

**Project Management**  
**Prof. Raghunandan Sengupta**  
**Department of Industrial and Management Engineering**  
**Indian Institute of Technology – Kanpur**

**Module No # 3**  
**Lecture No # 13**  
**Decision Tree Analysis and Risk Management**

Very good morning, good evening good afternoon to all my students at whatever time you are trying to listen to these lectures and hope all of you are enjoying this course and finding it interesting considering whatever concepts we are trying to cover in this course. So this is the thirteenth lecture in the forty lecture series in project management which is of twenty hours so each lecture is half an hour so we are trying to cover topics or portion of a topic or subtopics in those half and hours and planning it accordingly.

So in the eleventh and twelfth lecture if you remember we covered the concept of AHP done with the simple problem. Obviously there would be many queries from the students that how AHP can be utilized. So again I will urge the students the participants to send the queries to the forum and we will definitely take care of answering on an individual basis or an collective basis depending on the type of queries they are.

And the last lecture which is the twelfth one we started discussion two problems in decision trees in first problem we as we had the concept of unconditional distribution as the distribution value based on which he expected value of decision tree was found based on which you take a decision and in the second problem for the decision tree analysis we consider the conditional distribution conditional on some prior happening whatever event what was there and we saw in that problem for the drilling example.

How it solve it using the conditional values along the marginal values and conditional values of the probability along on the cells in that matrix and the marginal values along the bottommost row on the right most column now just to wrap up the problem. If you remember we found the expected value at all the values of C's and D's. C's and D's you know those are the folks or the



and maximum amount all considering you went from try structure to wet structure to the soaking structure. So if you start going on to the left to the final answer that means where you initially start.

So these red value is one as I am again repeating which cannot be taken and the blue one are the one which could definitely be taken. So if you find out the overall the diagram it is you definitely have to conduct a test and depending on the outcome which is not structure which is this part open structure which is small horizontal line where I am hovering my pen and this is what is this diagonal arrow which is going downwards is basically the flow structure and then you have the overall values which is given.

So these values are exactly the same for the last slide which we covered so to make things double clear for all the candidates or the participants this negative values are the outflows which are in the negative side and this point eight two point four which are plus are shown in blue. And again the value of zero point zero two and all those things are so called shunt cost.

**(Refer Slide Time: 05:35)**



So now if you consider this ways to manage the project risk so you basically juggle between the risk and the variable cost I am sure people can see this or else try to basically increase the font size to one sixty. So I am sure it is absolutely visible so you will have the pricing strategy of my mistake this is a small C missing here you have different type of concept of in sequential

investment improving information's that means you get better information's you have financially leverages.

If you remember the financial cost the internal rate of return the fluctuating rates on all these things we will come to that with set of five to six different type of problems you can take the concept of insurance into the picture such that you reduce your risk you can go for long term arrangements of borrowing and lending in order to minimize the overall risk.

So the risk profile may per method we must be aware of the probability distribution which is the net present value which is absolutely a fixed term and net present value is very simply the expected value and try to convert the same to the probability distribution of probability index of relative measure such that you are able to compare the risk based on the fact that the concept of interest rate can also be brought into the picture.

Now if you consider the risk adjusted when we were doing the CAPM model even though it is applicable into investments means financial investments it can also be replicated in the sense where the investments are more projects and each projects has each type of activities or jobs underneath it. So your risk adjusted discounting rate would be consisting of four terms which are the risk adjusted discounted rate or the kate project which need to find out and compare then you are the risk free interest rate again going back to  $r$  suffix F and the adjustment for the forms notional risk values whatever you have.

And DK is the adjustments for the differential risk for the kate projects which may be different so hence if I am able to find out the net present value would be given by this formula.

**(Refer Slide Time: 08:01)**

**Risk profile method**  
 We must be aware of the probability distribution of NPV (an absolute measure) and try to convert the same to probability distribution of profitability index (relative measure).

