Project Management
Prof. Raghunandan Sengupta
Department Of Industrial and Management Engineering
Indian Institute of Technology – Kanpur

Module No #5
Lecture No #22
Program Evaluation Review Technique (PERT) I

Welcome back my dear students friends so this is the twenty second lecture of the series if lecture for the project management which is a twenty hour course and the total number of lectures is forty number. So i keep repeating in order to make students to understand where we are how things are progressing. So we in the twenty first class we stopped at the point where how you calculate the forward pass method for the very simple diagram.

So for the first time in this whole series of lectures i will again urge the students first to understand the forward pass method the concept of how you utilize the early start? And use the concept of early finish and how you use the concept of late start and late finish? But remembering that in the forward pass method you will only start the job once all the preceding ones.

Which are in such a way that they are dependent on the jobs we are considering finish until and unless say for example in the last example that if four cannot start in one example at the end of the seventeenth day and another case it cannot start at the end of third day. So obviously we will consider seventeen because seventeen would mean that all the preceding jobs and activities of task corresponding to the forth job had finished.

So now let us basically use the same concept from the backward pass method but remembering that the concept was basically for the early start or the late start for the forward pass method the backward pass method it will be the finishing concept which will be utilized.
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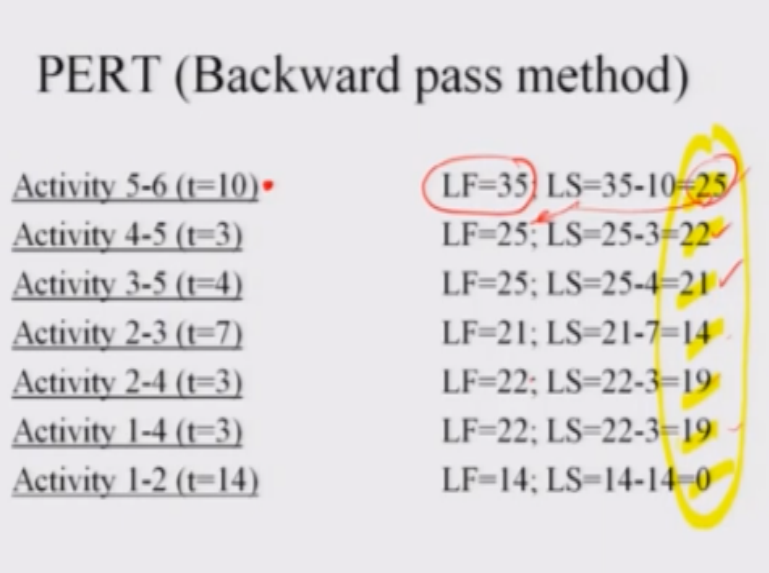

So let us co consider in the slide two sixty seven the PERT which is the backward pass method and activities in the initial forward pass method was from the left to the right from one two, two three, three four and so and hence so forth. Word basically put on the slide such that you do your calculations accordingly but now in the backward pass method you will basically note down the jobs from the right to the left.

So if you see the left most column which you have of the series of the activities it is not one two two three and so and also four but basically the activities are from five six till the last one which is one two and correspondingly to the right of the activities you have again written on the duration so for five six duration is ten days similarly for the last one which is activity one, two it is fourteen days.

Now let us proceed how we have basically done the problem for the backward pass method considering the duration of the total job is thirty five which is calculated from the forward pass method. So what was that that was thirty five so let us write down thirty five here. Which is the late finish for the activity five six so based on that we will proceeded to the late finish for five six is thirty five number of days now let us ask the question.

What is the number of days required for the five six job is ten days. So the late start which you can do for activity five six would basically be thirty five minus ten which is the duration which is X or D suffix A, D suffix B whatever you utilize and it gives you the value of twenty five. So note it down it will become much sense then as you proceed by the backward pass method.

Now for the activity four five you will ask the our self the question that four and five can ends only once the job five is six had started. So they take let us basically put it the other way five six cannot start until and unless four and five ends. So obviously the ending and the starting of four five and five six would be time accordingly. So if you consider the late finish for four five can only be twenty five which is coming from here.

Anyway i want to find out the late start of a four fifth activity it would be 25 - 3 which decrease the number of days which is twenty two. So note it down twenty two i know the calculation now let us come to the activity three, five. So three five again if you consider five is coming from here which is basically the first bullet point where I am basically marking my pointer.

