Project Managment
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Module No # 5
Lecture No # 24
Aspects and Applications of CPM and PERT

Good morning, Good afternoon, Good evening my dear friends, this is the twenty fourth, lecture for the project management course and as you know by this time my name I am Ragunandan Sengupta from the IME department IIT Kanpur India. So in the twenty third lecture, we rapped up the concept of considering the criticality index.

So this was basically the concept where if you keep simulating a job considering that you want to find out what are the set of activities which makes the critical path. So the number of times a particular job comes up in those simulations divided by the total number of simulation which you have done or total number of such instances of experiment you may have done. So that ratio basically the critical index.

And if you remember I did mention that you rank them from the highest to the lowest take the source sets of activities or job which have got the highest criticality index but remembering the fact that they make a sequence of events which add up and basically start from the source, which is job number one or the node one and end on to the last job which is the nth one or the sync, such that you are able to complete and have a feel that how the work progresses.

Now also if you remember I did mention that due date is also important to consider, that how we will consider the concept of due date along with expected time for the set of all the activities which makes the critical path and how the variance can also be considered in order to find out that what is the percentage of completion of any particular job considering T which is the expected time and D which is the due date are coming into the picture.

And I also mentioned in the last class and also beforehand that central limit theorem would be used irrespective of the fact that whatever the distribution for PERT word is considered. So we

will basically have the graph considering the central limit theorem to be true is this normal distribution whether the central line.

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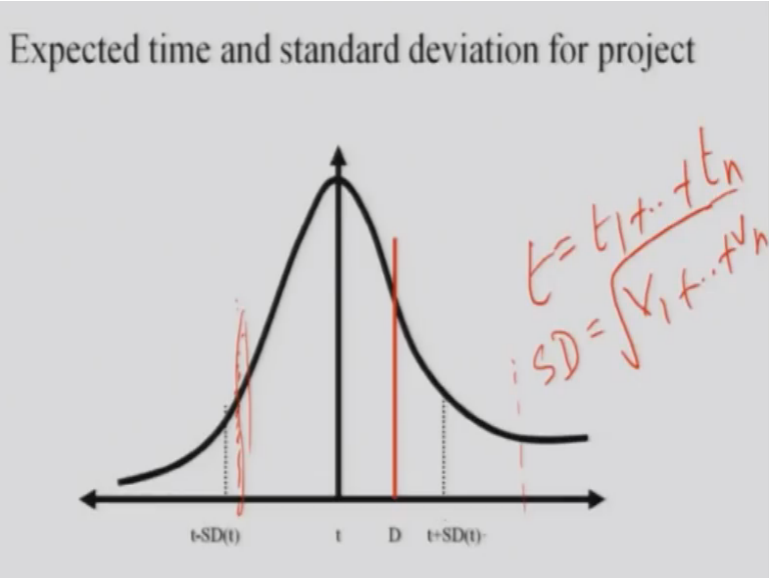

If you see it is T, so T is the if you remember is the summation of all the average time for the critical path and these standard deviations are basically the variances addition for all the paths and then you find out the square root and then if you go one sigma to the right, one sigma to the left, if you see my this pointed is hovering about the right hand side this is one sigma, on to the left is the one sigma to the left.

So now see the red vertical line which is their which is the due date, if you see it will give a feel that the due date can also be here. So the due date can also here but whatever concept I am going to mention just now will not be true for both the fact whether the due date D vertically can lie to the right or to the left, What they will have different conceptual consequence that, How you analyze the problem.

Now the due date can also be on to the right of T plus one standard deviation it can also be on to the left of T minus one standard deviation so this on this idea of where the due date is does not matter if you are able to analyze the problem in its true perspective from the practical sense. So this is what I will now discuss very briefly in the two eighty ninth slide.