**Risk adjusted discounted rate method**  
 $r_k = r_f + \alpha + \beta_k$   
 where  
 $r_k$  - risk adjusted discounted rate for the  $k^{\text{th}}$  project  
 $r_f$  - risk free rate of interest  
 $\alpha$  - adjustment for the firm's notional risk  
 $\beta_k$  - adjustment for the differential risk for the  $k^{\text{th}}$  project

Hence  

$$NPV_k = \left[ \sum_{t=1}^T \frac{\hat{A}_{k,t}}{(1+r_k)^t} \right] - I_k$$
 where

$NPV_k$  - Net Present Value for the  $k^{\text{th}}$  project  
 $\hat{A}_{k,t}$  - Expected or estimated cash flow for the  $k^{\text{th}}$  project at the  $t^{\text{th}}$  time period  
 $r_k$  - Risk adjusted discounted rate for the  $k^{\text{th}}$  project  
 $I_k$  - Net present value of investment for the  $k^{\text{th}}$  project

So NPV for kate project is given suffix K now what do you have is basically two terms inside the square bracket is the positive term and outside the square bracket is IK as a negative. So IK basically means the overall investment which you are going to do or say for example the shun cost fixed cost variable cost which are done at a particular time. So or different points of time I am not using the word only at a fixed point of time it can be of different points.

So consider that you are planning to invest trying to build up a project at the starting which is zero th time period  $T = 0$ . Then again at  $T = 1, T = 2$  this 1, 2, 3, 4 can be their months or can be weeks, can be years quarters it does not matter. And if the time frames are different for the fixed cost the investments considering the returns are at different points of time at different intervals it does not matter you just need to find out the time value of money at a particular time which is  $T = 0$ .

So this IK is the sum of all the investments in the negative sense that means money here out it in the project at different points of time. So this is the value of money at time  $T = 0$  while which the for simplistic case have not written down into the equation format while the set of values which you have inside the brackets.

So AK is basically the expected estimated cash flow for the K project at different point of time T is equal to 1, 2, 3, 4. So hence the suffix A case for the project T is for the time and in the

denominator if you look is the exactly the same formula we have discussed few slides back for in in one other classes.

Where  $R_K$  is the risk adjusted interest rate and  $T$  is the time period so say for considered that if you consider the problem for the moped the decision tree analysis problem which we did for a company trying to start with moped just before the oil rig drilling example there you had a time period of twenty years for twenty million per year and thirty million per year and the interest rate was twelve percent.

So that interest rate  $R_K$  suffix  $K$  it may change from project so this whole formula is exactly the same what we have done in that problem. So now we can formulate the problem accordingly where the value of the net present value or NPV can be done considering that you multiply the overall so called payback which is happening by a value of  $\alpha_K T$ .

So the  $\alpha_K T$  very simply  $K$  and  $T$  means for the project  $K$  and  $T$  is the time period is basically the weightage you are going to put for the first project second project third project whatever it is for time period  $T = 1, 2, 3, 4$  accordingly. So here  $\alpha_K T$  in some sense that the overall weightages why weightages because there would be some certainty value.

So if I am certain that for the project I am with probability ninety percent that the payback would definitely be positive then that value of  $\alpha_K T$  would be point nine corresponding to the other projects it would be denoted accordingly. So again  $K$  again repeating you that net present value of the investment  $A_K$  the expected or the estimated cash flow which is coming back  $R_F$  is the the risk free interest rate and  $\alpha_K T$  is the certainty equivalent.

Now only one thing between change  $B$  is these formulas if you see this formula which is  $R_F$  and  $\alpha_K$  and tried to compare with value of  $R_K$  the relationship between the certainty value or say for example overall net value which you are going to get from each investment. Payback at time  $T = 0$  would be calculated based on the fact that  $R_F$   $R_K$  and  $\alpha_K$  are related.

So higher the value say for example alpha A certainly equivalent corresponding to that fact you will try to find out RK value given RF is known to you or vice versa whatever it is but generally we will try to basically bring the same datum where you will have RL being replaced by certainty value alpha and RF for the risk free interest rate. Because our risk free interest rate would be same for all the projects then the overall risk for that project considering the interest rate from point of view of the interest rate would be basically subsumed another value of alpha for each project for each time.

