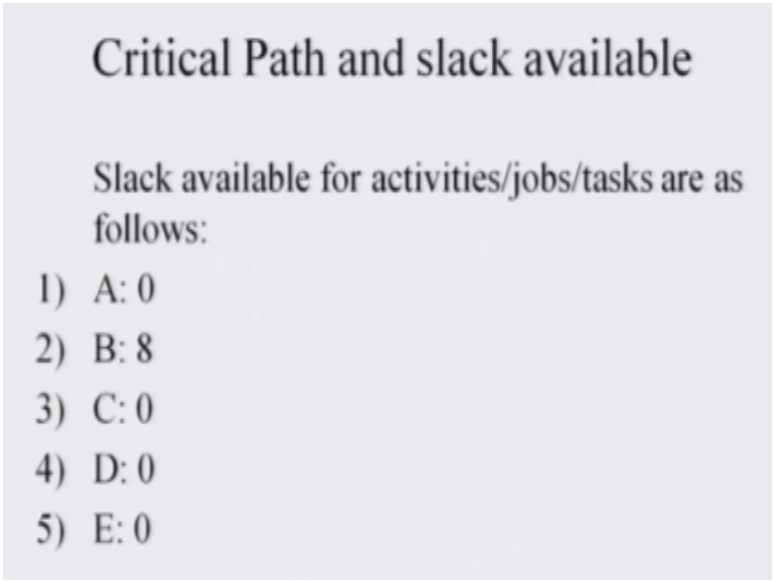
Project Management
Prof. Raghunandhan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology – Kanpur

Module No #5
Lecture No #23
Program Evaluation Review Technique (PERT) II

Very good morning and good afternoon and welcome my students for the project management course this is the twenty third lecture and iam sure you have understood the problem for the PERT which is project evaluation review technique that considering the job timings are given how you can find out the overall critical path? and then take decisions accordingly even though i did mention the crashing the jobs depending on resource constance.

All these things can also be done considering costs are linear in nature. So linear cost would basically mean trying to reduce the number of days would have a linear increase in the cost structure and in non linear increase would basically mean that trying to find out which job basically take first to crash in very complicated project sometimes becomes very difficult. So if you continue with the last example where we left at the end of twenty second lecture.

So once you find out use the concept of forward pass backward pass concepts and you find out the total slack the free slack. So at last we have as per the problem the slacks available.
(Refer Slide Time: 01:30)



For the activity job tasks are as follows A does not have any so obviously means zero it has to be on the critical path B has eight C, D, E all of them have zero which means as you actual can understand and you can rightly point out the critical path would consists of the job in the activities which are in sequence as A, C, D, E so this eight means the number of slack. So the cushions which have been mentioned time and again would be eight number of days for job B.

So as that it can be slided or moved or broken down accordingly. So as that it does not exceed that eight day cushion which you have in case if it increases obviously it will have an effect on the critical job later on point number one. Point number two is that, if you are able to basically crash this job. Say for example, in job B then with an also you are in a position to crash job C also so job B crashing may reduce the cost but would not basically have any effect on the critical path point one.

