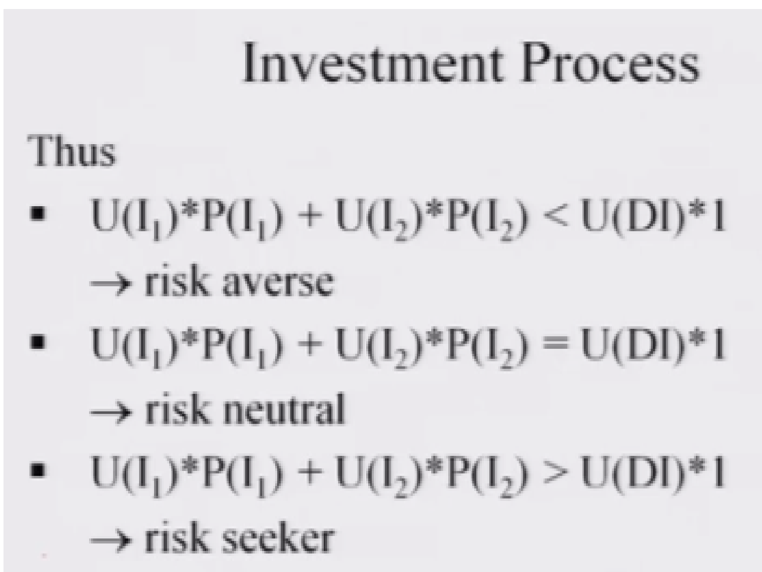
Project Management
Prof. Raghunandan Sengupta
Department of Industrial and Management Engineering
Indian Institute of Technology – Kanpur

Module No # 3
Lecture No # 15
Application of Utility Theory in Project Management-II

Welcome back a very good morning, good evening, good afternoon to all my dear students this is the fifteenth lecture on which is as per the norm the lecture for the third week related to the course which is project management. And we have just started the concept of utility analysis and then the last slide before we ended the fourteenth lecture I gave the example.

That how a person depending on his or her utility can make a decision such that the overall picture we get about a person can be the risk loving person, risk hatred person (()) (00:57) different person depending on the gamble and the sure event particular person is trying to compare so let us continue where we left.

(Refer Slide Time: 01:09)



So if we see this slide this is the investment processes and we did mention about again I am repeating it the two characteristics one was the concept of non-citation that means more I give the more the person wants which means the first derivatives of the utility function is positive.

And the next point was as I mentioned risk aversion properties either I love I hate or I am in different to the concept of risk.

So if you see this slide it basically means if I am trying to balance two slides of the equation in the first one in the first equation it means that what is the expected value of the gamble. The fair gamble example it can be the gamble depending upon the what the outcomes are but our example was basically related to with the fair gamble. So the first term which is the multiplication of two terms is U I1 into PI1 where I1 is the investments.

So the first term is the utility multiplied by corresponding probability second is the second outcomes utility per corresponding multiplied by the probability. And in the right hand side of this less than sign equal to sign greater than sign is basically the share events. So you have basically the deterministic event or investment DI multiplied by 1 which is the probability.

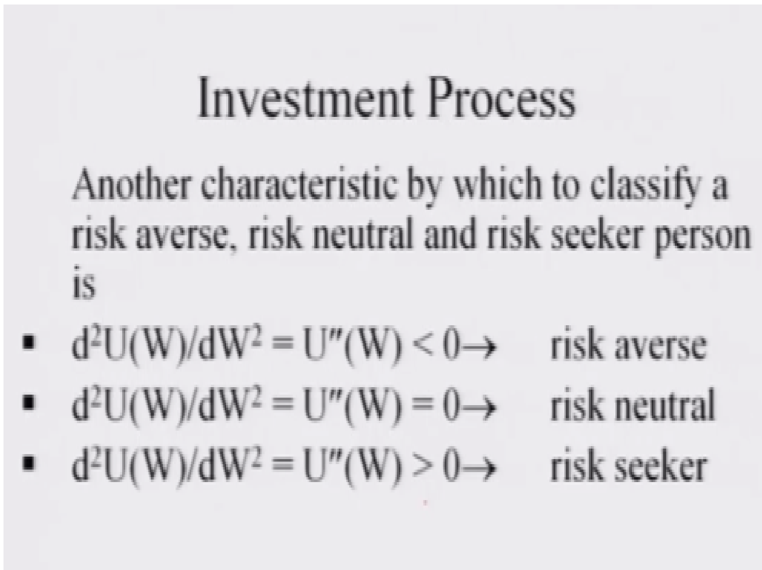
So in this case if in the first equation of first bullet point if this is less than this right hand side which is sure event is less than the concept which is there on the right hand side left hand side which is the gamble. Which means the person is more inclined to take the sure event so he or she is risk covers in the same notion if we consider equality on both side for this equations it means the person is indifferent and if it is greater the means the left hand side is more then we have the person who is a risk seeker or he wants to take a risk.

Now risk seeker or the risk averser is neutral person is trying to analyze the problem from a very typical point of view. Now if you see the gamble the last example if we consider a fair gamble and the sure event if I am risk averse person. I am always thinking that the chances even the probabilities are exactly the same half and half in the last example or in probability is PI1 and PI2.

