Project Management
Prof. Raghunandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology – Kanpur

Module No # 4
Lecture No # 17
Application of Utility Theory in Project Management - IV

Welcome back a very good morning good evening good afternoon to all my friends and students who are taking this course this is the seventh lecture each lecture as you know is for of for duration of half an hour and this is the fourth week we are into second class for the fourth week. So as we were discussing the concept of utility what is the concept of expected value what are the different utility function which you have.

How the certainty value is calculated how you make a decision depending on different decisions are probabilistic so we have covered all these things. So let us continue and discussion and we will basically expand our discussions in a little more interesting practical applications from the conceptual point of view and once that is over we will definitely go and solve few problems. So to give a brief outline of what things are going to come we will try to finish off weekly analysis in in as much as detail as possible and then start off the concept of precedence diagram.

So by that way we would generally go into the network concepts and then in between we solve few problems and also come back to the different back to the different ideas how ratio how financial concepts are used in order to take decisions related to projects and investments like IRR then interest rate and all the things.

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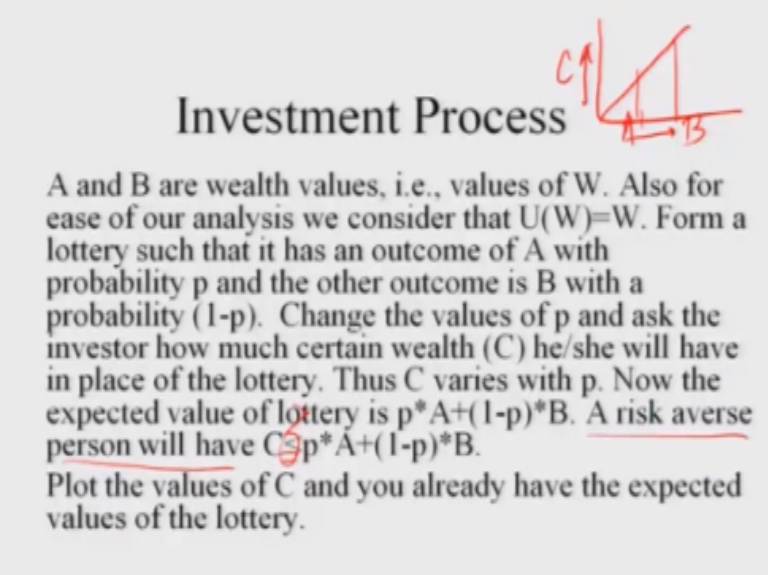

So consider again A and B are the wealth values if you remember if you remember I did draw the diagram and explain to you how we can find out the certainty value certainty value are measuring along the Y axis and the investments so here was a here for B and you made a decisions and this was the forty five degree line. So what I discussed in the last slide of the sixteenth class I will just start of the seventh class with their discussion of that slide.

So if you see this this red color diagram where I am basically hovering my pen so based on that the discussion starts. A and B are wealth values that is the values of W which is the wealth in Rupees, Dollars, Euros, Yen and Deutsche mark Deutsche mark is not there in existing now but Canadian dollar or Australian dollar. Also for ease of analysis we consider the utility analysis is W which is the straight line forty five degrees line form a lottery such that outcome of A with probability P.

So you can make it much simple by considering probability and half and half so A has a probability of P this A and the other outcome B would be definitely have a probability of 1 – P because the sum of the probability would be one change the values of P and as the investor how much certain wealth see he or she will have in place of the lottery. So the concept which I discussed was trying to change the value of A and B you can also do by changing the value of P.

So let us continue as it is written and it will absolutely make perfect sense what I discussed in the

last class the sixteenth one what time discussing the seventeen th the lecture thus as see very varies with P as a probability changes now the expected value of the lottery which is there in front of you would be A multiplied by corresponding probability P + B multiplied by 1 – P. A risk averse person will have this value was this so if there is equality sign here it means that I am indifferent if the value of C is more means I am more risk averse and will and the person just one minute or risk averse person will have this one.

So this should be technically be greater sorry it basically means that if I am risk averse I will shun or shy away from the gamble and if I want risk I will basically go towards the gamble and shy away from the certainty value. So if it is equal that means the value of the gamble with respect to the C value are exactly because now there is only one point which you have already you have understood the value of C is basically technically you see you see is certainty value.