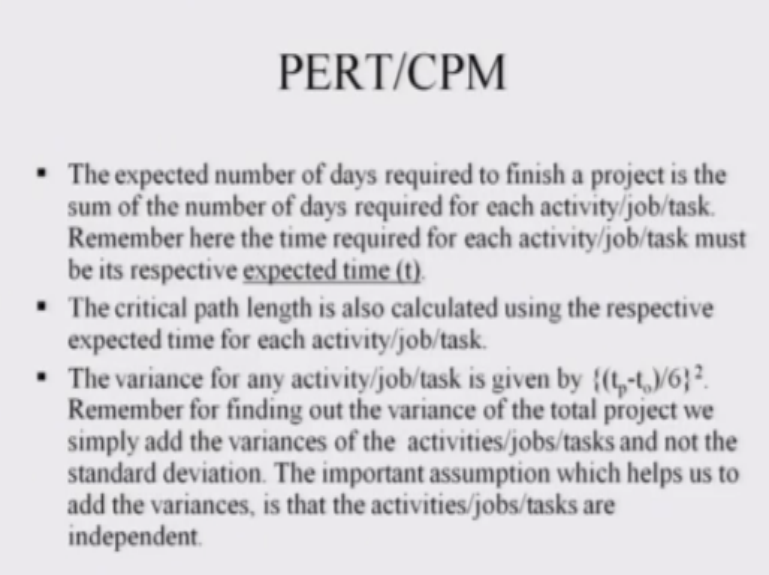
Point number two, if you basically concentrate only on C the overall positive effect in the sense that you able to decrease the total costs for the whole project would be much more substantial if you take care of the critical path because that would be difficult because crashing one of the critical activities or the task along with the critical path may be difficult to find out.

May be difficult to handle but if you are able to do that then the end of the day the result is the overall crashing of the total costs is much more substantial and it has a much more positive effect on the overall project. So thus before i just read the points in this two eightieth slide the concuss with based on which we find out the critical path for the PERT method would be exactly the same for the CPM method which is critical path method the only difference.

If you remember that PERT has optimist time pessimist time and the most likely time based on that we find out T and based on that we do the calculations point number one point number two is that there would be a variants concept also coming from the PERT because if it has a distribution the random variable based on which the distribution time varies then you will have the variance and variance can be utilised in order to find out that what it is the overall dispersion or overall shifting of the percentage of the job been completed.

When you are considering the PERT method for the CPM you would not have those luxuries in the sense the randomness is not there with respective time but obviously it will also mean that finding out what is the probability that you are able to finish ninety percent of the job or based on the dead line which you have what is the percentage of the job still left may not be possible under the CPM method so that i will continue the slide.

(Refer Slide time: 04:55)



The expected number of days required to finish a project is a sum of the number of days for each and every activity which is there on the critical path remember the time required for each activity job task must be respected to the expected time considering that is a PERT method and for the CPM method is the time which is as given the critical path length is also calculated using the respective expected time of each activity job.

Now i have to find out the variance which i just mentioned because variance would be required to find out that what is the percent of work delay or if you think that the dead line is given what is the percentage of the job still to be done then we need the variance or the standard deviation for each and every path or activity now for each and every path and activity of the job which constitutes the whole project.

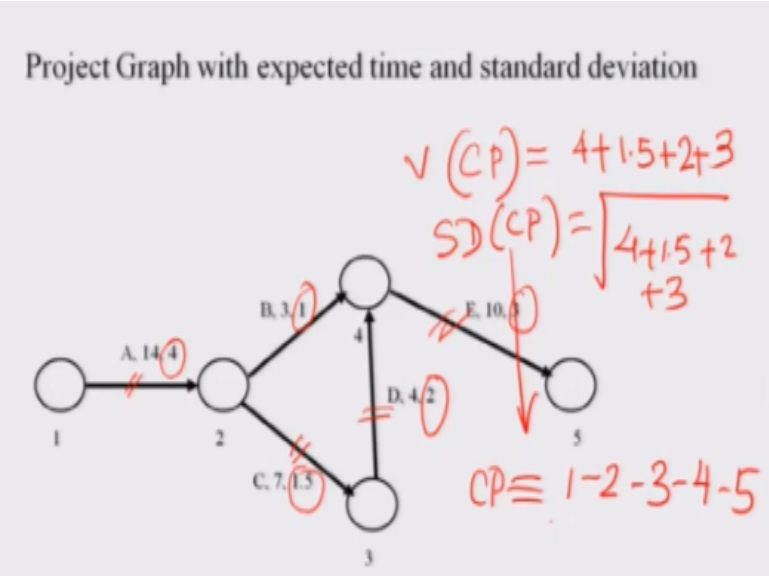
You will have the optimist time the pessimist time and the most likely time based on that you can find out the variance which is as given in the third bullet point. The variance of the activity job task is given by tp minus t0 which is a pessimist minus optimist divided by six whole square this is basically you can use the distribution as given and then calculate the variance remember for finding out the variances of the total project.