The person is thinking that he would definitely get because as he basically risk averse he would be in a position that he forced to take or the overall event would come in such a way that it will be negatively beneficial for him or her. So it is best for the person to take the sure event in case if is a new risk neutral person he or she is trying to balance what is the expected value or the net output both for the gamble and the sure event and if it is a risk seeker person he or she is thinking of that probability and the corresponding utility which would come out and benefit him or her.

Even though in a long run if you keep playing a gamble obviously the expected value in both the cases is for the sure event and the certain event both are same.

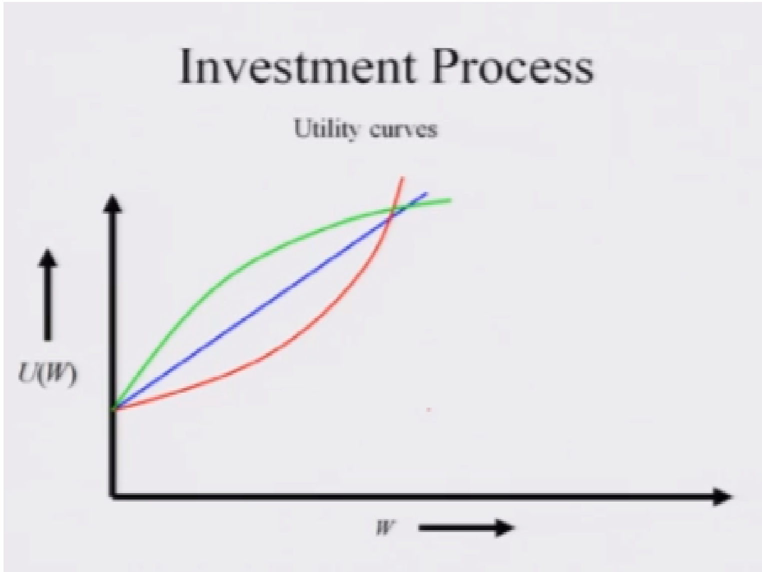
(Refer Slide Time: 04:41)



So another characteristic by which we can classify a person as a risk averse, risk neutral and risk seeker is by considering what is the second derivative of the utility function. So is the second derivative of the utility function is in the first bullet point it mentions if it is less than zero it is a risk averse person if it is in the second bullet point it is basically equal to zero the person is risk neutral.

Similarly for the third bullet point it is greater than zero the person is risk seeker we will see that using the concept of absolute risk aversion property and the concept of relative risk aversion property. So now to give a very simple conceptual notion on the graphical notion such that it it really make sense for the people who are taking this course.

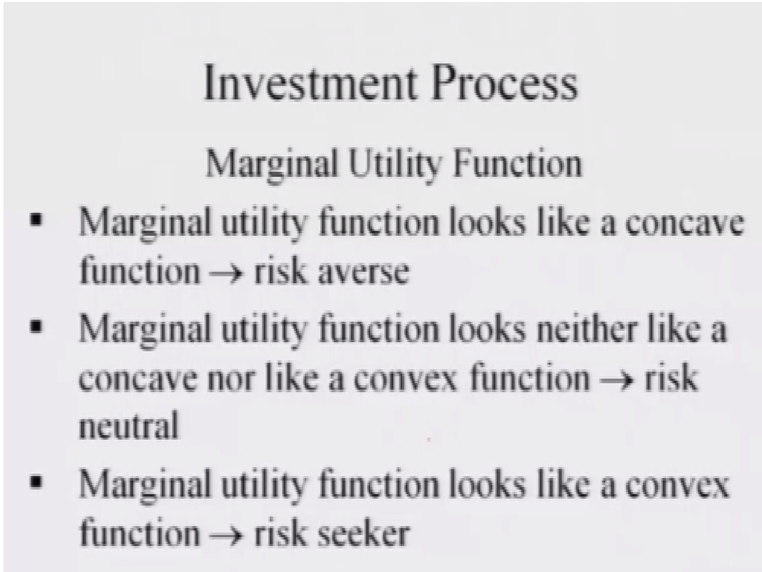
(Refer Slide Time: 05:32)



Let us consider the three graphs which are there shown here along the Y-Axis we have the utility. Utility of the decision the project the investment whatever it is there another X axis is there wealth this amount of investment you are doing. So there is a green curve which is slowly decreasing the rate of change of the function I am mentioning the rate of change of the function because it should be useful in the later few slides is decreasing.

Decreasing in the sense the decrease is there but it is increasing at a decreasing rate that's I wanted to mention if you concentrate on the blue one it is increasing at a constant rate. If you consider the red one it is increasing at a increase rate. All these three diagrams would definitely immediate very easy concept that how the concept of risk aversion property neutral property risk seeking property would come out from this.