So activity three to five would only end based on the fact that after that five and six can start. So let me understand that what is the late finish for this activity three five would be twenty five which is coming from here. And if i want to find out the late start it will be twenty five minus the number of days which is required for activity three five so it is 25 - 4 it is twenty one.

Let us come to activity two three so two three can only basically end based on the fact that activity three and five basically would have started after that only. So the late start is twenty one bring this twenty one here which is late finish and then if i want to find out the late start activity two three it will be twenty five minus the number of days which is been consumed for doing the activity 2 and 3 which is 7, 21 - 7 is 14.

Then let us come to two and four in two and four. If you consider let us go where four has just started. So it was late start was twenty two. Twenty two comes here 22 - 3 is 19 and i find point the late start of activity two four. When i come to one four so one four i find out is basically coming from the fact when four and five would start. So that was basically twenty two which is the late start for activity four five.

So this twenty two comes here 22 - 3 which is the number of days for the duration for doing the job activity one four was three. So 22 - 3 is 19 and the last thing which i do is basically for activity one and two. So one and two for that i now go which is the late finish now here is what i want to point out if you see activity two.

So these would end and after that only activity two three and two four can start. So let me note down what are the points let me highlight it for our better understanding. So one is this one one is this one so there are two jobs fourteen and nineteen. Now if it is fourteen and nineteen i want to ask myself that one and two what would be the late finish. So if I am considering the late finish for one and two considering that two and three and two and four can start.

You will see automatically the late finish if I am going from there backward pass method would be fourteen such that the late start would exactly match the day zero based on the fact when the actual activity or the set of activity for the whole project starts. So as you doing the backward pass method remember you are main important fact is late finish and as you are doing the forward pass method you are trying to basically concentrate on the concept.

As you proceed your early start early finish late start late finish would be considered in the same sequence of ways such that logically it means that a job can only start after the fact that all it is criticizes had finished. So consider if there are three predecessors i will only consider the job after all of them has finished. So what is important to know is that in the PERT forward method you will do the calculations and find out the jobs.

Where I am pointing my finger in the last slide two sixty six slide. You saw that similarly when i come to the backward pass method again i have the set of jobs. So now let me point it out for this set of values would be needed for me this one similarly they would be needed for the forward pass method also. So once i have the forward pass method values and the late and the backward pass method value.

I need to find out what is the total slack? What is the free slack? So now if you remember i had mentioned what is the concept of total slack and free slack. So let me go back to the slide where we had considered that.

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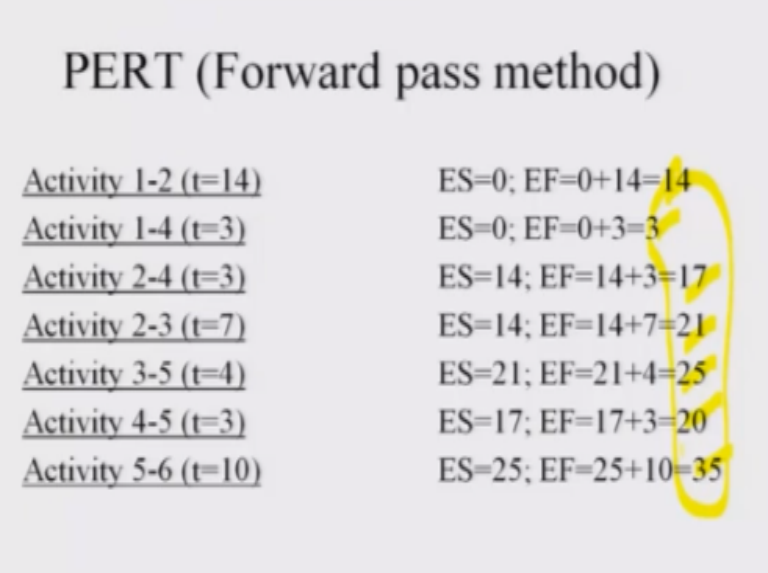