So this is point is important iam a going to again read it remember for finding out the variance of the total project. We simply add the variances of the activities don not add the standard deviation we add the variances of the activities and not the standard deviation the important assumption which helps us to add the variance is that the activities in the jobs are independent of each other this is point number one for this third bullet point another point which will be useful when we find out.

To find the relationship that for this queries that what percentage of the job is finished we would not be using the distribution for the PERT each and every individual activities because if you remember in one of the classes either in the twenty second or the twenty first concept of central limit theorem which means that considering the distribution being either gamma or beta or whatever it is as the number of observations basically in cases.

If you are tempted to take and with theoretical background for that the normal distribution comes into play with the respective mean and the variance being for the original distribution based on which you work so we will see that within one or two minutes so again the important assumption being that the activities are independent each other hence you add up their variances not the standard deviation

(Refer Slide Time: 07:40)



So consider the graph is again the exactly the same thing only thing which has been added here in this case is this part numbers. So these are very expressingly it should mention in the problem whether the other variances or the standard deviation. So if the other variances then

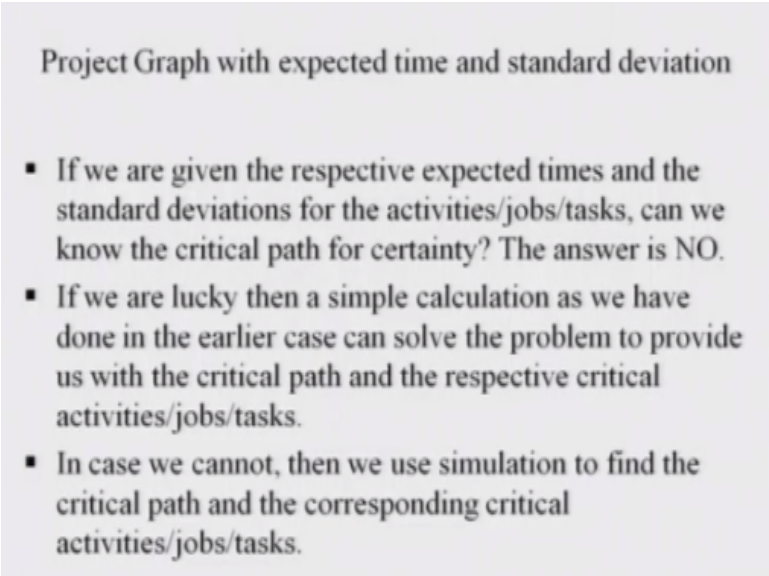
it would mean if i want to find out the variance of the critical path this one, this one, this one and this one which is one two, two three, three four and four five.

You will add four plus one point five plus two plus three so the overall variance for the critical path would be this V is the variance and CP is the critical path would be four plus one point five plus two plus three and if i want to find out the standard deviation which is SD is the standard deviation then CP is again the critical path. So this critical path actually is let me write it down critical path is equallently one, two, three, four, five.

So the standard deviation will be the square root of four plus one point five plus two plus three and you can find it out accordingly in case if four, one point five two and three are the standard deviation then you have to basically square them up and find out the overall variance and again then find out the square root to find out the standard deviation of the critical path.

Now what is important to note which i mentioned in the two eightieth slide is that whatever the distribution is when you want to find out the overall completion or percentage of completion will use the central limit theorem.

(Refer Slide Time: 09:37)



So if you are giving the respective expected time and the standard deviation from the activity task. Can we know the critical path? For certainly the answer is no reasoning that as we know the standard deviation. Standard deviation means dispersion. So if i have this expected time

and not the standard deviation in case if it is not given it means that each and every activity would be finishing by that exact expected time or the time for completion of the task.