When converted to a utility is you with the variables as taking the value of C now that is a straight line because the utility function is W hence the value is C. Similarly for A it is UA which is again back to A and B where the utility function is UB is equal to B again because the utility function is linear.

So as P values are changed or as if P values are fixed and A and B values are changed you plot the values are C they can be the above the straight line below the straight line and based on that you can find out whether it is concave or convex or straight line and find out whether the marginal rates are increased and increasing rate increasing at a constant rate increasing and a decreasing rate then corresponding to that you can find out A, A prime R, R prime. A is basically absolute risk aversion property and R is relative risk aversion property

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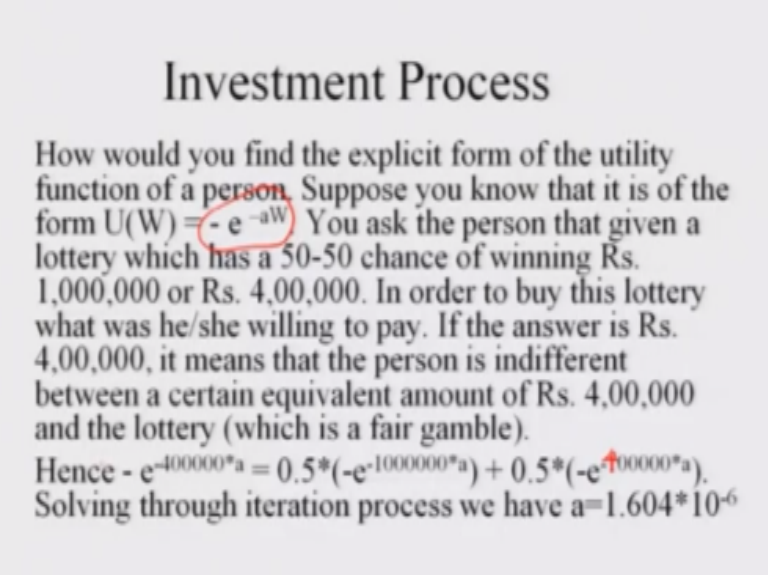

So how would you find the explicit form of the utility function suppose you know that it of the form UW is exponential in nature which is this function which is there in front of you. You asked the person that given a lottery which is fifty fifty chance of winning that means you are not tossing an unbiased coin an unbiased. So the chances are the amount of money is one million or ten lakhs than that amount is four lakhs in order to buy this lottery what will that person he should be willing to pay if the answer is four lakhs.

It means the person is indifferent between the certainty amount of four lakhs and the lottery. So when you are trying to find out is basically considered you place it in front of the person and you do not know the value of A. Now here the value of A has to calculated now if you put this value ten lakhs and four lakhs in the gamble. So correspondingly you will have these values that we corrected it will be for next year 1, 2, 3, 4.

So what N this is ten lakhs so this value is the utility corresponding to ten lakhs this value is utility corresponding to four lakhs this point five point five probabilities and if the certainty value is four lakhs you put it here. So here one is there is not written here because certainty value is probably one based on that you can find out that what is the value of A you iterate different values find out the though certain values and equate to the gamble slows you put it put it out or basically equate it you will find out different estimates of A and take the and the best estimate

considering the statistical problem test you are doing and you can basically find out the value of A.

Similarly you can do for the quadratic utility function you can do for the log it you could be utility function you can do for the power utility function and correspondingly to other utility functions which you may also find. So utility functions and some axioms or the rules so the actions are very conceptually very interesting so let me read then and explain it very simple terms they would have not consequence on how we solve the problems.

Solve the problems mainly utility would be from the point of view of decision tree analysis and general utility as concepts and then we will be always those concepts in the areas of project management.