Because interest rate is changing from time period to time so that in the again going back to the moped problem it was twenty twelve percent for each time period consider it is not ten eleven nine thirteen whatever it is. So these interest rate have to adjust it considering RF is there which is fixed and this adjustment would come into the factor which is alpha as I mentioned few minutes back it is the certainty equal.

So if alpha KT is one then the person is risk indifferent we will consider that if it is less than one it is a risk or avoider and greater than one basically means that person is risk love the risk. So this word would be love the risk.

**(Refer Slide Time: 14:07)**

**Certainty index**  
 In general we may term it as the probability of being positive, i.e., if we say the certainty index of procuring raw materials on time is 85%, then it means that the probability that the raw materials are obtained in tie are 85%.

Thus in a problem we can have the following information

Item	Certainty equivalent coefficients			
	1 <sup>st</sup> year	2 <sup>nd</sup> year	3 <sup>rd</sup> year	4 <sup>th</sup> year
Research and Development	0.68	0.65	0.55	0.50
Marketing	0.78	0.73	0.66	0.61
Starting the project	0.90	0.84	0.76	0.70
Procuring raw materials	0.85	0.80	0.77	0.74

Note: Remember the time frame for each item/sub-project is different

In evaluation remember that if there are three stages for the completion of the project such that we can have A leads to B then to C and then D or A to C then to B and then to D, then the corresponding probability would be calculated utilizing  $P(A) \cdot P(B|A) \cdot P(C|A \text{ and } B) \cdot P(D|A, B \text{ and } C)$  and  $P(A) \cdot P(C|A) \cdot P(B|A \text{ and } C) \cdot P(D|A, C \text{ and } B)$

**Assignment # 1**  
 A large steel company has three (3) options with regard to production, which are: (i) produce

So if I consider the concept of certainty value index in general we may term this as problem which are going to consider in in in very simplistic sense so I would not going to assignment as

such I just go through the concept and close this this this topic in order to be dealing that is in the utility concept. So this assignment which is there in the last part is not to be done as of now we consider that later on.

So in general we may term it as the probability or being positive that is we say the certainty index of procuring the raw materials identifies person it means that probably the raw material are obtained is eighty five. If we remember the weightages we gave is that if it is one it is hundred percent certain. If it is not obviously to mean that there is a change in the certainty value for different small event or our activities with respect to time.

Thus in a problem we can have the falling one so if you see his table you have the research and the development the marketing and starting the project and procuring the raw materials along the first column and along the other second, third, fourth fifth year wise first year second year third year fourth year you have the certainty value which is given say for example for us research and development this is just a value are given point six eight or say for example starting the project at time T and for the kate project.

Say for example this is for the third project and I am trying to find out the certainty value of starting the project in the third year then the value for the alpha would be  $\alpha_{KT}$  for  $K =$  third project  $T =$  third year it would be seventy six percentage or point seven six. So you have the values accordingly in evaluation remember of the there are three stages completion of the project such that we can have a case like this A leads to B which leads to C and then D to A to C and then to B.

So there are two consequences of how things are being done like you procure the material you do the initial processing of the raw materials then the raw materials is basically worked on in in the final stages and then sold. So if I consider procurement initial processing final processing selling fourth stage.

So A which is the first one would affect B, B would affect C which is the third stage and D would basically affect the last stage of the selling. So if I am not able to get the good quality



products raw materials is poor still I go into producing then what effect it would have in the overall probability sense would be the probability or the chance of selling the product in the market would be definitely be less.

So the probability of D which is the last one dependent on CBA has already happened would be very low. So what you will have if probability of A if you look at this term the probability of B provided A has happened and the third term is probability of C provided A and B has happened and the last term there you have is probability of D provided A, B and C has already happened.