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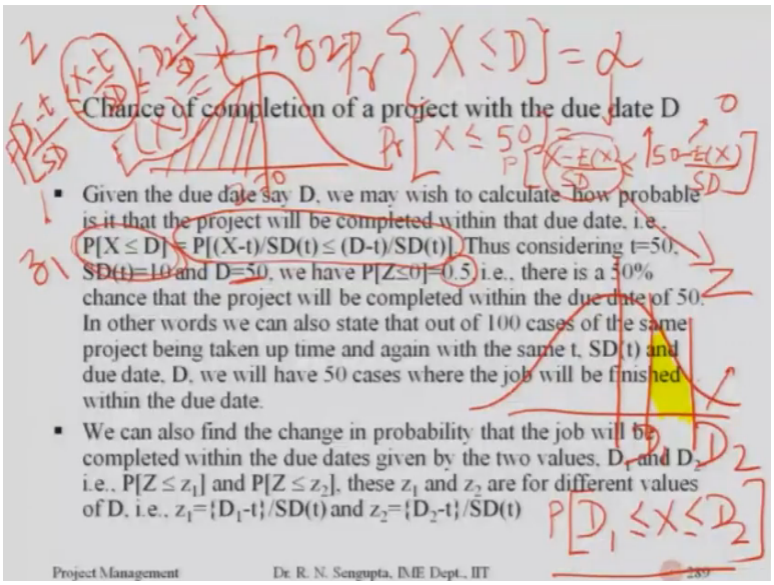

So now it sense the chance of completion of project within the due date, so due date is d given, the due date say D we wish or we may wish to calculate how probable on the probabilistic sense on a non teachable sense, is it that the project will be completed within that due date which is what we need to find out. Is this is very important? So what we need to find out is PR is probability which is P here it does not matter.

So we want to find out X is the random variable which denotes the overall distribution of the time for the overall projects for which the expected values is known to which is T as mentioned in the normal distribution as shown in the normal distribution and the standard deviation was SD in the bracket T. So my main concern is this I want to find out what is the probability alpha?

If alpha is very high it means that I am able to finish on an average the job before the due date on a very high probability note which would mean two things, that I do not incur extra cost very simply mentioned here because I am considering cause can give come due to moreover time given if the job duration increases more than the due date because I want to assign more and more people in order to finish it.

Or I want to assign more work to the same set of people giving them over time it may also mean that I have to resources extra some machinery extra some land consider land I utilize it can be try

to utilize some extra help from other organizations from vendors. So these are very simple points which I mention the other important fact which may be negative is that.

If I am the vendor I may need to supply and deliver to the government of India a road considering the road connects the city of Bhopal and Indore, now if I am able to finish that work within the due date of consider three months then obviously I do not pay any penalty, but if I deliver the overall project after the due date then obviously as per the clause and as per the contract I may have to pay some penalty.

So you may be asking that if that is the case if I am able to deliver the job before the due date, obviously I should get some benefit also. So those can also be brought into the picture considering the overall contract how it is signed. So If I consider A, as the T A the overall random variable depicting in the time and if I consider central limit theorem to be true then X would have a random variable distribution as normal.

Then the expected value of T and the standard deviation as SD bracket T or variance as calculated. So variance calculated were basically some of the variance for all the path which are on the critical path. So using simple standard normal concept, so this is the formula which I have which is X minus the expected value divide by the standard deviation is less than equal to D which is the due date minus T which is again the expected value of T divided by standard deviation.

Thus considering T as fifty and standard deviation as ten and D as fifty, we would have basically that now this is considering that the Z is the standard normal. So how it came about is exactly this probability of fifty of sorry less than equal to fifty. So this fifty is this one is equal to, I want to find it out so what I need is probability X minus is expected value divided by the standards deviation less than equal to fifty minus the expected value divided by standard deviation. This expected value E X is equal to T.

So I am using general notion, so this fifty and expected value is also fifty this fifty -fifty becomes zero. So whatever it is this is now becomes a standard normal deviant z. So hence it is written as

probability or z is less than zero. So if you consider the normal distribution the overall area for sum of the properties for a any distribution and assuming for the minimum value to the maximum value.

We know it is one and normal distribution being and the symmetric distribution of above and below its mean, median and mode. So this value z is equal to zero which is the mean of the standard normal deviant is zero. Standard deviation for this is one so the overall coverage till the mean value is fifty that is why it is written as fifty. So this is point five so its fifty percent so the chance the project will be completed within the due date of fifty is fifty percent

In other word we can also state that out of this hundred such simulation we need to do before you basically commit your resources and you want to find out where the over run could have happened or where they can happen. So out of hundred such cases hundred can be two hundred, can be one thousand, can be one million.