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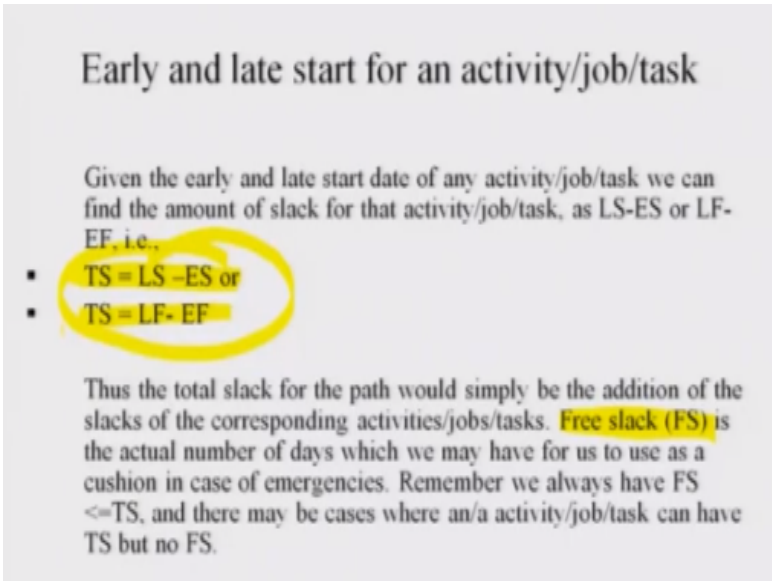

So these were the forward method and these values were needed for me. So these values are needed and if i go to the forward method and. So the total slack or the free slack was basically this so this is in slide two forty eight. So i did not consider that considering that people had picked up. So these are the formula which is going to use so total slack is late start minus early start and total slack is also late finish minus early finish and the free slack would be basically given accordingly. So let me again jump back to slide number two sixty seven.

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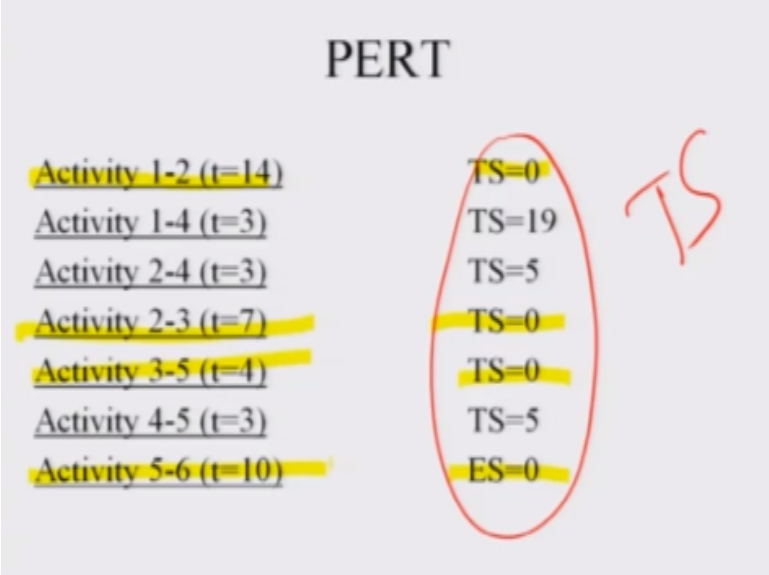

So these were the values based on that once i do that i have the total slacks for all the activities. So i would use the formulas as total slack i would not do it i will strongly urge the students to do it .So what is best is that write it down using the formulas of the forward pass backward pass and use the concept of total slacks once you find out the total slack values the values are given here activity one two total slack is zero.

So if you remember free slack would be less than equal to total slack which means total slack is zero free slack is zero activity one four total slack is nineteen. So free slack can be there positive or less than if is less than equal to nineteen can be any value what is the value iam going to come to that two four has a total slack five two three has total slack of zero three five has a total slack of zero four five is the total slack of five and activity five six is the total slack of zero.

Now again iam mentioning that the total slacks and the free slacks are calculated as denoted in just few minutes back in the slide and noted down. So now i want to stress the fact to the students pay attention to the values of total slack being zero. So let me highlight it. So what does it mean? It means whether you are going from the late start late finish or consign the early start early finish.

Whatever concept the amount to leave way for these four corresponding activities one two two three three five and five six is zero that means you do not have any leave way if there is no leave way it means they are on the critical path.

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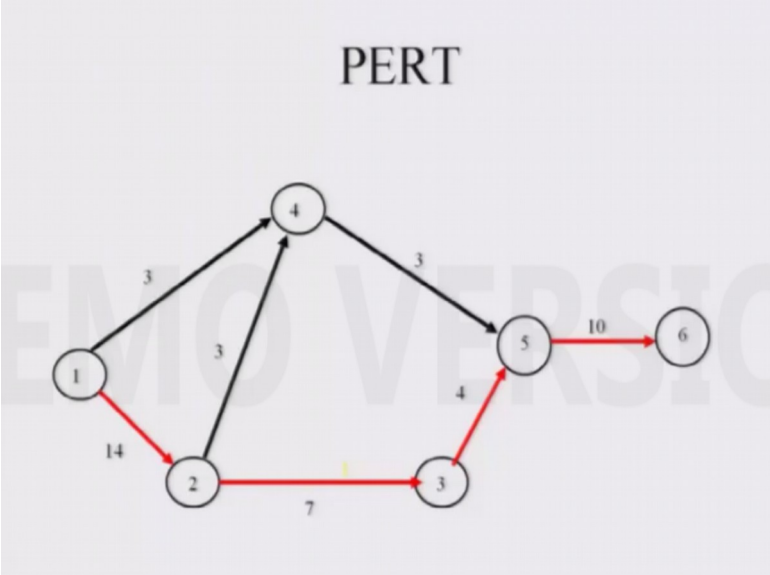