So you can find it accordingly in other sense if the consequence is something the flow of the of the jobs or the activities of the investments are like this it will happen as probability of C is dependent on A. Probability of B is now dependent on A and C which is already occurred and probability D would basically depend on A, B, C has already occurred that I am using the sequence of the events as A, B, C but it technically if you see the last two terms in these two lines it is D dependent of A, B, C and in other case in deeply dependent on A, C, B as the consequence of the events which basically occur.

So now with this I will basically start off the concept of decision trees and utilities we have already have done the problem in decision trees utility analysis and how they can be utilized in order to make a different type of decisions for the projects. So let me change from word document to the PPT for the further discussion.

So now we will consider concept of decision analysis so you are the CEO of a company which manufactures three different ratings of electrical motors and they have the falling this is just a makeup or trying to basically build up the motivation for why decision tree analysis where you used and how it can be utilized for different type of problems later on.

**(Refer Slide Time: 19:04)**

## Decision Analysis

You as the CEO of a company (which manufactures three different ratings of electrical motors) have the following information in front of you

- 1) Motor rating 75 KW with a certain unknown demand,  $d_1$  (remember this is in units)
- 2) Motor rating 150 KW with a certain unknown demand,  $d_2$  (remember this is in units)
- 3) Motor rating 200 KW with a certain unknown demand,  $d_3$  (remember this is in units)

So here the following information's front of you have three different type of motor ratings so in building up a motor it can also be a project. Motor ratings as seventy five kilowatt with certain unknown demand of numbers consider is D1. Note motor rating one fifty kilowatts has a demand of D2 and motor rating two hundred kilowatts as a demand of D3. So there is no relationship as such we are considering between D1, D2, D3 it can be independent but they can be dependent also.

**(Refer Slide Time: 19:42)**

## Decision Analysis

You have the SP for these ratings as Rs. 15,000 for 75 KW, Rs. 35,000 for 150 KW and Rs. 50,000 for 200 KW

You have the selling price for these ratings rupees fifteen thousand for seventy five, rupees thirty five thousand for one fifty one and rupees fifty thousand for two thousand for the kilo watts.

**(Refer Slide Time: 19:57)**

## Decision Analysis

You are interested in finding these numbers,  $d_1$ ,  $d_2$  and  $d_3$  in order to find your total sales value. To ascertain these numbers you give this task to an industrial marketing firm and they supply you with the following information

- 1) The optimistic demand for 75 KW is 300 with a chance of  $7/10$ , while the pessimistic demand is 200 with a chance of  $3/10$
- 2) The optimistic demand for 150 KW is 210 with a chance of  $5/15$ , while the pessimistic demand is 100 with a chance of  $10/15$
- 3) The optimistic demand for 200 KW is 90 with a chance of  $1/5$ , while the pessimistic demand is 30 with a chance of  $4/5$

So what you are interested in finding out the number  $D_1$ ,  $D_2$ ,  $D_3$  the demand with certain probability says that if you want to find out the overall on input the front considering we are not going to consider the cost structure for time being. So what we you would need would be like this let me write in a simple way manner. So this is this I will just use this pen for the making for the time being but I will try to write in accordingly.

So you have the selling price given as fifteen thousand thirty five thousand and fifty thousand. So what I want to do is sum up for the three I am using the mouse for the time being and then we will check once it is stabilized. So you will have basically a demand  $D$  so  $D_2$  with  $D_1$ ,  $D_2$ ,  $D_3$  you have that you will have the probabilities  $P$  which will be  $P_1$ ,  $P_2$ ,  $P_3$  and you will basically have the selling price which is  $SP_1$ ,  $SP_2$ ,  $SP_3$  which is fifteen thousand thirty five thousand and fifty thousand.

Now this is basically the sum now remember one thing considering that the probability is there for each and every item we will consider simplistically that the sum of the problem sum is probably is one they can be conditional probabilities also and they can be lost cells also that means product are not sold but we would not consider that we will consider the demand as deterministic and that has some distribution and the selling price which you are considering it does not consider the concept of cost.