So I am just giving an example hundred cases of the same being taken up time and again with the same time which is the expected value of X which is fifty standard deviation, which is given as ten and due date d which is again fifty we will have the fifty chance cases whether the job will be finished within the due date and fifty per chance of fifty such cases that jobs would not be done by the due date.

We can also find the chance in probability that the job will be completed in within certain due date which is given by two values D one and D two consider like this, you are the vendor and again coming back to the laying of the road from the city of Bhopal to Indore and the overall average time decided due date is three months and within the three that months if it exceeds the three months but less than say for example four months you pay a penalty of consider two lakhs per day.

Now after the four month your penalty increases to five lakhs per day. So what you want to find out is that what the probability is, that would I have would finish. I as the vendor would finish the job within that time frame of three to four that means, I am not able to deliver over the job

within the three months but I am able to deliver it within four months. So I know on an average what is the cost which is two lakhs per day.

I want to find out that what is the probability of paying that amount because in case if I exceed the four month the overall cost structure with the penalty structure cost structure in the negative sense for me, would now become not two lakhs but five lakhs per day. So in that case I need to find out the what I the probability that exceed finishing the job for a time period more than four months and what are the consequences I have to face.

So based on such practical example, so let us just give the concept very briefly so we can also find the chance in probability that the job will be completed within the due dates given by two values D one and D two as just mentioned. So I need to find out that what is the probability? So technically it is like this. So my job would be completed within some D one and D two this due dates are given I need to find it out.

So again going through the same simple concept was central limit theorem the concept of standard normal. So what I do is this I will now just write down the formulas it would absolutely make very simple sense to all the students who are well versed in the concept of probability. So it will be basically D one minus T which is expected value by the standard deviation less than equal to X minus T by standard deviation which is less than equal to D two minus T by standard deviation.

Now these value D one is known T is known standard deviation is known, so what I have is one small Z one which is the standard normal divide value realized value which I know I can check in the standard normal table here also D two is known, T is known, standard deviation is known, from there I found out the Z two. So this value which is X minus T by standard deviation is basically the standard normal divide.

So I basically check given D one and D two and T and standard deviation and check the small Z one values small Z two values, find out what is the overall probability such that I will be able to finish the job within the stipulated time of D one and D two. So if I go back to the graph in this

becoming a little bit guttered but please bare with me so what I actually have is this graph this T and these are D one and D two.

So based on that I have this formula which I have given here. So if I highlight what I need to find out is that what is the probability inside the region as shown between D one and D two based on that I can find out at that that what is the probability that I am able to deliver because based on that I will basically recalculate my overall budget and take a decision whether I can delay or whether it is best to finish of that job.

Before the D two period using some extra man material, giving overtime, trying to take an take a help of other vendors, trying to pay them extra amount of money, such that I am able to overcome the overall loss which I may face if I do not utilize this external resources or man material and other things such that in that case the overall time taken to finish this the overall project now is more than D one D two.

Because if it is more that D two what happens is that it will go into this region where if I mention in the example that triangle finish off the road connecting to two different city is in India the overall cost is now five lakhs per day. So if it is exceeds four months which is D two then the overall cost I have to bare is very high with respect to the fact that if I am able to deliver before the time period of four months.

Then my overall cost to be paid by me per day two lakhs such that able to offer said that utilizing extra amount of man materials such that my overall cost budget image is under control. So I am sorry for gluttering this two eighty nine slide, but I am sure you would have understood the two important concept.

One is what is the probability of finishing within the dead line or D and what is the probability of trying to finish off certain percentage of the job between two deadlines D one D two, such that I am able to recalculate my overall cost structure for the project so the penalty structure for the project.