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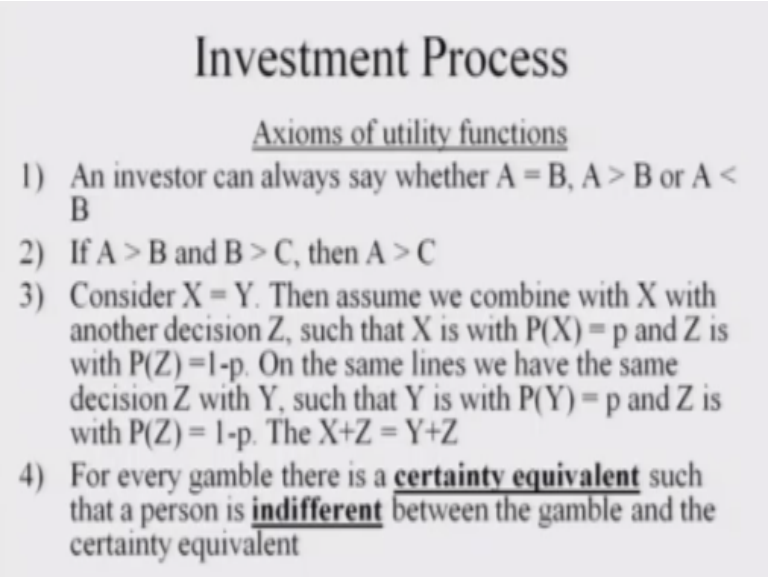

So an investor can always say that whether A is equivalent to B where A is better than B and where A is not as good as B or that means put it putting it it another B is better than a that means my charges are I mean different name B I am inclined to take A or in the last case I am inclined to take B. So in case if I prefer a with respect to B I prefer B with respect to C and then logically I would always prefer A with respect to C. So this this is a axiom but in general many of the practical cases we may not be true to consider the X is equal to Y the third axiom when we assume we combine X with certain decision Z.

Such that the that X is with probability P and Z with probability 1 – P there are two outcomes on the same line we have the decision Z combined with Y Y and in that case if probability of 1 – P then the probability of Y is P then you assured the value or the equivalent between X + Y + Z that means the decision outcome is equivalent to Y + Z such that you can say what is the outcome by combining X and Z is equivalent to combining XZ with Y.

For every gamble or every decision which you make or every say for example investment which you make in a project there is a certain equivalent such that the person would be indifferent with respect to the gamble and the certainty value which we have already discussed into the one or two very simple problems.

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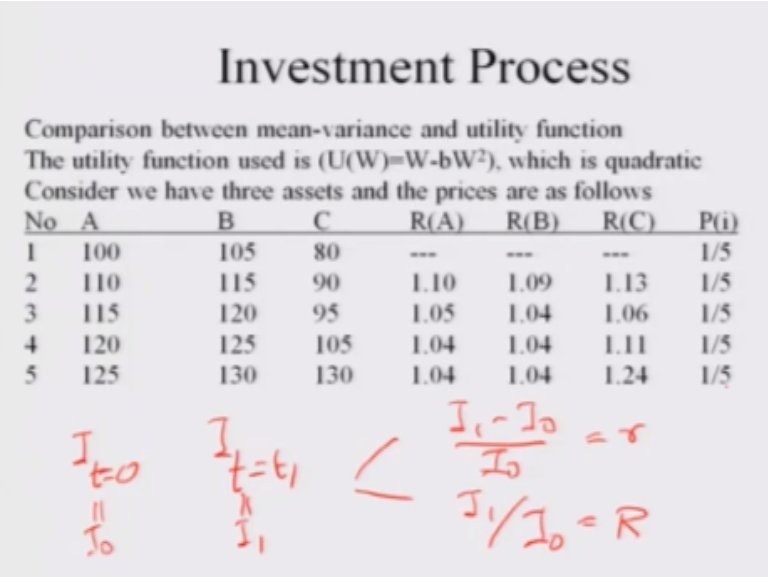

So now comparison we come to companies in between the mean variance concept and the utility concepts so let us pause here for one minute if you consider the capital asset pricing model and the linearity prices and the concept which we discussed.

So it generally means the fact that we are considering in a sense the return distributions are elliptical in nature means they have some properties i am not going to go into that but normal distribution is one distribution which I did mention time and again when we doing the quadratic

utility function that if the returns are normal distributed then the actual utility function is quadratic and vice versa.

Because there are some very nice properties about that so an mean variance theorem which was basically the work done by pioneering work done by markwitz for which and other work he wants the nobel prize. So we will consider the mean variance concept from the point of view investment and the utility function consider the utility function as a quadratic where B is the parameter for quadratic utility function and we have three assets and the prices are given as as in this table.