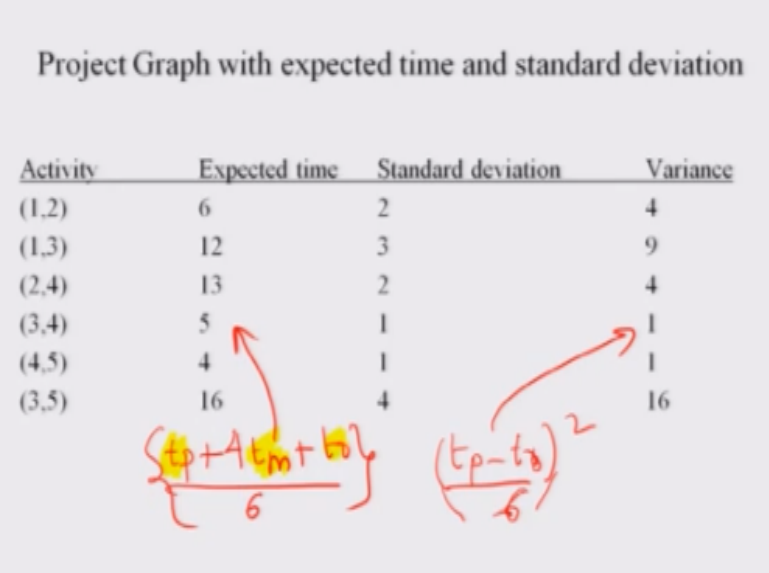
But the moment you have the variations or the variants or the standard deviation. It means that there is a probability of trying to finish of that particular activity or job based on the expected time. So when we are to find out the exact time for the whole project we are trying to basically find out by adding up the sums of the expected value but in that case the variance that is just mentioned in the two eighty first slide that how you calculate the overall variance for the overall project.

Then it would be in the there is some variance of the some amount or dispersion that some probability of the overall job with respect of time would be finished within that certain number of days which is there so expected would basically means on an average. If you continue doing that particular job time after time the average time taken to finish that project or that particular set of activities would be given by the expected time but if there is a delay this delay should be taken into consideration.

Considering the fact that variance has come into the picture if you are lucky then a simple calculation as we have done in the earlier can solve the problem to provide us with a critical path and the respective critical activities jobs and the tasks in case we cannot then we use stimulation to find out that what is the critical path and what is the average value of the critical path and what is the variance of the critical path?

So stimulation concepts can be utilised in order to find out that which jobs and activities would be appearing the maximum number of times as you do stimulation and then you find out that what is the average time based on which overall project would be finished such that is the expected time and then we find out the variances accordingly.

(Refer Slide Time: 11:53)

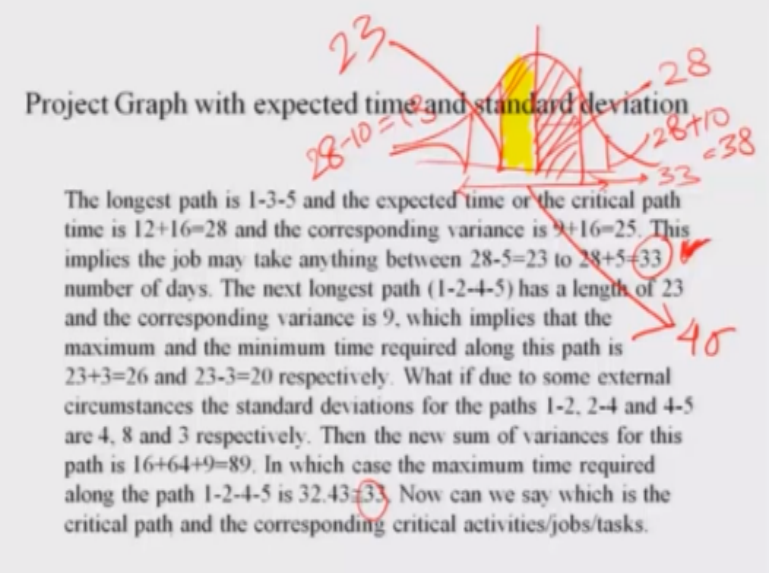


So in this problem let us consider the activities are giving on the first column which is 1 2, 1 3, 2 4, 3 4, 4 5, 3 5 the expected time is given this expected time considers that for the PERT method the optimist time the pessimist time and the most likely time were given based on those informations given. You find out the second column which is the expected time so those are there then based on the pessimist time and the optimist time.