So if i draw it using the diagram which is there and now the critical path is being denoted by the red lines. So the critical path is basically one to two, two to seven, three to five and five to ten. If i consider the total number of days it will be fourteen plus seven twenty one twenty one plus four is twenty five, Twenty five plus ten is thirty five. So the total number of days is required critical is thirty five.

Now if i consider the overall amount or leave way which i have consider thus if iam going to four. So four can only start four to five can only start after one four and two four is finished so now consider one four is three number of days is finished but to reach four to the other route which is basically one two two to four you need fourteen plus three number of days which is seventeen.

Now seventeen means basically if one and four is finished three days it will have a huge amount of quotient. What is the quotient? That seventeen minus three that total number of days would be there such a quotient as that in case is this a delay you can delay it by a total quantum of fourteen days which means one and four. Similarly, if i go to five so five can be basically only start fourteen plus seven which is twenty one twenty one plus basically four is twenty five.

But if i consider five or the other work which has to be finished it can be fourteen plus three seventeen seventeen plus three is twenty. So here is twenty here is twenty five that means you have a so called quotient of five days which can be utilised how you utilizes that that will depend on what are the Constance which you are facing.

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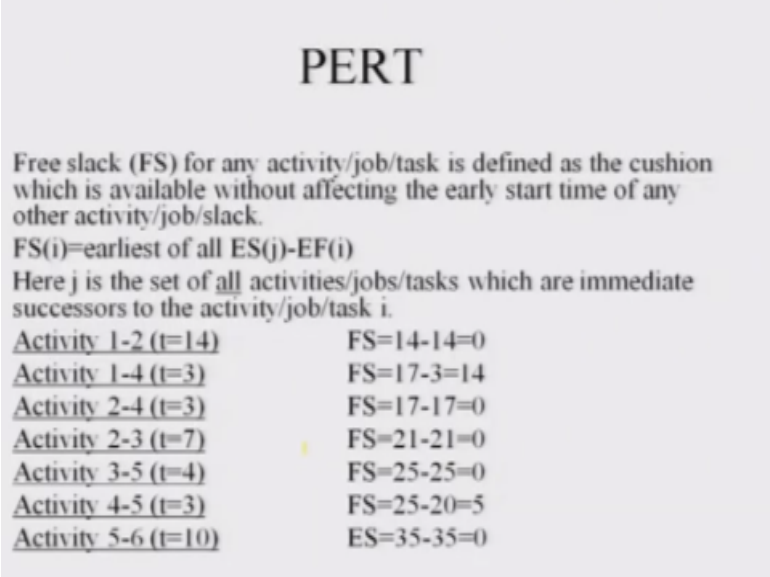

So now what i explained i will try to basically explain in more detail in this two seventieth slide. The free slack for any activity is defined as a cushion which is available without affecting the early start of ending other activity. So free slack you want to find out considering the concept of total slack would be earliest of E, S, J which is the early start of the J job minus early start of the I job so iam going in the consequence of I before J.

So iam basically going from the left to the right I would come J would come and correspondingly whatever variables which you get here J is the set of all activity job which are immediately successor to the activity of job activity task I. So I would be predecessor of J is the successor all sets of jobs which are there. So if i consider activity one two which is a duration on fourteen so i will basically consider the free slack which is fourteen the first term is fourteen.

The right hand side is basically the early start of all the sets of jobs which follow one and two. So that is fourteen the early finish for that particular job is basically fourteen. So hence i have the free slack for one two as zero. Similarly, if i go to basically activity one four the early start for all the set of jobs which are after one and four is seventeen minus three which is the early finish of the (()) (15:58) corresponding to that the fee slack is fourteen.