So A, B, C are given I find out the returns so returns can be found out if you know in simple status in finance. So returns can be say for example I invest at time T is equal to I0 so this T is just for the notional concept and then at T is equal to t1 I get my value. So consider this value as I1 this value as 0. So returns can be calculated according to this concept I1 – I0 by 0 this is R and another one is I1 by I0 which is capital R returns. So you can do it accordingly to whatever you consider.

So A, B, C prices are given based on that I find out the return for A, B, C individually and the probabilities are given is basically are equal. So there are five outcomes and considering one fifth for each of them.

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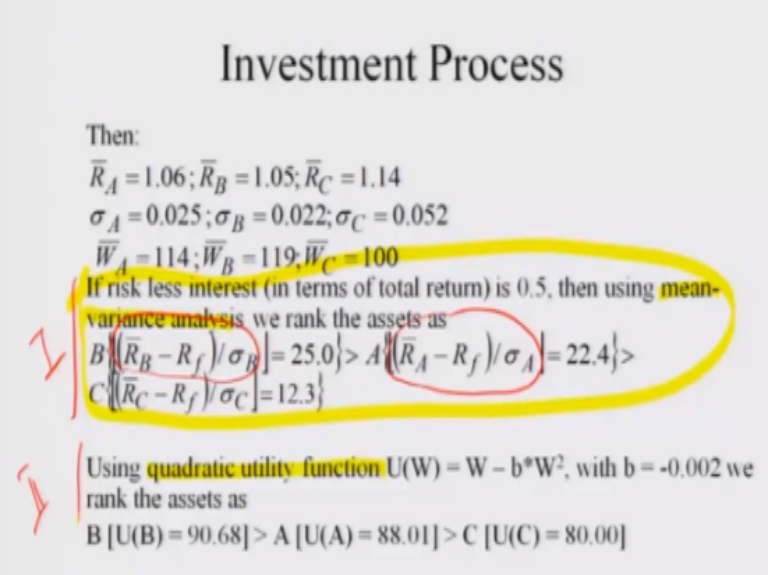

So this is a distribution is one fifth means it is a uniform discrete distribution just for a interest of the of the students who are interested I will just mentioned that now what I do is that I find out the average values of returns that means sum the R A values use for all the values and divide by number of outcomes similarly i do for B and C. So the average returns with the bar is given for A as 1.06 for B as 1.05 and for C as 1.14 which technically means I am getting a extra plus return of six percent plus return of five plus return of fourteen percent.

So it was in the other side so it would be less than one and similarly I find out the standard deviation using very simple calculations as prevalent in statistics I find out the standard deviation for A as 0.025 be as 0.022 and C as 0.052. So they can be converted into percentage sense also now I consider say for example the prices average WA, WB and WC as hundred and fourteen hundred and nineteen one hundred.

So if risk free interest rate in terms of the total risk so if you remember I did mention R suffix F which is the risk free interest rate for investing in a government bond or security for which technically we consider these no risk. So there is a average return but conceptually the risk is not there and we consider this S is 0.05 then using the mean variance analysis.

So what we do is that we will concentrate it here this formula it basically normalized the return of B with respect with average value and standard deviation. Similarly for B A we basically

normalize the return of A with respect with average value and standard deviation and similarly we normalize C with respect with norm on average value and it is a standard deviation. So the normalization is being done based on day term whether the return of A or B or C more or less than RF which is the risk free interest rate.

So if RB is more than say for example RF it make sense to invest so technically what we would generally do if we are not thinking of risk for the project we will first basically analyze the returns as the primary motivation based on which we will rank them.

Then if you are only considering this nor the return is to the maximum value of the risk and if you basically trying to combine them and find out the ratios then we will consider if we remember consider the ratio of mean to risk or risk to mean and rank them from respectively from the maximum value to the minimum value or from or for the second case from the minimum to the maximum and take it correspondingly.

Now why RF is coming is because the risk free interest rate is being considered as a data based on which you will basically analyze the investment for the project is right. So if you find difference of RB over RF normalize with respect to the standard deviation which is the dispersion which is the twenty five percent for A it is twenty two point four percent and see these twelve percent point three percent and sense technically for project B where you will consider as twenty five percent as the return is the highest.