(Refer Slide Time: 06:36)

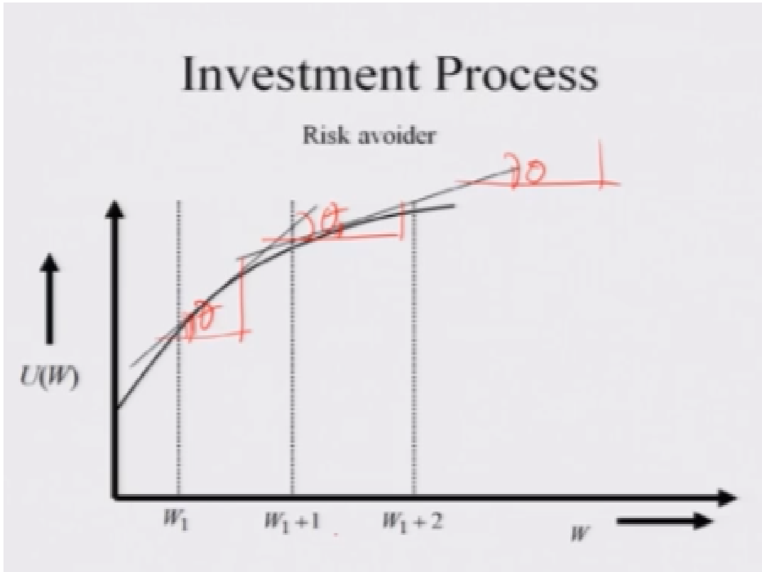


So now let us go into the concept of marginal utility function and what we mean by the marginal rates. So the marginal utility function basically means the first derivative based on that. So if the marginal function looks like a concave function it is a risk aversion property. So if you see you back to the last slide and in in in just covered last which is the one forty nine slide.

You see the red one which is increasing which is obviously mean which is the marginal utility function is look like a convex function which is the person is a risk seeker. If you see the blue one which is the second bullet point which is the marginal utility function neither concave nor convex which is the straight line which is the straight line which is just neutral and if you see the green one which is dipping going like this.

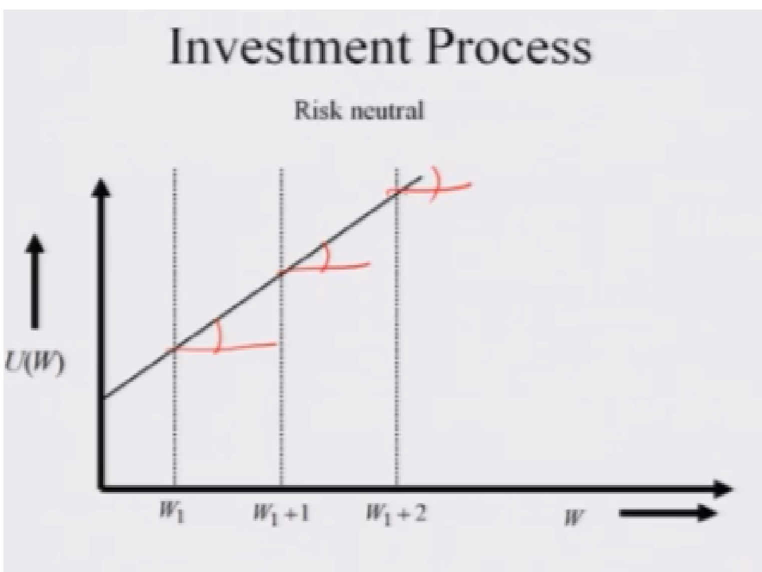
Then it is a person who is a risk averse person so marginal rate increasing at a decreasing rate now means a risk averse person just mentioned one minute back. Marginal rate increasing at a constant rate which is a straight line is risk neutral marginal rate is increasing rate red one is a risk seeker so here what I was mentioning a DYDX

(Refer Slide Time: 07:51)



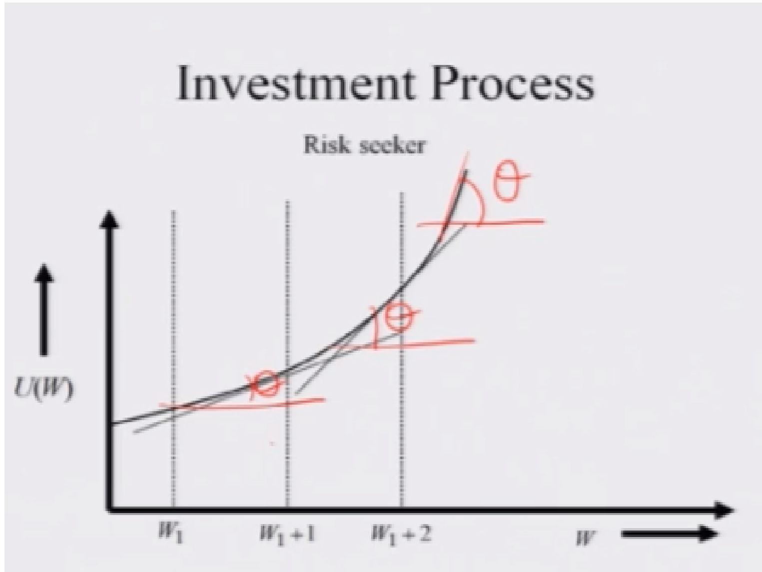
So If you consider the blue one here the color has been removed only drawn one curve. So if you follow the pointer if the curve is like this which means if I draw the DYDX of this at different points then you see the theta angle is decreasing. So here is the theta here is the theta here, here is the theta. So it is decreasing means increasing for the rate of change is increasing. So hence it is basically risk avoided and for the values are taken arbitrary show it very clearly W1, W1 + 1, W1 + 2 and so and hence so forth.