Similarly i found the free slack of two, four, zero which is zero two there is zero three five is zero. Four five is five and activity five six is zero. So total slacks are calculated free slacks are calculated we have found all the critical path also so now what is important to know let

me just sorry let me go back to the two seventieth slides free slacks are given it me go to the total slacks. So let us consider here total slack for one two activity was zero here.

See here the free slack is zero if i consider one four free slack is fourteen or the total slacks nineteen which means there is a cushion of five days which we can utilise. So depending on the total slack and the free slack you can find out the cushions and do our calculations accordingly so where there you can shift a job in block or in parts if parts are allowed such that it does not affect your total critical path and the total duration crashing is not being considered now.

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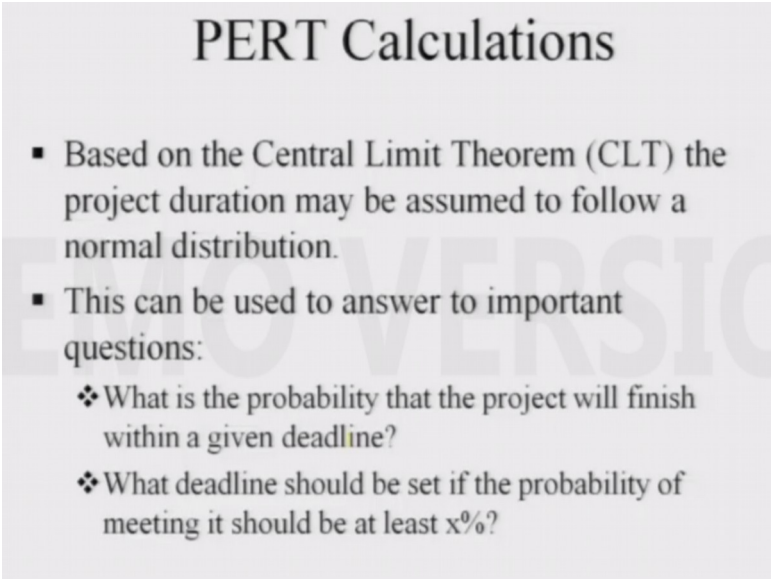

So now based on the central limit theorem the project duration may be assumed to follow a normal distribution now you may be guessing till now we have been considering the beta distribution now we are basically telling it is very normal distribution what we are considering that if we are keep repeating and doing the stimulation concept we will see that the distribution which will be utilising in order to find out the overall distribution of the stimulation concept.

For the time duration for any set of activities for the job would basically be normal distribution using the central limit theorem. It is a very important concept in statistics so this can be used as two important questions or several set of important questions. One is what is the probability that the project well finish within a given deadline based on whatever the deadline is given.

Why it is necessary? Because many of the contracts, many of the projects, many of the works being done set of activities is done they would be a deadline such that the contractor would be panelised if the dead line is exceeded but on the other hand the contractor would get a benefit. If the dead line is not exceeded in the sense the work is finished before in the dead line. So obviously you have to have a cost benefit analysis based on whether the deadline is exceeded or not exceeded.

Now you will also try to answer that what dead line would should be set if the probability of the meeting should be at least X percentage. So if i want to find out that within the dead line or within say for example fifteen days start of the work. What is the probability that what portion of the job is finished or say for example we can also try to answer that if there is a probabilistic frame work of time duration and the resource consistence are there or if time durations are increase or decreasing for different type of activities .

So you may try to find out that within the dead line off consider the contract had been signed for nineteen days. So within that nineteen days what percentage of job has been finished? So if say for example ninety percent of the jobs are been finished we will only concentrate on the rest ten based on the fact we will try to find out that those ten would consume what amount the extra amount recourses and what is the corresponding cost?

Or else we can also consider that those ten percentage have to be crashed then what is the overall cost i have to bare in order to meet the dead line.

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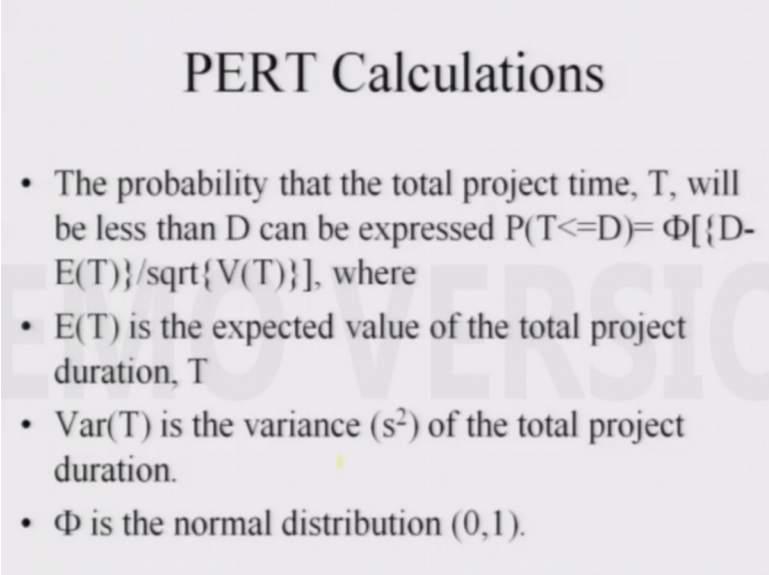