Cost of the raw materials and considered in the problem so in case we have the cost concept I would become like this now in this problem again let me go back. So basically selling price minus the cost price. So say for example so this would be cost price for each and every item which a conserving. So if selling price is fifteen thousand and the cost price for the seventy five kilowatt is is consider five thousand it will be fifteen thousand minus five thousand for the first time inside the bracket multiplied by the probability P1 multiplied by D1.

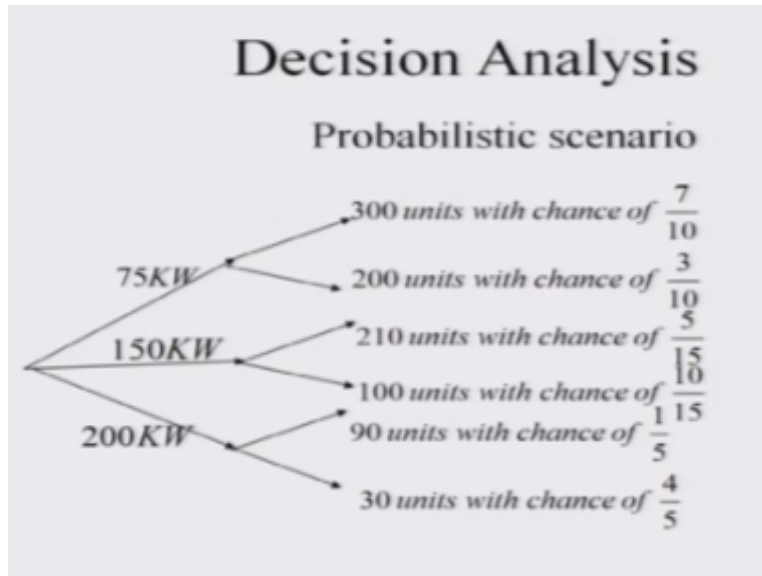
Similarly it will be selling price thirty five thousand minus some cost price is CP 2 inside a bracket multiplies by the demand D2 probability P2 and for the third case it will be again if you see this slide this is fifty thousand minus the cost price CP3 inside the bracket multiplies by demand 3 D3 and probability P3.

So if you are interested to find this number D1, D2, D3 consider that the optimistic demand probabilistic demand and so called specificity 1 it can be any combinations we consider very simply that they are two outcomes of the demand very good, bad or average man or it may be good or bad like basically you are tossing the coin. So this head and tail there were two outcomes. The probabilities are with optimistic seven by ten for seventy five three by ten for two hundred.

So okay it is my mistake for the seventy five one the demand value which is given is value wise is D1 is given is three hundred with probability seven by ten two hundred with the probability of three by ten. So now demand numbers are also changing similarly for one fifty and two hundred kilowatts the corresponding probability if you see in pint bullet point number two and bullet point number three it is five by fifteen and ten by fifteen for the one fifty kilowatt and similarly the values for the probabilities are one by five and four by five for the two hundred kilowatts.

The corresponding numbers demands D3 for the third type of motors for these two cases of one by four and five and four by five or sixty and thirty. Similarly for the one fifty kilowatt for probabilities of five by fifteen and ten by fifteen this it is two hundred and ten and eight hundred. So this based on that you will just put in the formulas and calculate accordingly.

**(Refer Slide Time: 24:36)**



Now this is the diagram tree decision which we have try to analyze using the simple problems it is given like this the arms are the seventy five one fifty two hundred. The corresponding units are given corresponding probabilities are given.

**(Refer Slide Time: 24:54)**

## Decision Analysis

For the probabilistic decision process the value/units for any particular rating of motor would be found by the expected value, which can be calculated by  $n_o * c_o + n_p * c_p$

So for the probabilistic decision problem what we need to find out is that the expected value as already done in the problem and already showed in the last two slides.