Hence we will basically allocate our amount of money from the project accordingly. Using quadratic utility function if you now basically have so this was done the first part where I am now marking it as one was done based on the fact it can let me highlight. So this part was done based on the fact that the mean variance concept was being used which was by the Markowitz. Now in part two if we consider this part what we are doing is that we are considering the quadratic utility function to be true.

So quadratic utility function is given where B is minus zero point zero zero two and what we do is that based on the price is here returning the standard deviations are coming into the picture

based on the prices we found for the utility function they come out to be ninety point six eight for B. So the B prices are given hundred and ninety for eight comes out to be eighty eight point zero one for A hundred and fourteen the weights are not the weights the wealth is given.

For C is eighty point zero again for C it is given as hundred so based on that one will tank them you will find out the quadratic utility functions concept being used then the utility of B is highest than A and A is better than C. So rank them so if you compare the first concept being utilized for ranking and the second concept being utilized for the ranking generally they match because by due to the concept the underlying assumption being fundamentally true for both of them.

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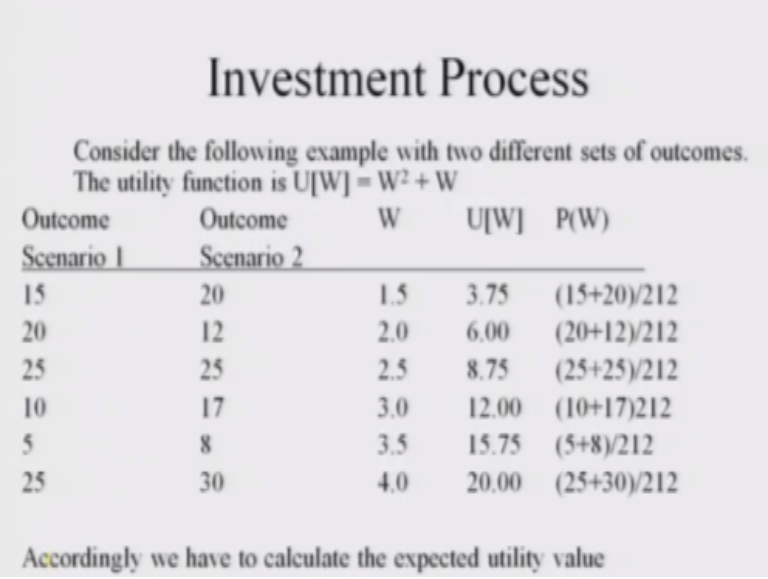

Consider the two different sets of outcomes again coming back to similarly type of problems which we which we started with for discussing utility functions. So scenario's are given outcomes was scenario one which is fifteen twenty twenty five ten five and for the next one against the scenario's are twenty till the last one is thirty.

Now what we have done here is that we have taken again the wealth is the same time type of problem just a reputation in order to like kick start or thinking process in a much more positive light. So W's are given based on that we found utilities and the probabilities are given probabilities are basically simply that you find out the total sum of the scenario's which is scenario 1+ scenario 2 and then find out what are the corresponding probability. So for scenario

one and scenario 2 there are two outcomes based on that you find out the probabilities which as given in the last column again accordingly we find out the expected value and rank them.

If there are more than one projects and if both of them are same we then basically go the risk if again if risk is same say for example. So we may be tempted to rank use the concept of risk to return, return to risk. How good is better than risk free and all those things.

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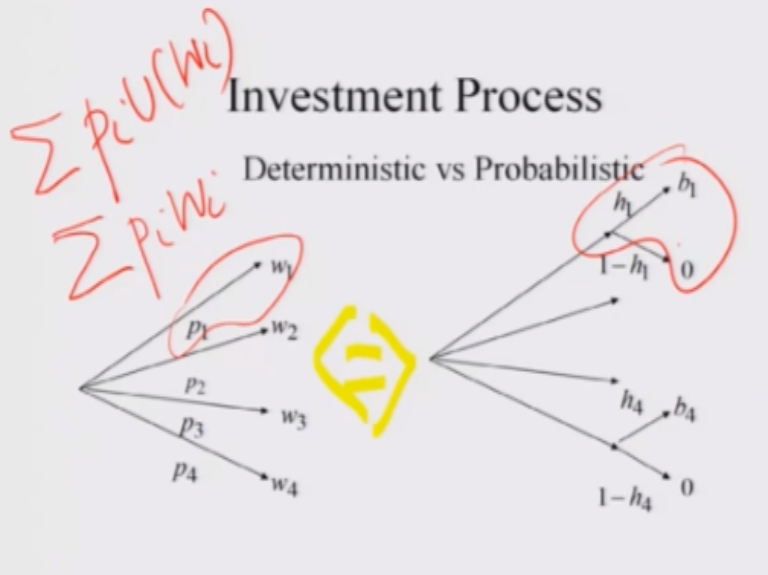