(Refer Slide Time: 08:37)



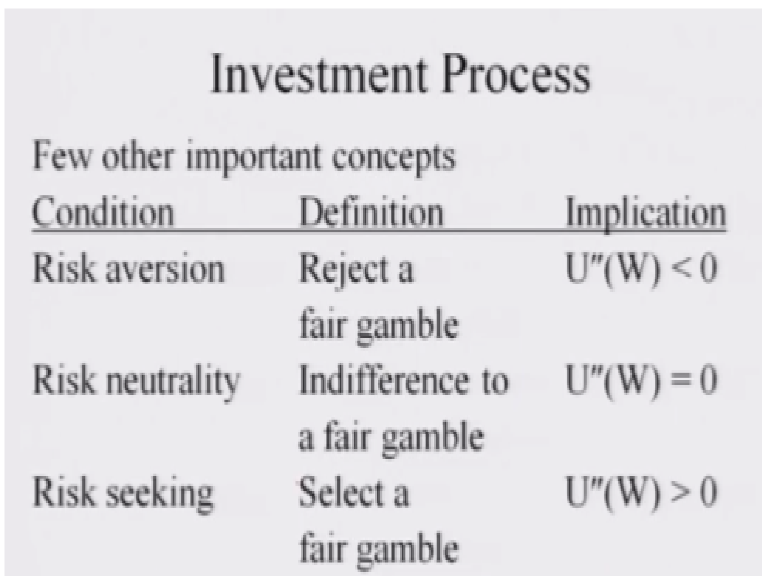
For a risk neutral person is a straight line blue one which you saw here the rates the DYDX is constant Tan of the theta is constant which is this one. So hence it is a risk neutral person.

(Refer Slide Time: 08:54)



And for the last curve which was the one which was going up the red one if you see the DYDX is slowly now increasing. So this theta this is the highest then next one then this one. So hence the DYDX is increasing hence the person is less seeking.

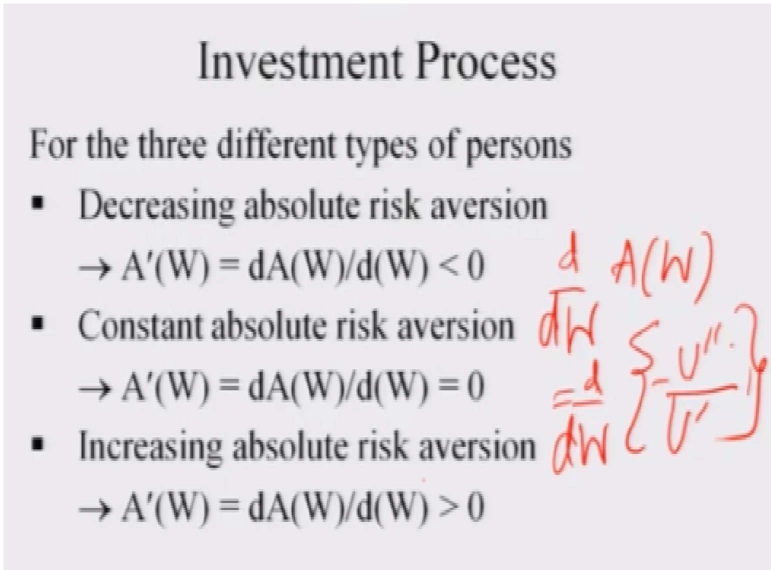
(Refer Slide Time: 09:15)



Now as I mention the second derivative which is second derivative in utility function which respect to the wealth which is UW prime. So if you consider the first column second column and the third column if you give you immediate sense that how it is been explained. So risk aversion probability would be a person who rejects a fair gamble hence the rate of change of second derivative is less than zero because if the curve is like this.

So it is increasing DYDX is DU W is increasing it is increasing at a decreasing level hence this is true. If a risk neutral person at the straight line it is in different to the person in different to the fair gamble with respect to the certainty event. Because here so the rate is double derivative is zero similarly for the third example if it is going up then the double derivative is positive.

(Refer Slide Time: 10:18)



Now we will two important properties they are very heavily used in any decision making process whether in a project and investments or trying to do the optimization problem whatever it is. The first probability risk aversion property without the proof I am giving the formula. So AW which is the absolute risk aversion utility function is given by negative of you double prime divided by U prime.

Now if you see this term U prime are less positive so the property of AW will depend on U double prime. So if U double prime is positive so positive multiplied by negative one it will be negative in nature in characteristics. If U double prime is zero, Zero multiplied whatever is zero so which will give you the concept is risk neutral concept and if U double prime is the third one is negative so negative would become positive you will have the concept of absolute risk aversion property correspondingly.

So here I mention so now what we do is that we want need to find out the derivative of A also which I need to find out the DYDX of D of this function with D W. So AW you want to find out which is equal to DW multiplied by U double prime and not writing W here. So what my actual notion would be I need to differentiate this function which is AW and then find out how it behaves with respect to the rate of change of W. So A prime if it is less than zero this is risk aversion property.