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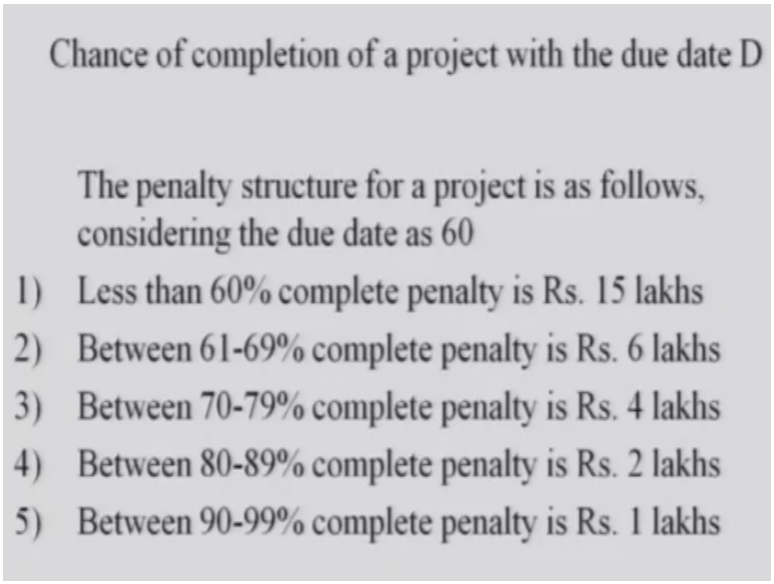

Can be say for example as follows that if you finish less than six, so coming back to the exact things example which I finished in the two eighty ninth slide, between D one and D two the deadlines and the due dates. So if you are able to finish only sixty percent completion of the jobs, you pay a penalty of fifteen lakhs. So this is exactly fixed penalty that is what I mentioned it was two lakhs per day between sixty one and sixty nine percentage is complete.

You pay a penalty of six lakhs between seventy to seventy nine, pay a penalty of four lakhs if between eighty to eighty nine is complete, you pay a penalty of two lakhs and between ninety to ninety five ninety nine is complete, you pay a penalty of one lakhs which means more complete less the penalty.

So we could also added if you see this slide sixth seventh point accordingly that if you are able to finish it before the deadline obviously. You do not incur a loss or do not basically pay a penalty but it would be a negative of penalty in the sense, you gain some from the actual customer to whom you are supplying that project or to the government of India considering that you have signed a contract that you get some benefit.

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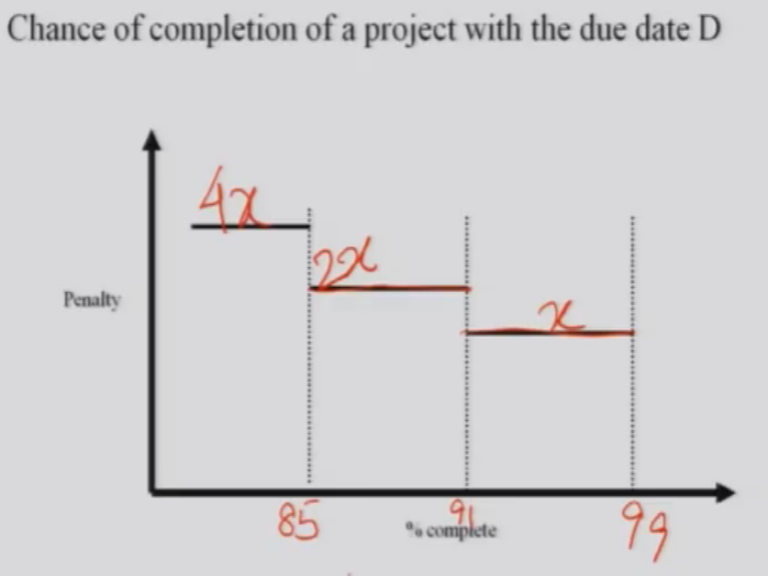

So the chance of completing a project whatever it is there now this is percentage completions are given on a penalty on a linear scale or a step five function, which means that if percentage completed say for example between ninety one and ninety nine percent, the cost is I am not giving the values which I written I am just giving the any sum or bit variables. So this is X, so you would basically pay an amount of X if it is less than ninety one.