We find out the standard deviation using this formula so your actual formula if you remember iam again repeating it the expected time is found out using the pessimist time four times the most likely time plus the optimist time all these things should be divided by six and in case if i want to find out the variances. Variance would basically be the pessimist minus the optimist time and then you basically square it up with this divided by so let go back to the formulas.

So it is divided by six whole square so use these formulas to find out this and so these are found out given whatever informations which are there which let me highlight is tp tm to.

(Refer Slide Time: 13:26)



So the longest path i will just remove this on the left hand side is easier so the longest path is one, three, five and the expected time or the critical path is given by adding up the sum which is twelve plus sixteen which is twenty. And the corresponding variance for the path which is one three five would be given by adding the variances. So what are the variances it is nine and sixteen add them up.

It is becomes twenty five this implies that the job may take anything between now it is important to note what iam saying. So average time is twenty eight variance is twenty five so if i want to find out that the distribution whatever it is we will take the central limit theorem to be two now this five is one standard deviation. So twenty eight which is the average value in between which is the normal distribution and this is twenty eight.

If i go onto the right that means onto my right five units it means that one standard deviation plus the work would basically be finished with a number of days being twenty eight plus five which is thirty three. So this would be basically thirty three and this value is twenty eight and if i go let me use different colours to highlight. So this value this one is twenty eight minus five is twenty three.

If you look at this distribution and if you any follow the simple concept of standard deviation. it would been that within the number of days between twenty three to thirty three were assured that you will basically be able to cover in a very theoretical sense iam just first mentioning in the theoretical concepts that the overall area of coverage is about two standard

deviation onto the right and the left. So the overall coverage is let me go into the just the values which may not exact it is about sixty seven.

So which means that plus minus five the overall concept of the number of days is coming out to be twenty three and thirty three as said the overall coverage of the area is sixty seven percent and if i go one step further if i go plus two sigma and minus two sigma which would be here and here so the overall coverage now is four sigma so this value would be twenty eight plus five plus five which is ten which is thirty eight and this value would be twenty eight minus five minus five which is ten is eighteen.

So this eighteen to thirty eight basically would mean that with plus four sigma that is plus two sigma and minus two sigma iam able to finish the job within a stipulated time. So again continuing what is written here this implies the job make take anything between twenty eight and thirty three days the next longest path as length of twenty three if you go back to the figure which is in two eighty three number of slide.

So it means that the next longest path after the critical path has a length of twenty three and the corresponding variance is nine now here we should pause and try to understand if it is twenty three then plus minus one sigma would become twenty three plus nine which is thirty two and twenty three minus nine would basically means fourteen which implies that the maximum and the minimum time required along this path would be ok.

My mistake iam sorry the variance is nine so standard deviation would be calculated by square root of nine my apologies. So it will be twenty three plus three is twenty six and twenty three minus three is twenty. So which means that what if due to some external circumstances standard deviation of the path one two two four and four five which is there in the next longest path which is one two, four five are given as four, eight and three.

Then the new sum the variance path would basically now become four square which is sixteen eight square is sixty four three square is nine so it is basically eighty nine in which case the maximum time required along with path would be correspondingly found out as thirty two point four three which is thirty three. So iam just trying to bring some information into this concept.