If A prime is zero is risk absolute risk aversion property is not the risk constant the person neither risk lover nor a risk hater risk in different person. And in the second case which value as an absolute risk aversion property that is the first derivative A prime is zero then correspondingly V we allocate the property to the person who is taking the decision.

(Refer Side Time: 12:43)

Investment Process

	Condition	Definition	Property
1)	Decreasing absolute risk aversion	As wealth increases the amount held in risk assets increases	$A'(W) < 0$
2)	Constant absolute risk aversion	As wealth increases the amount held in risk assets remains the same	$A'(W) = 0$
3)	Increasing absolute risk aversion	As wealth increases the amount held in risk assets decreases	$A'(W) > 0$

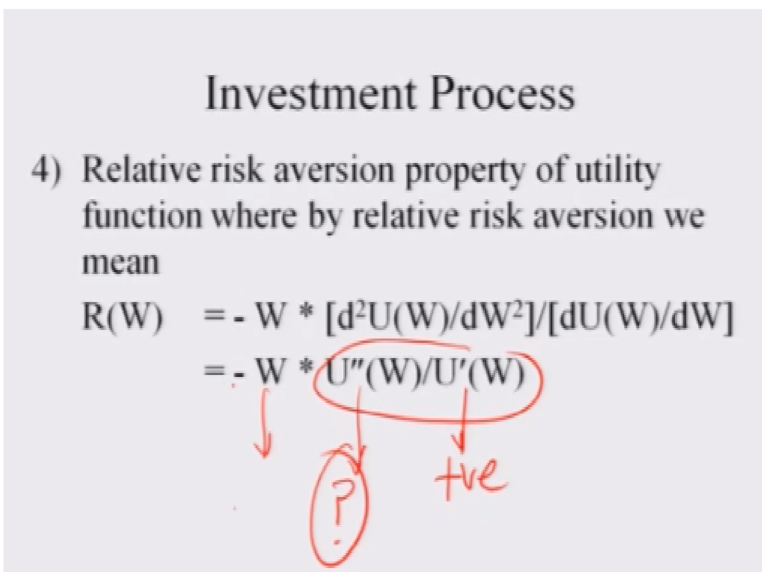
Now if you want to understand on qualitative notion so these are all mathematics very simple mathematics. So if I want to understand the concept of the qualitative notion this is how it can be stated. So again the conditions are given on the first column that is decreasing absolute risk, constant absolute risk and inclusive absolute risk. So if you consider the third column is exactly what I discussed in the last slide which was one fifty seven.

So now what it means is that consider only one property which is A prime is less than zero which is decreasing absolute risk aversion property. Which means that as wealth increases the amount health in risk held in risk assets is increasing now which means my absolute risk aversion property is decreasing that means I am willing to take the risk which means I am becoming more and more of the characteristic were I am risk seeker.

Because it would mean that if you if you read that that particular statement and as wealth increases the amount of the healthy risk increases that means I am willing hold more and more of overall portfolio of the project of the investment whatever it is in risky assets. So it means I am willing to take the risk if you consider the second point it clearly states as wealth increases the amount held in risky assets that remains the same that means the total quantum remains the same that mean I neither willing to take the risk nor willing to basically hate.

But I want to basically continue with my same concept but I am indifferent hence the first derivative of A is zero. And if I come to the last point which means A prime is greater than zero it means as wealth increases the amount health risk as decreases which means I am not willing to take the risk as I increase more and more of my wealth.

(Refer Slide Time: 14:44)

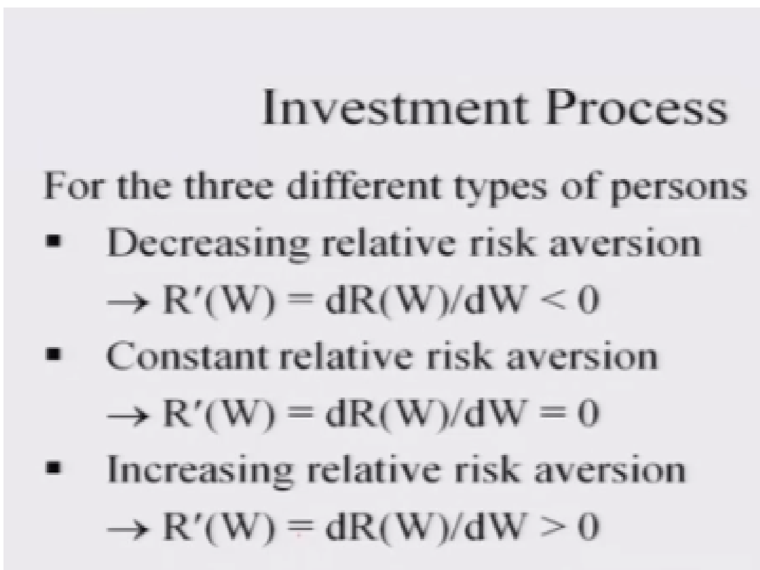


So now let us come to the second concept which is the relative risk aversion property the first one is absolute it is a relative risk aversion property. So before I disc I come to this same sequence of how the explanation was done for A. Let me mention the relative risk aversion property is very simply in laymen terms use at the concept where in the relative sense whether the wealth in the risk asses is increasing or decreasing or constant.