So deterministic probabilistic we have already considered the diagram so I am just again highlighting it the equivalence between them would be true if the overall expected value which we have overall expected value this case would be PI into WI. So PI how many I's are there one, two, three, four blue I I am considering as the utility so incase if utility is A is linear then it is W incase if it is not linear then I have to use the utility function as given so I basically sum up.

In this case what I am doing is that for each arm that means the PI and PW I am taking them into a fair gamble and then trying to find out if the utility concept to the on the certainty value is equal. So in this case what you will happen is BI into HI plus zero into 1 – H1 should be exactly equal to P1 into W1 but again we are considering the B1 and zeros are the corresponding utilities based on which we are doing out calculations.

If there is some other utility function you will basically use this utility functions but multiplied by the corresponding probability by which in this case H1 and another arm as a probability of 1 – H1. So generally for the investment processes or the project management concept of investing in projects on making a decisions.

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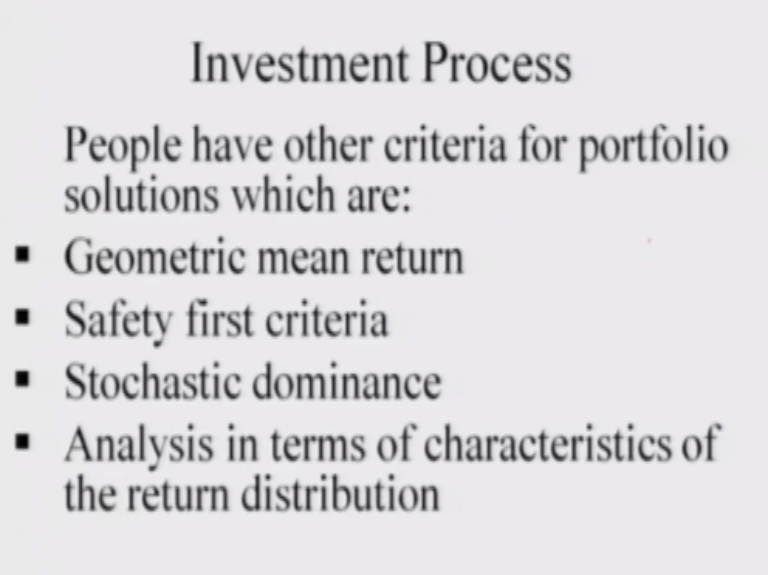

People have other criteria of selections also one of them is the geometric mean return one is the safety first criteria third one is the stochastic dominants concept and analysis of terms and characteristic of the return distribution. We will consider on detail the first two and I just mentioned about the third and the fourth and then proceed so I will try to as I mentioned as we started this lecture will definitely go through one are two very simple problems for these concepts as we proceed with the project management course.

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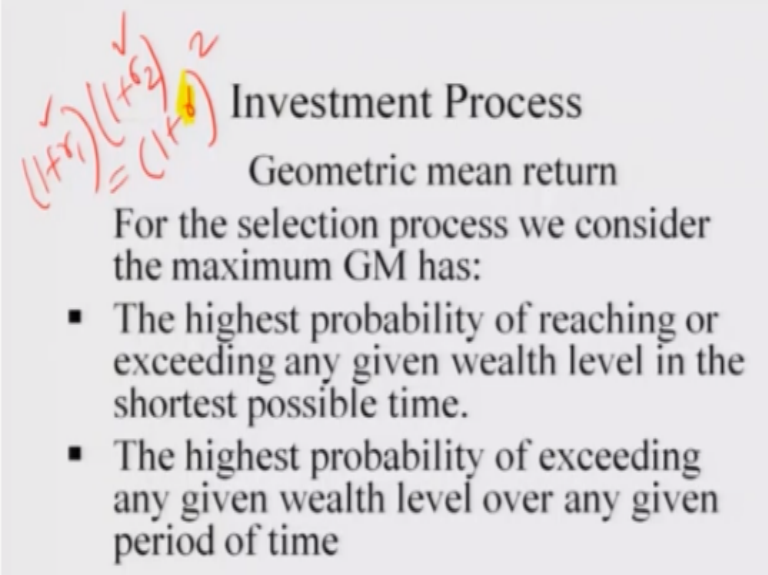