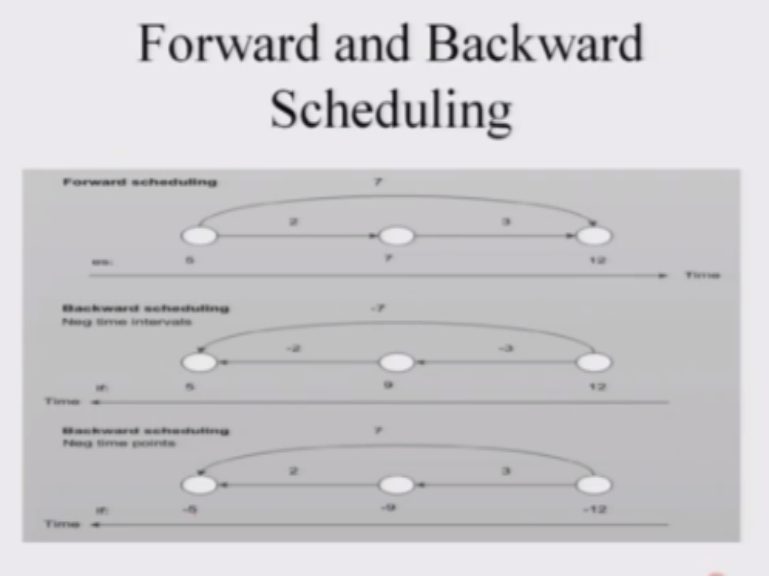
So if you see this thirty three value and this thirty three value. So this thirty three value the first one which is here where iam now trying to point my the pointer this thirty three with is with respect to the critical path and this thirty three is with respect to the other path which is not critical now where does the problem raise if w have the PERT concept actually taken to consideration the practical application which are there which would been that now you will see that the critical path is or may not be the actual critical path.

Which you have found out using the concept of backward pass method and the forward pass method the next longer path is coming to the picture because the overall standard deviation the set of activities it is very high or the number of such stamnda5d deviations are much more in number because the number of such activities and jobs are much higher in number than the critical path.

So when you basically square up the standard deviation find out the variance for the not critical path hen adding up plus minus one sigma on to the average time for the not the critical path method would basically mean that it may become if dispersion comes into the play in the practical sense and if there are dealers then that set of activities which is not on along the critical path may become important.

Such that you have to be a more attention to those activities which are now basically going to create a problem in the sense that in any reduction in the number of days or any exceeding the number of days for those non critical activities would basically have a huge amount of consequence in trying to understand what is the critical path and what is the overall time required for that particular project .So this

(Refer Slide Time: 20:29)



Forward scheduling and backward scheduling iam just bringing into the picture the reason being that when you are trying to do the forward scheduling and the backward scheduling i did mention even though i did it may be repletion if you remember w mentioned the concept of the CPM and the PERT is based on the fact that ones one job ends then only the next can start which is the end to finish end to start concept.

So ES concept but there are other three other concept also which is end to end then it is start from start and one is start to end. So if you have those three concept being brought into the picture then the overall calculation of trying to find out which is the overall algorithm which you had for trying to find out the forward pass method and the backward pass method and trying to find out the critical path and the slacks and the free or the total slacks.

Whatever it is meet are now to be different from what we have as seen point number one point number two is that as you try to utilise those concept in trying to calculate what are the critical path then using the central limit theorem as we just saw in the example in the two eighty fourth slide would have a consequence as that the non critical path becomes important for you to consider considering variance is very important and add on to that.

If you use the concept of not the end to start concept then the slacks and the total slacks which are calculated using the end to start concept may not be true hence trying to find out that what is the critical path is. What is the delay? What is the dispersion? Would give you a totally different picture if your overall add-ons based on which you are trying to do the slacks and the total slacks are not end to start by the combination of the four.