So you will see that within the next two slides so relative risk aversion property RW is now given so this part which you have with the minus sign is always A W that mean multiplied by W. So again if I want to draw some conclusion about the property of R we know this is always positive because as per the concept of non-citation this is always positive because wealth has to be greater than zero. It is one rupee two rupee or ten dollars or twenty dollars or hundred euros, two hundred euros whatever it is.

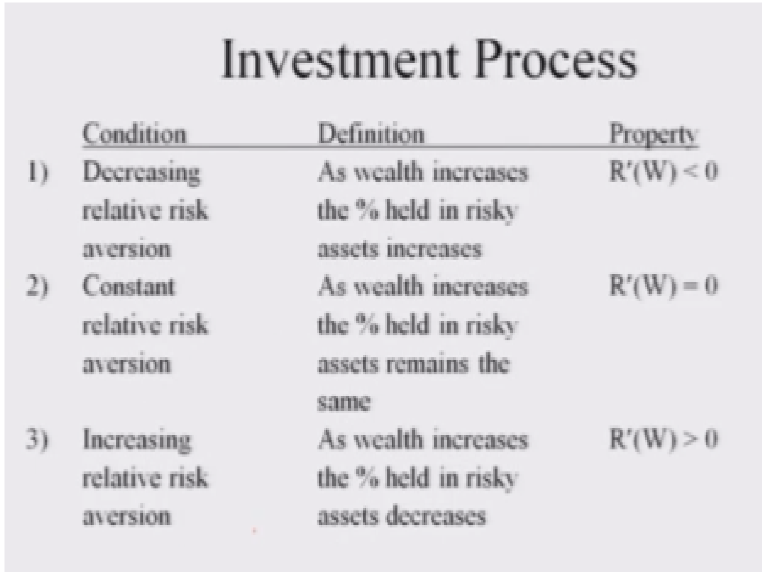
So again the property of RW would basically depend on this which is U prime U double prime sorry. So U double prime which would definitely have effect what is the property of A and what is the property of R which is absolute risk aversion property which are there.

(Refer Slide Time: 16:08)



Again falling the same sequence of (()) (16:10) R prime being less than zero which means decreasing relative risk aversion R prime being zero it means relative risk aversion of property is constant and R prime being greater than zero that means risk aversion property is increasing. So let us again explain it has we did for A.

(Refer Slide Time: 16:31)



Investment Process

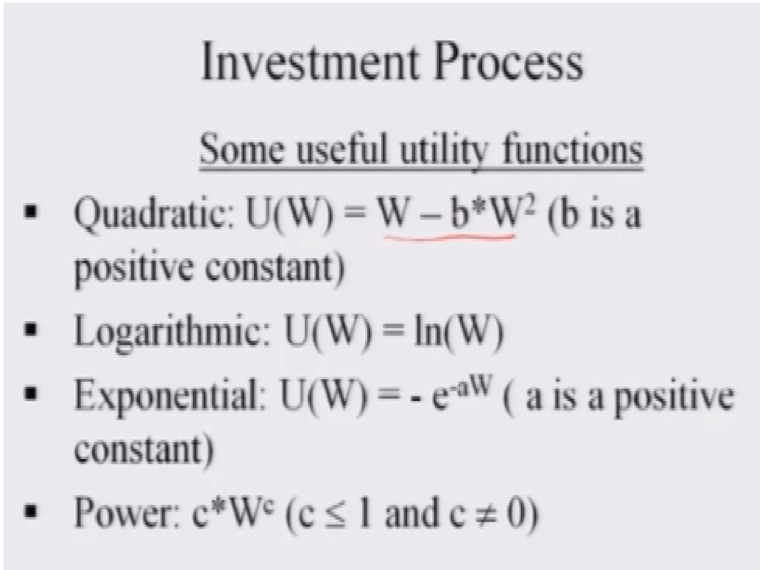
	Condition	Definition	Property
1)	Decreasing relative risk aversion	As wealth increases the % held in risky assets increases	$R'(W) < 0$
2)	Constant relative risk aversion	As wealth increases the % held in risky assets remains the same	$R'(W) = 0$
3)	Increasing relative risk aversion	As wealth increases the % held in risky assets decreases	$R'(W) > 0$

Here the first column and the third column are same only in the third column in place of R prime or for A prime we have replaced sorry my mistake it is A prime has been replaced by R prime.

Again R prime is less than zero equal to zero and greater than zero and again the condition are exactly the same. So here the absolute word has been replaced by reality which we noticed yet so this is relative or relative word. But what is interesting to note is that the definition so as wealth increases the percentage held in risky asset decreases. In that case it was the quantum sense now it is a percentile sense so again it would mean as wealth is increasing the percent wealth in risky asset increasing it means I am willing to be more risk taker.

Exactly the same as we discussed for A prime for the second point it means in the in the percentage sense it is constant as it is mentioned remains the same. So hence R prime is zero that means I am risk neutral neither willing to take the risk not willing to basically avoid the risk I am basically different. And in the last point where the R prime is greater than zero it means that as wealth increases the percentage health in risky asset decreases.

