupees to rupees so in the numerator you have rupees in the denominator you have rupees so that cancels each other outside the bracket is t. So I am able to find out the waited sum of the time such that the waits would be coming from the fact that it is the net present value divided by the value of the project so further it is closer to my time period zero it is basically the waits would change accordingly.

So I find out the some of those waited some of those times based on that I say this is my time period waited some such that I am assured that my overall return has taken place to the exact amount.

Now this if you know the term in the bracket is the ratio in the present value of the payment of the project at time t1 to the project cost and if I am able to find out the dy dx of that it comes up to be delB the rate of change of B divided by B which is the price of the project equal to negative of D which is the duration into delY which is the interest rate which you have.

So I would not go into details of how these things are derived but slowly solve problems and discuss that later on. So with this I will end the twenty seventh class and then start of the twenty eighth related to the PERT problem. Have a nice day and for any queries as mentioned please get in touch with the forum. Thank you very much