So the geometric mean returned it the distribution means for the selection process we consider the maximum geometric mean such that is the highest probability of reaching or exceeding any given wealth left in the shorter possible time or it is not returning maximum the high probability of exceeding of given wavelength over and above the given mediate time.

Now why geometric mean is used so if you consider very simple statistics we know that we there are generally there are three type of average measures. So I am not considering mean median mode and the median I am considering the average characteristic one is the arithmetic mean one is the geometric mean one is the harmonic mean. So when I am considering the arithmetic mean is just the simple average so if I have ten jam bottles and I want to find out the and the weights of all of them are given I want to find out the average weight.

So I will just sum them up divided by number ten which is the number of jam bottles if I want to find out the height of the set of students I add up this height divided by number of students. Now if I want to basically use the concept of harmonic mean then the harmonic mean is a very interesting example which we all know consider a car travels form Delhi to Bombay at a certain speed and the comes band take certain time and then it again travels back from Bombay to Delhi at a certain speed.

And we are considering a certain time and you want to find out what is the average speed of the car travelling between Delhi to Bombay and back and then we will try to use the concept of harmonic mean so I am not going to give the formula but just for people who are interested they can check up any simple class eleven or twelve book or even class ten book. Now if you come to the concept of a geometric mean in generally geometric mean and used in financial concepts because interest rate are accordingly calculated.

So here I will try to give the formula because this is of interest for us to this course and I will as I mentioned that for arithmetic mean and force basically the harmonic mean people can check simple examples very interesting examples. Now when you have two interest rate R1 and R2 for year one and year two when I want to find out what is the interest rate which is being applicable for year one and year two combined then I be actually I will be using the concept of geometric mean.

So how it is done is like this say example interest rate for year one is R suffix one interest rate for year to is R suffix and I want to find out what is interest rate between this two years combined. So if I have year one, one year, year two second year and this is equal to interest rate for two years what I do is that I know R1 I know R2. So if need to find out RR and then I simply out it in this formula and calculate so this is the concept of geometric mean averages so generally geometric mean returns and average is concept are used in these cases.

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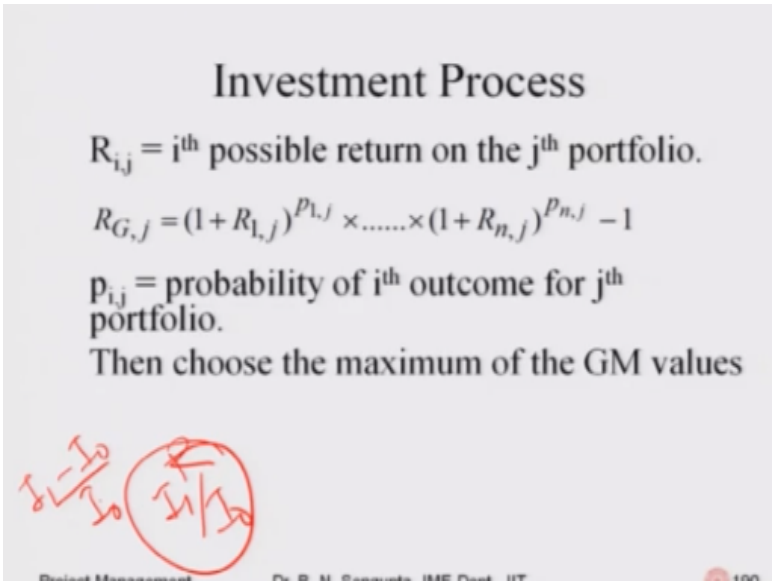

Consider the IR this capital R is the intake if you remember again just for the collection if we remember we have two values i1 – i0 by i0 and another was i1 by 10. So this is capital R which we are using here in a calculation so capital R suffix I and suffix J where I possible returns for the J portfolio of Gth investment or Jth project is given and I want to find out the geometric mean averages of portfolio or a project set of conglomeration activities which are there.