So the probability distribution that the project has if it is T then obviously using the simple concept of normal distribution. We can find out like this .So if the probability is say for example T is less than equal to d where D is the dead line then you can basically find out T minus the expected value of T which is the average time divided by the standard deviation. So this is standard deviation for the time duration.

So i want to find out D minus E by T divided by sigma T. So this whole value says for example the probability is alpha. So this alpha value if it is given as nineteen percent we will try to find out that using standard normal deviation tables at what percentage and what is the time take n or the dead line such that we are able to complete the ninety percent or if you are going the other way round like not fix the percentage of finish but we want to finish.

Fix a time duration then you can find out what is the percentage of the value of alpha based on which we can take action accordingly. It is the expected value of the total project duration VT which is sigma T here is the square root square of sigma and the variance if the variance of the total project duration and this capital value of this the normal distribution which we consider with a mean value of zero and standard deviation one.

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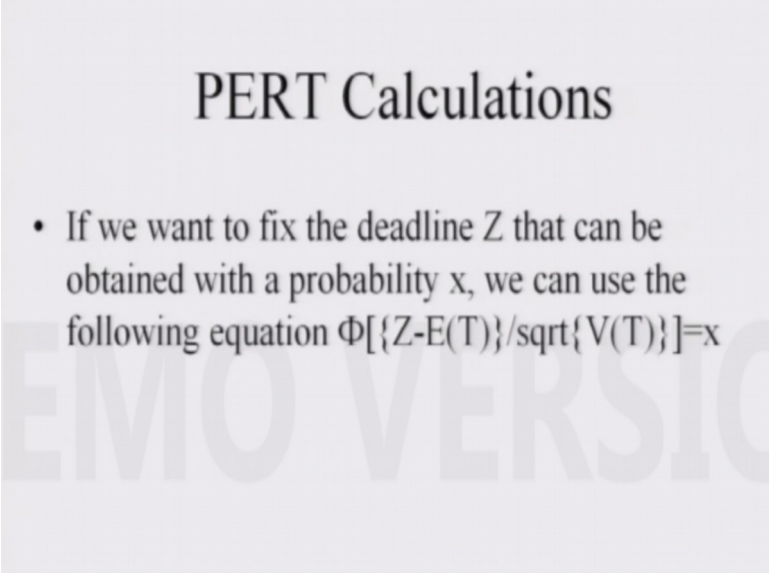

So if you want to fix the dead line Z that can be obtained with probability X that can be need to find out this so whatever the formula i just wrote in the two seventy second slide is exactly this. So i mean to find out T less than equal to D is equal to alpha. So if i know alpha then i can find out T minus expected value of and this variance of T probability of this D minus E of T variance of T and do your calculations accordingly.

In case if it is not there. So alpha is fixed. Then you find it out or say for example if alpha is not fixed now. So what i will do is that i will find out the probabilities in the same formula put it to some alpha. So alpha is not fixed but B is fixed based on that i can find out alpha and do my calculations accordingly and use stimulation central limit theorem standard normal tables to do the calculations. So again i will come back to the GANTT chart.

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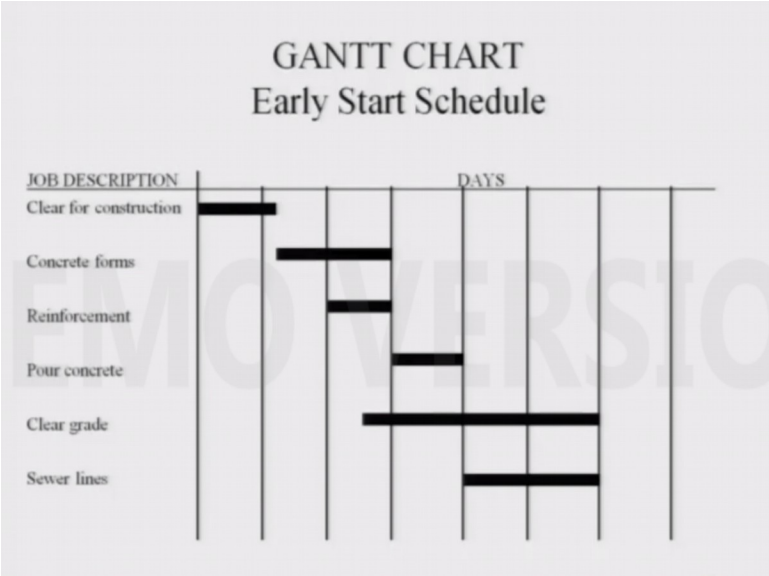