Say for example greater than eighty five, so these which I have taking percentage complete are just arbitrarily they can be equally dispersed or not depending on how you have framed a problem or what information which you have. This would be higher, say for example it is two X and in this case if it is a less than eighty five percent complete you can say for example four X so this X, two x, four X, are again arbitrarily.

So what is important to note is that in this region between eighty five to ninety one, or in this region from ninety one to ninety nine, the cost over should be remain fixed that means if you are able to finish that that particular percentage of work either it is ninety one point five percent complete or ninety eight point nine percent complete you basically pay a fixed amount but it may so happen that this fixed amount may not be true.

In some of the context where they would try to basically try to find out that what is the overall percentage you have finished and based on the percentage it may be a non linear function it can

also be a fact that depending on the percentage they may also be like, they may mean the customer to whom you are supplying the project. They would also like to know that what is the number of days which are left to finish the project.

Based on that the cost structure for the penalty may be calculated or it may so happen that percentage wises important but they may like to look into the fact that what are the critical job which are finished. So if the critical jobs are finished then the rest assured that the job would definitely be completed within a certain deadline, new deadline of D three D four.

Whatever it is so if those critical jobs are finished then your total penalty structure which you as the vendor are going to face would be much less in case if those critical jobs in the critical path are not finished. So there can be different scenarios by way of which overall cost structures can be calculated. Based on which the overall cost for the projects can be calculated a penalty structure need not.

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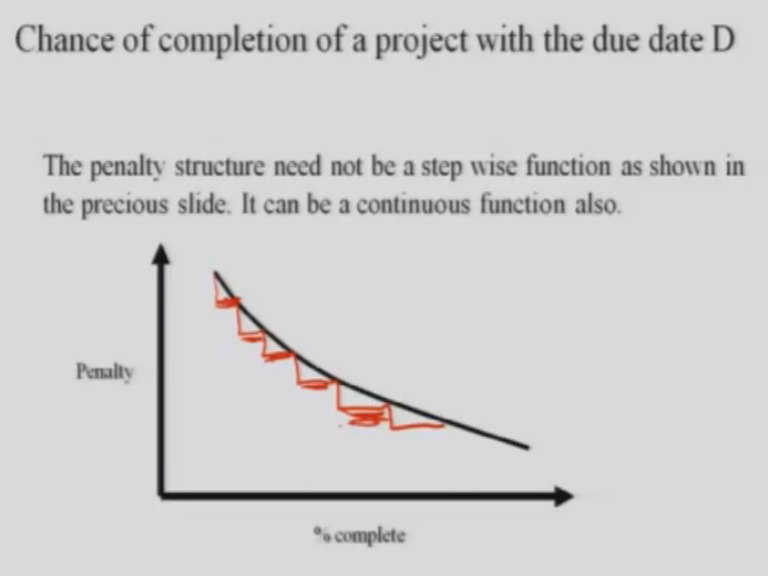

Be a step wise function as I just mentioned. So as shown in the previous slides, so it was step function. So within ninety one to ninety nine, some cost within eighty five to eight ninety one, some cost and based on that you calculate. But in case if you have a cost structure which is non linear, so what would be important is on two accounts number.

One whether we are able to replace this non linear cost structure by a step function. So closer your step function is to say for example, so what will be required is that whether these lines are able to replicate this horizontal lines are able to replicate your overall scores. Cost structure which is non linear in nature. So if you are able to do that then trying to find out the calculations for this small quantum increase or decrease of the overall percentage.

Finish of the job or the due date being exceeded can be calculated in much better way that is one another can be say for example, if I know the marginal rates of increase in the cost structure for the whole projects if that is known to me that means it either it increasing at increasing rate, increasing and decreasing rate or increasing at a fixed rate. Fixed rate is basically the concept which we generally try to utilize in order to make our life simple and try to find out good results.

For the overall projects which are very complicated in nature. So in case if the marginal costs are increasing at increasing rate and increasing and decreasing rate then try to find out the DY, DX of the cost structure needs to be brought into the picture that we are able to calculate the overall cost structure which is there for the project or for each and every activity based on which you can proceed.