So it is given by to notice 1 + R suffix 1J is basically for the first possible return of the for the J project multiplied by to the power corresponding to the probability for the first similarly we continue doing it till the last one which is basically R1 + R suffix N comma J for the Nth possible return for the Jth project multiple to the power PN I suffix N comma J which is the probability and -1 because you have to find out the percentage.

So based on that I can find out the return and the geometric mean concept for the projects and then rank them accordingly. So this PI suffix IJ probability Ith income for the Jth portfolio then choose the maximum value for the geometric mean values and then tank them accordingly.

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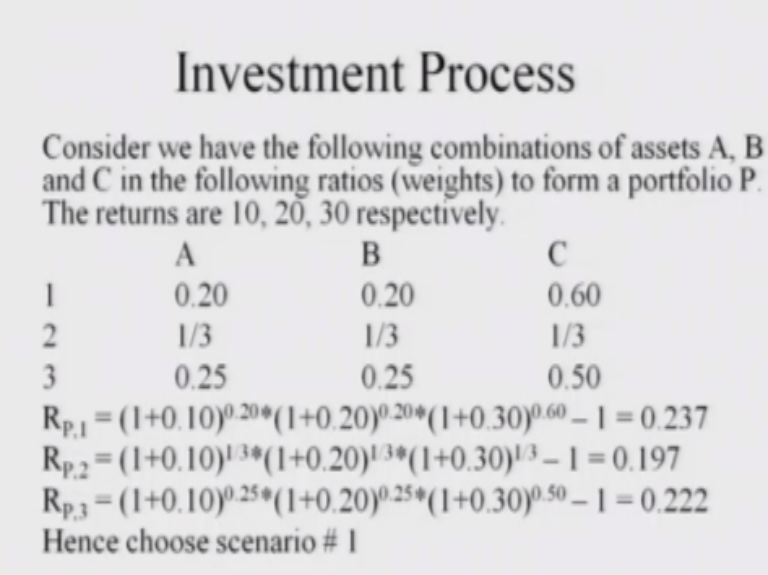

Consider we have the following combinations of assets ABC are in the following figures as they are giving this table ratios are for one it is given as twenty twenty sixty some is one. For the second one which is one third one third one third for the third one is twenty five percent twenty five percent and fifty percent and the returns for ABC are given as ten, twenty, thirty can be in percentage term it can be short terms whatever it is I am just giving the values.

So if I want to find out the return for portfolio one portfolio two portfolio three scenario one, two, three with the suffixes it would basically make sense. So where is my finger is PV is the portfolio comma one means the case so in these case for the first one it will be one plus point one because there is a return to the power twenty. So this twenty is coming the second one will be 1 + .2 to is the return the power twenty and the third one is ten plus point three to the power sixty minus one it comes out to be point two three seven which is twenty three point seven percentage.

For the second one which is there in the calculation which is ten twenty thirty remains same ten twenty thirty and if I want to find out the probabilities are given as one third one third one third so here the probability are one third one third one third minus one you are return of nineteen point ten percent. And if I go for the third one again returns are ten twenty thirty probability are twenty five twenty five fifty and I put them on to the power as given here I will find out it as twenty two point two percent.

So if I again look it into in this diagram choice is scenario is one considering the concept of geometric mean return you can could have done using the utility analysis can have done using the concept of mean variance for concept of the portfolios for projects. So this is just an example to give you that how different concept can be utilized to rank projects rank decisions accordingly.

So with this I will end the seventieth class and then again consoles this finish the first portion of the utility analysis and the concepts in the eighteenth and most probably we may go into the nineteenth one but I will try to finish in the eighteenth one such that we can cover a whole lot of new concept which are there which are exciting for the students and I am sure they are going to benefit it they study again I would urge I am bringing this point again please study the slides understand the concept right to the forum get in touch with us.

The TA's and the instructor and please read the books there are lot of books which are available and the old set of books which is given they are also available in PDF format in the net you can definitely search the net or lese get the hard copies of the book from the liability and I am sure if you understand them read it do the problem many of the concept interesting concepts actual practical incremental of concepts of the project management would definitely get to clear thank you very much have a nice day