So consider you are doing a construction job so the construction job descriptions are clear for the construction concrete forms, reinforcement, pour grade, clear grade and sewer lines of they fix for building and civil engineering building and the durations of the days are mapped along the horizontal line from left to right and the bas which you see the solid horizontal bars these are the durations which are there for the construction.

So what is again i want to highlight even though we may have gone through that is that what is the relationship between these activities which are there on the first column like clear for construction, concrete forms, reinforcement, pour concrete .What is the relationship between the activities? Considering time duration is very important. So if i consider the time duration. What is important?

Is that whether i will ask myself? Whether they some delay between the end of activity A? Consider this is A, B, C, D, E, F. So if there is a delay between A and B or else there is a delay between the start of B and start of C. So this is what we are trying to bring into the picture that the GANTT chart would also consider the concept of end to end. End to start. Start to end and start to start. so this is basically end to start.

So end to start would be here this is basically if i consider this one and the second one B and C it will be start to start so start to start would be here so correspondingly the GANTT chart has been shown here. Where pouring concrete which is deactivity would only start after say for example reinforcement and concrete forms are done so relationship between B and D and C and D would be end to start.

Similarly if i go to clear grade it may so happen that clear grade would have a relationship such that it would be an end to start also from point of A and E or it can be basically start to start between corresponding to job activity C and E. Similarly for the F which is sewer line we have the activities marked in the bold horizontal line and it will also mean the end of E and end of F have to be same. So in that case it would be end to end concept which will be utilised.

So similarly i can consider that trying to basically move the jobs so if you consider the two seventy forth slide and two seventy fifth slide which is there in front of you can see many of the jobs can be shifted so as that the concept of slack which we consider in details using the

forward pass and the backward pass method is making sense once if you GANTT chart. So initially the GANTT chart.

If we consider was to only depict what is the GANTT chart later on here in the two seventy forth and two seventy fifth slide iam trying to portrait a GANTT chart such that it will be also be able to portrait concept of end to end, start to start, start to end and all these concept in a very nice manner but using GANTT chart in random with a precedence diagram gives a much better (()) (26:25). So that is what is the main focus of the discussion for the last two minutes.

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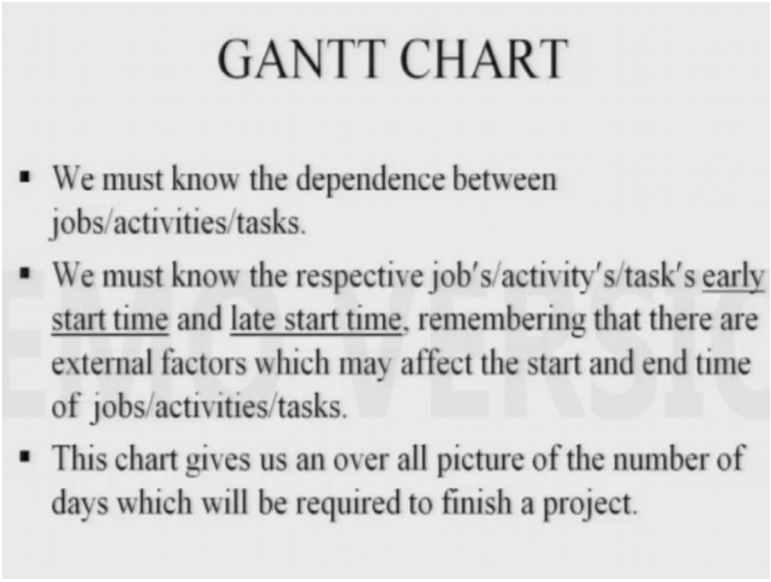

In the GANTT Chart we must know the dependence between jobs and activities and task so those dependence structure considering what concept we are using whether is end to end, end to start, start to start or start to end should be considered in a very clear cut manner. We must know the respective job activity task early start and late start time so these are important considering which concept you have following.