Another important fact is that if you remember in the two or three slides before I did consider one critical path and one set of non-critical path but showed that if the variances of the non-critical paths or the set of activities which are there on non-critical paths is very high, then it may so happen that trying to exceed the overall due date may have an effect such that the non-critical path activities or the set of path which are there not in the critical path may come into the picture.

Such that we need to calculate that that overall extra cost those non-critical paths and also see whether the non-critical paths does really effect the overall critical path and the cost if it does then your cost would be two fold number one coming from the fact that non-critical path are being effected due to overall delays due to say for example, high cost of some labor or high cost from some input material.

And such things plus the fact that if it effects the critical path the two things would happen number one, if I need to utilize resources then it the cost would increase as it would be for those non-critical paths, plus the effect is that if the critical path is exceeding in the sense the time delay is happening then the as per the contract what are the actual cost which have to be worn by the vendors which is you would also be considered in the picture.

And then the critical index may change because if the variations are happening in such a way due to external factor in happening in such way that may affect the overall working of the project. Based on which you are want to do your scheduling for the set of activities which are there in front of you. So now I will just consider very simply the concept of the projects life cycle.

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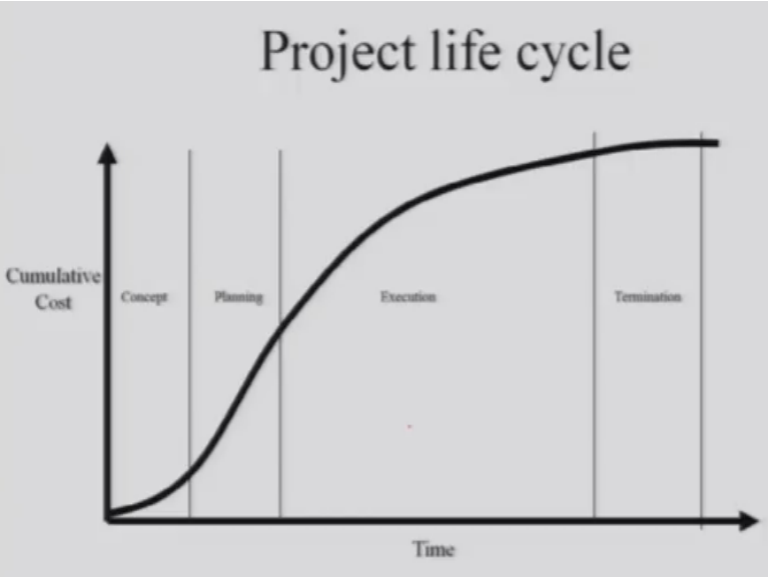

And how it effects the overall planning of the projects and cost would basically come into the picture as a sub part of the overall project life cycle. How it is considered, so if I consider the concept of time along the X axis and the concept of cumulative cost or the cost on the Y axis generally the project overall development in a whole happens is like this as shown in the two ninety third slide.

In the initial state you basically conceptualize try to find out what is the effect to the project and how should be taken to consideration then you go into the planning stage and obviously as you

proceed from the concept which is the left most bar or the vertical line which you have, then you go to the planning stage, then slowly the overall cumulative cost increases.

Then the cost in case may not be linear in shape which has to be basically non-linear in shape and then we go into execution phase there will be fixed cost there will be sum cost there would be variable cost there will be marketing cost there will be cost related to man to labor to design to intellectual property rights and all this have to be taken into consideration.

So as the execution phase ends and then the overall work for the project starts considering is the ongoing project then obviously it will depend on town in sense increase would basically from stagnant and there would be straight line, almost a straight line that means the cost are on a fixed scale.

And if the projects end then and there then obviously the project has to be round up and then cost structure of the overall project will basically, start decreasing. So here if you see my hand, so after the termination phase actual cost of the project should basically started and should technically be zero.