(Refer Slide Time: 17:55)



So now let us consider four different utility function which are used very heavily in in decision making process where whether for the portfolio whether for a project where for a optimization problem or whether for buying a car buying a fridge whatever it is. So this is on theatrical notion so obviously there are other non-parametric method are which are used for such qualitative decision one was AHP there are other method also which is not under the ambit of project management.

So some useful functions are generally what is use the most important one is quadratic utility function. So let us pause here for two or three minute let me explain about quadratic utility function. If you see the function is a quadratic function but what is very important and interesting about the fact is that this quadratic function have some function to normal distribution function.

So if it can be proved it in finance literature can be proved in in economic literature in different investment process. That if your returns are normally distributed then the utility functions are quadratic in nature and vice versa. Now normal distribution has very nice property is like finding out any combination of normal distribution always results in a normal distribution like combining to normal always lead to a normal distribution.

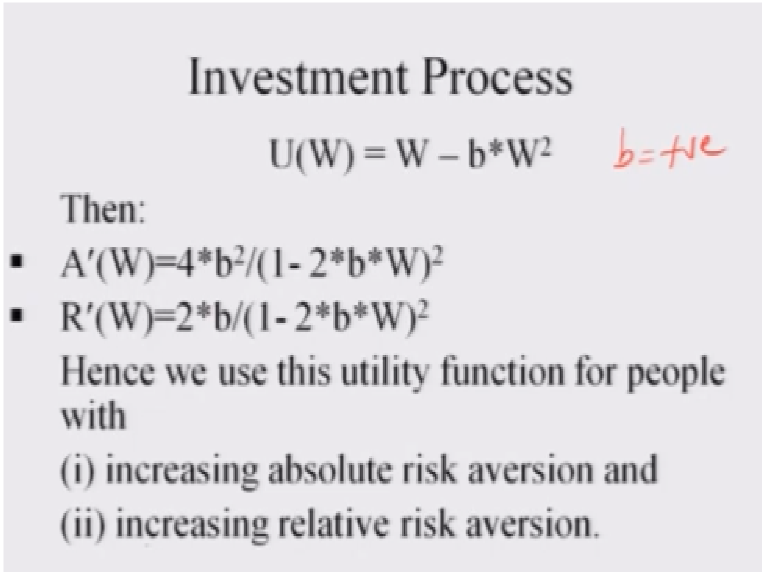
So if you remember the example of the risks which we mention the RF which is the risk free interest rate or RI or RK comma J depending on where the projects R is the Kth one A is the Jth period of time whatever it is. If the returns are normal distributed then then you can assume that the utility function based on which the decision has been taken for those type of investment are quadratic in nature. So we will consider this quadratic utility functions in general.

The next point is the logarithmic one which is that utility function is given LN of W (()) (20:07). If you go back to one of the slides earlier on where investments where given the prices where given and we wanted to find out what was the utility of other expected value of the utility we did use L, N, F P, P was a price. So that was just pre cursor based on the fact that logarithmic utility function which can also be utilized. Then we have the exponential utility function given by this formula of minus E to the power minus A W.

W is the wealth and A is the parameter is a positive constant and while coming back to this sorry I missed it. So this value B is also parameter which is constant in its value and the power function is given by a C into W to the power C where C is less than zero and C is not less than one and not zero so this is also parameter for the power function. So let us consider the concept of U prime U double prime A and A prime, R and R prime for all this four utility function and let us spend some time in discussing that.

So the first one was the quadratic utility function so without going to the calculations I have just written the bullet points and I was strongly urge the students to do it on their own such that they can get a feel that how this this concepts comes into the play.

(Refer Slide Time: 21:37)



So if I consider A prime the A prime value is given by four B square in the numerator divided by one minus two B into W whole square. So now if B is as I mentioned if it is positive or negative even it is negative it does not matter. So the numerator is always positive because of square and the denominator is also positive because is a square. So if you consider this property for A prime then you can without doing any calculation without going to any details of trying to understand and you can immediately mention that is what an increasing absolute risk property.

If I go to the R prime the value is given by two B divided by the same denominator which was there in A prime. So the denominator is basically positive and this two B the value in the sign for concept would definitely depend on what is the value of the B. So if B is positive 2 B is also positive so again we see it is an inclusive relative risk aversion property if B was negative consider hypothetically.

Then the first concept A prime would always be positive because it does not matter square it is always have inclusive absolute risk aversion property. But if B was negative then this term which is in the numerator on the right hand side of this equality for R prime will be negative hence would have a decrease in risk aversion property.

(Refer Slide Time: 23:18)

Investment Process

W	W-b*W^2	A(W)	A'(W)	R(W)	R'(W)
2.00	3.00	-0.25	0.06	-0.50	-0.13
3.00	5.25	-0.20	0.04	-0.60	-0.08
4.00	8.00	-0.17	0.03	-0.67	-0.06
5.00	11.25	-0.14	0.02	-0.71	-0.04
6.00	15.00	-0.13	0.02	-0.75	-0.03
7.00	19.25	-0.11	0.01	-0.78	-0.02
8.00	24.00	-0.10	0.01	-0.80	-0.02
9.00	29.25	-0.09	0.01	-0.82	-0.02
10.00	35.00	-0.08	0.01	-0.83	-0.01
11.00	41.25	-0.08	0.01	-0.85	-0.01

So let I have done a very simple exercise I have just given an example for the quadratic utility function and i have done it with excel but I have just I am giving you some set of values so for the participants can understand. On the left most column are the W values arbitrarily taken from two, three, four, five ratings each quantum of increase in WS by one unit in can be any other one unit.