If it is EE which is end to end or start to start you will consider those accordingly and now also do the calculations based on the fact that what precedence concept are you using. So based on that we will do the calculations for the problem which we solved for the forward pass and the backward pass method we simply use the end to start concept which is true for the critical path and the PERT method.

So you should remember that there are external factors which may affect the start and end of the jobs based on which we can do the calculation. So the start which we just completed for

the GANTT one gives us a overall picture of the number of days which will be required to finish the project.

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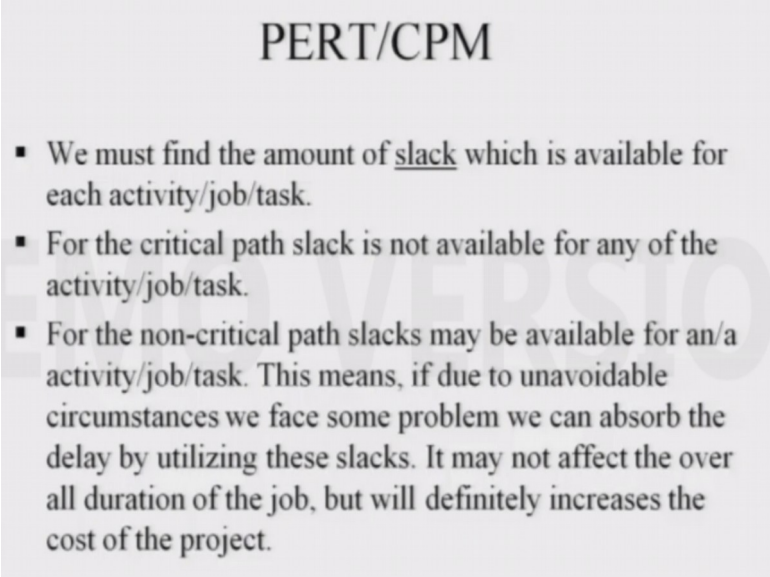

So for the PERT and CPM we must find the amount of slack the total slack and the free slack which is available for the each activity job for the critical path it is slacks is not available which means that that do not have any leave way to change the start and the finish of the jobs in the critical path but obviously you can crash. So as that the overall cross would come into the picture.

For non critical paths slacks may be available for an or a set of activity this means that if due to unavoidable circumstances we face some problems. We can observe the delay by utilising the slacks from this non critical activities in some way if possible in the critical concept but it will definitely increase the cost of the project so crashing would then be required.

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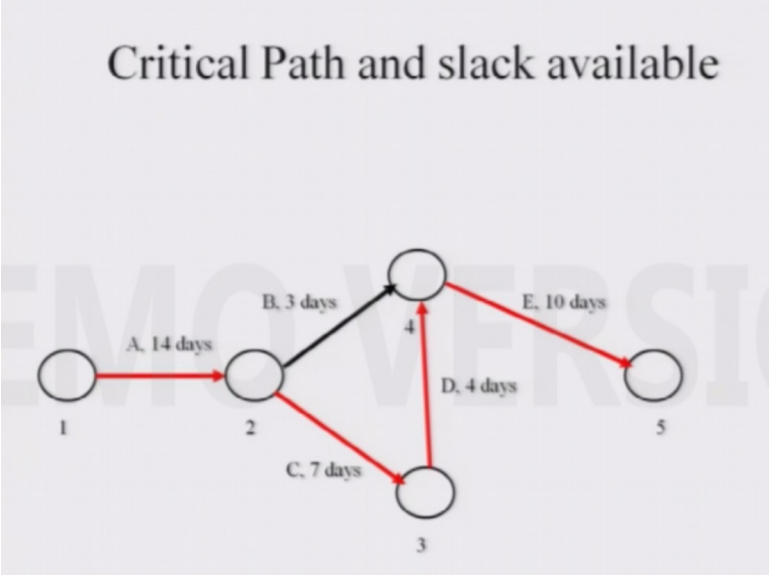

So this problem which you have already done is if you see the critical path. So here we have fourteen seven four and ten for the critical path. So this three means that you are trying to basically consider in such a way that there is a cushion. So that any add-ons here in B or trying to delay may not affect the critical path but if does affect then obviously crashing and such delays and cost over else would be there.

So with this i will end this lecture and then consider the concept of critical path and critical method CPM method and PERT method with more problems and then slowly going to the crashing of the jobs and explain this concept in detail. Thank you very much and have a nice day.