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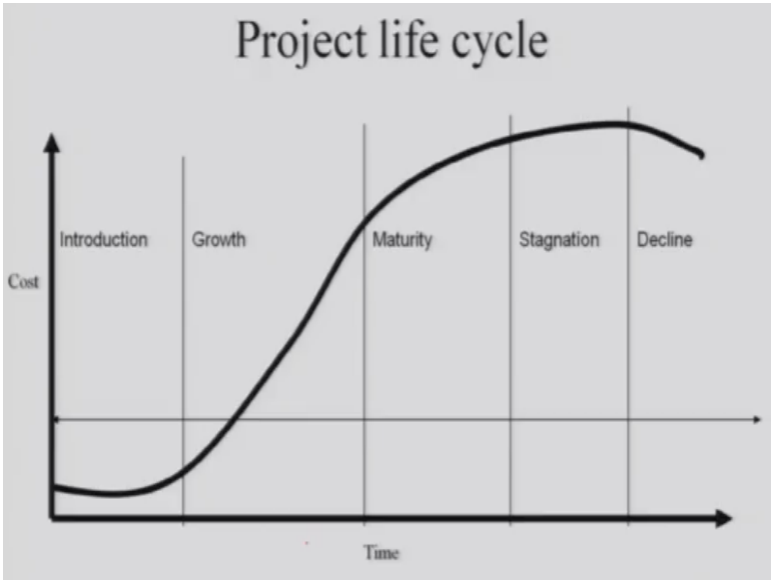

So if I consider the cost structure considering the introduction growth maturity and the stagnation or the decline of the project, then again you will see the overall characteristic of the curve is

exactly similar what you saw in the two ninety third slide. So the here the cost structure on the left hand side or the Y axis is the cost there it will be the cumulative cost.

So they are generally the almost the same. So the total cost the cumulative cost whatever it is so here also it starts slowly and then exponentially rises then at the stagnation and the decline phase it starts decreasing.

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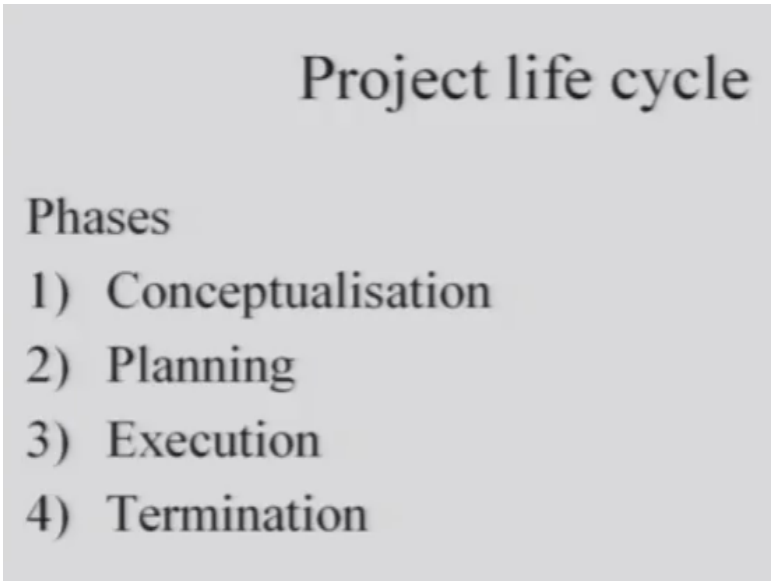

So the project life cycle basically consist of I will come to the project life cycle in detail with one or two simple ideas and reading to be done. So the project life cycle basically comes to the conceptualization of the project the planning which is needs to be done the execution part and the termination of the project.

So basically if you have a big project which is one time like building up rig, building a factory or trying to basically my rig the oil and in either Gujarat basin or north sea, or in say for example in Assam or you want to build a tank considering for the Indian army, or you want to basically start of a school. It can be one as a standard on project or in it can be a continuous project. So they can be execution phase how you execute it.

And then in once the work is done considering the overall the benefit of the project from the project point of view, from the organization point of view, from the social point of view.

Basically terminate the project and see how the overall cost structure the benefit of the project has at grown to the society as such.

So with this I will end the twenty fourth lecture and continue in the discussion using the project life cycle and how the concepts are used in the twenty fifth and the twenty sixth lecture, onwards and then later on come to the concepts of GERT, QGERT and other things in between as if you remember, I did mention I will do few problems in the area of trying to utilize the financial concepts and trying to basically rank a project or try to find out what are the returns for the project based on which you can take the decision. Have a nice day thank you very much.