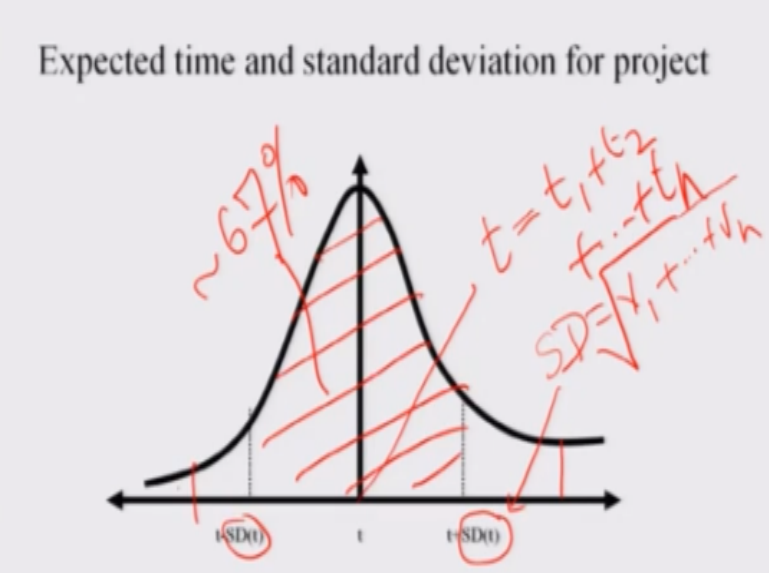
Then doing detailed analysis may become very cumbersome that means it is a difficult problem to solved and you have to do whole lot of cal calculation using some stimulation technique like slam or trying to use any optimation tools or trying to utilize any different type of softwares like MATLABS, CPlex so solving such problems using excel may be difficult so you will try to use some softwares like that.

So in those cases you have to very careful that what are the actual concept which i have been utitilised to find out the slacks the total slacks and the critical path and what is the variance what is the standard deviation whether you are able to use the central limit theorem to calculate all these things would become necessary and then next on you will try to sorry my apologies.

So next you will be able to go into the discussions that after able to solve that question would be that what if i try to crash one of the jobs which are not in the critical path which are in the critical path. Whether the cost structure is liner or non linear so those things which are basically pile up in such a way that are trying to basically solve a problem in its whole gambit where the problem have different nuances may become very interesting but also difficult to solve.

So if you remember i did not mention about the central limit theorem and how whatever the distribution is your actual dispersion based on which you will try that what is the percentage of the job? Which is finished? You are trying to utilize the concept of the normal distribution so this is how the normal distribution look like.

(Refer Slide Time: 24:12)

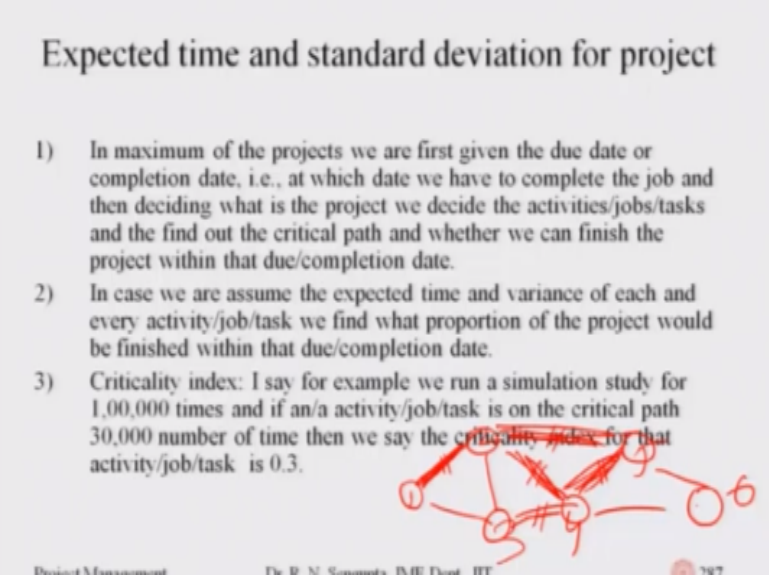


This have iam trying to point it out this T for the whole project or for the critical path and this is basically the sum. So iam using t1, t2, t3 till say for example Tn were t1, t2 ,t3 till Tn are the expected time taken to finish activity one two three four based on the fact for t1 we have the optimist the pessimist and the mean time. Similarly, for t2 you have the optimist the pessimist and the mean time and correspondingly to the last job in that critical path.