And if I consider the utility function UW the UW values are given so what values are B are considered you can immediately find from here. So if this value is three means W is to 2 – sum B value into W square is four. So four basically one if B is one by four so if I have W say for example the first equation 2 – 1 by 4 into 4 because it is two square oh sorry my mistake should basically be I have taken negative my apologies here to make you understand.

So if I consider two minus of minus one by four into four which is square this four four cancels this minus minus becomes plus two two plus one becomes three so let us come to the second one three minus of minus one by four into three three nine so becomes four two are eight. Eight and two point two five so it becomes five point two five as it is given so I have taken negative just for explanation it is taken as positive.

So this U values are given the second column then I find out AW so AW formula which we all know is minus of U prime by U double prime by U prime. So how do you find out U prime so U prime I have not shown it here so you can just put some extra columns and do the calculations. So U prime between value of W going changing from two to three would be five point two five minus three which is the difference in the inclusive of the utility function divided by the change of the utility change in the value of W.

So it will be five point two five minus three in the numerator and in the denominator the change of the W value is three minus two. So you basically have 2.25 which is difference divided by one which is basically U double prime U prime. Now if I want to find out what is U double prime again you do the same calculation accordingly and you find out U double prime. So I have omitted that and I would strongly urge the students to use this values like use the first column and second column.

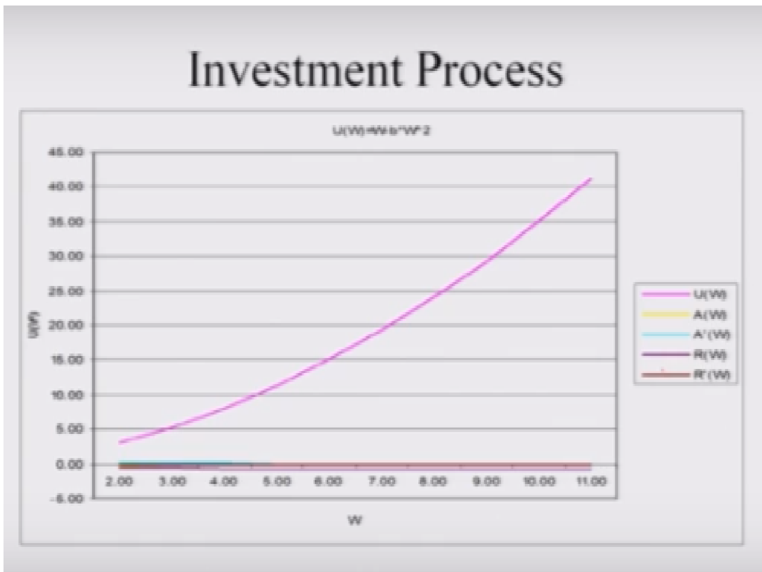
And re do the calculation considering there are third and fourth column which are not clear which would be U prime and U double prime based on that you do the calculation. Hence you do the B value is minus one by four so A prime AW is given I want to find out A prime so how do I do. So these two difference between minus point two minus of point two five divided by three minus two. So this is in the difference is there in the denominator and the values of difference of A is there in numerator.

So based on that we will find out the value of A prime now if I want to find out the value of say for example R. R as you know basically A multiplied by W so if you A multiplied by W2 will be two multiplied by minus zero point two five which is minus two point five. If you find out for example let us take another value if it is coming out to be let me check yes so if I want to find out. Say for example the value of R or 5 so it will be almost five to six point minus zero point seven zero but I have taken two or three places of decimals.

Hence it is coming off minus zero point seven one so all this values are given so once you know you which is not there then you find out this value of U prime U double prime based on that you find A U prime R, R prime. So once R prime are given so what you need to do is that plotted so let us go one by one. So if I want to find out the characteristic of R prime and A prime just have a look at this values. So it basically have a range it is positive so you immediately go into the characteristics of what is A prime it is greater than zero equal to zero less than zero give you a comments accordingly.

Similarly for R prime the values are given immediately if you come with the concept where R prime is greater than zero, less than zero, equal to zero based on that you give the comment. So I have to draw it for the same graph which may not be very clear possible but if you do in excel immediately understand.

(Refer Slide Time: 29:15)



So the pink on is U then the yellow one is somewhere here if you draw it you will understand accordingly and strongly urge and request the students to draw it you have the value of A, A prime R R prime so this actual excel sheet will give you the all the characteristics of the curve which is there considering the utility function is quadrative.

So with this I will end the fifteenth lecture the sixteenth lecture would basically start with the other three different type of utility functions which are there which was logarithmic, the exponential and power function and in the similar way I will give you an excel sheet which will explain the how the graphs are drawn. Once you start you will be able to appreciate the examples later on have a nice day thank you very much.