**(Refer Slide Time: 25:05)**

## Decision Analysis

Thus

- $d_1 = (300*7/10+200*3/10)$  units of 75 KW motor
- $d_2 = (210*5/15+100*10/15)$  units of 150 KW motor
- $d_3 = (90*1/5+30*4/5)$  units of 200 KW motor

So now if I want to find out D1 considering that I want to find out on an average. So that if you see for the seventy five one fifty and two hundred they were for different probabilities. So you multiply the demand in the probability demand in the probability you find it out for motor first which is seventy five motor two which is one fifty motor three which is two hundred and you find out the corresponding values this this very simple way of trying to solve it.

**(Refer Slide Time: 25:36)**

## Decision Analysis

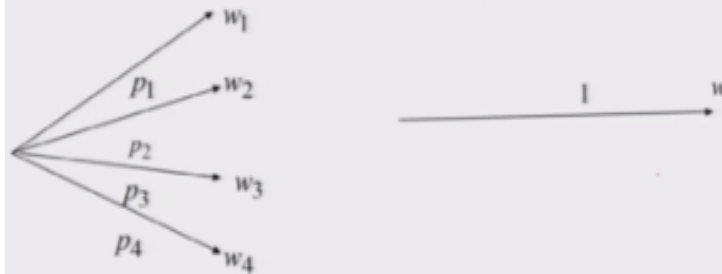
Hence expected sales figure is  
 $(300*7/10+200*3/10)*15000 +$   
 $(210*5/15+100*10/15)*35000 +$   
 $(90*1/5+30*4/5)*50000$

Hence the expected value figures are given you multiply inside the bracket the number multiplied by the selling price and you find it out. So exactly the same only the the quantum of number showed are changing.

**(Refer Slide Time: 25:52)**

# Decision Analysis

## Probabilistic versus Deterministic



So what is the actual analysis what we are trying to aim for the decision tree analysis is that the probabilistic framework has to be changed into the deterministic framework in order to find out what is the expected and based on that we take a decision. So if I see the probabilistic framework on the left hand side where I am pointing there are probabilistic  $P_1$  to  $P_4$ . The corresponding value are  $W_1$  to  $W_4$  what are  $W_1$  and what are  $W_2$  two values which I will come to that later.

$W_1, W_2, W_3$  you want to equate it with the corresponding to a certainty value with probability one and what is the value of  $W$ .

**(Refer Slide Time: 26:32)**

# Decision Analysis

## Utility analysis

Consider the same type of construction project is being undertaken by more than one company, who we will consider are the investors. Now different investors (considering they are investing their money, time, energy, skill, etc.) have different attributes and risk perception for the same project

That is to say, each investor has with him/her an opportunity set. This opportunity set is specific to that person only.

So I will just give the brief definition you play analysis and then end this lecture for this thirteen lecture series and the continuing all the discussions in the starting from fourteenth one. So what we mean by decision analysis and concept of you in this like rather paying attention to the slide I will just request to the students to like listen to what I am saying consider that you want to basically invest in trying to buy a car.

Or say for example want to invest in trying to the invest you time in trying to basically get a good education or buy a house or invest in trying to basically build up a project. So in utility analysis you main values want to find out what is the network and the utility or the value or than or the concept so called positive sense which you try to get by trying to invest in certain project or certain decision or certain investments.

So say for example if a person wants to buy the best shirt at a very high price you may as a other person may think that what is the value of trying to basically buy a such a high priced shirt but it may mean or it may imply to the person who is trying to buy that that high priced shirt. Is that it gives him or her a certain sense say for example being very smart or say for example trying to use branded product or say for example it is very comfortable or made out the best cotton.

So whenever you are trying to analyze a problem the concept of beauty and unless think carefully that what is the net worth a person is trying to get in order to make that decision worthwhile for him or her. So with this I will close this thirteenth lecture and start you from same slide considering the fact that what I have just mentioned about utility from the very qualitative perspective would slowly be transferred in the quantity feel.

Such that all that all the students who are taking this course and trying or understand this concept of utility analysis and how it is used in project management would be appreciated that thank you very much and have a nice day.