Which is tn which has got pessimist optimist and the mean time then obviously you will have the standard deviation so the standard deviation would basically be the square root of the sums of V1 till Vn. V so, this is basically till Vn where V1 is the variance for the first job which has got the average time t1 similarly the last one Vn is the variance for the last job in that critical path for which the average time is Tn find out the standard deviation.

So this standard deviation is what is here. So if you consider again the overall area it is approximately sixty seven percent and based on that you will do the calculation. If it is plus and minus the overall area would be correspondingly found out using the standard normal table so standard normal table would have all the values. So in maximum

(Refer Slide Time: 25:55)



The projects we are first given the due date so now we are trying to basically come into the concept of the due date so in the last slide which is the two eighty six slide you saw that how the normal distribution was done. How? Where T was which is the average time and the expected time for the whole project considering the critical paths then you found out the standard deviations and adding the standard deviation on the left considering standard deviation over overall coverage of area.

So in maximum the projects we are given first given the due date so due date is not coming into the picture so now the due date which is given mat be more than T less than T so you have to basically find out that how the due date will affect now your immediate question would be that if due date is onto the left of T obviously it will should be very fine but the problem is that if there is dispersion.

So there would be some variance in trying to finish of any of asset of activities which is there on the critical path which are not there on the critical path so if the overshoot due to some reason or the other then obviously it would be that the time taken on an average would be much more. So in that case the effect on trying to finish of the job would basically have a devastating consequence on due date and the overall cost structure.

In case, if say for example the due date is onto the right then it would mean that on an average that if you are able to finish the less or more amount of variance in the total overall set of activities then also it may have a negative impact. So in maximum the projects we are

first given the due date which will denote by D or the completion date that is at which date we have to complete the job and deciding what is the project?

Deciding the activity job task and find out the critical path whether we can finish the project within the due date in case we assume the expected time the variance of each and every activity which is T and Vt the variance T. we find out what pro proportions or portions of the project would be finished within the date due date and the completion date. So now we tried to basically being into the concept of the critical index.

Which i will explain in the later class which is the twenty fourth one because this twenty third is almost the at the fag end it need to finish it but i will just give a detail explanation of critical index in a qualitative sense and then carry in on in the twenty fourth class. So criticality index would also be required to found out for the jobs i can say for say for example we run a stimulation study.

If you remember we said that the more you basically stimulate you will able to find out what are the jobs which are coming out maximum number of times in the critical path based on that we say that we are on the on the critical path and the other critical activities so we run the stimulation for say for example hundred thousand and time which is one number of times if an activity job is on the critical path thirty thousand number of times.

We say it is critically index is thirty thousand by one lakh. So it will be thirty percentage which means that job would come up in the critical path thirty number of times. So basically out of stimulation or say for example so if we have the critically index of many of the paths what we will do is the simply rank them from the highest critical index to the lowest one and choose those paths which actually make a sense to complete the job because it may so happen if we add up the top most critical path they may not make the overall job complete.

So it may happen that i will just spend one or two minutes. So say for example the critical index of this path, this path, this path and this path and if i consider the tough five they are coming into the picture. So this added up you will see they are actually some activities logical activities but they do not complete the project that means they are not as per the concept to the critical set of activities which makes the critical path because we have to start at one and finish at six.

So if this four six or five six is not there so obviously it does not make any sense to consider them and as important for the critical index concept. So with this i will end the twenty third lecture and in the twenty forth lecture i will start discussing something about the critical index and then very forward the concept which i have been mentioning about the critical in path concept the variance. How they can be brought? considering the central limit theorem to be true thank you very much and have a nice day.