

**Project Management**  
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**Module No # 3**  
**Lecture No # 12**  
**Decision Tree Analysis**

Welcome back this is the twelfth lecture in twenty hour course so twenty hour basically mean forty lecture each of thirty minutes. So in the last class we briefly discuss in not all the technical details but the concept of the AHP and how it can be utilized of a to solve the problem where the decision is to buy the car and it can be expanded for very complicated problems also and it gives very good results.

So now if you remember in initial part when we just started we have completed three or four lectures we did discuss about decision trees and also I am repeating again we did discuss the methods of how different type of financial concept can be used to make a decision for a project. So today in this twelfth lecture we will consider that how decision analysis can be used. So what are the salient points of the decision tree analysis the key steps in decision analysis are as follows you should identify in the problem and the alternatives.

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## Decision Tree Analysis

The key steps in decision tree analysis are as follows

- Identify the problem and alternatives.
- Delineate the decision tree.
- Specify the probabilities and monetary outcomes.
- Evaluate the various decision alternatives.
- Remember the decision points, also called decision forks, denoted by D, and the alternative actions are available for experimentation and actions are possible at these points, while the chance points, called chance forks, denoted by C, are the points where the outcomes are dependent on a chance process and the likely outcomes at these points.

Delineate the decision tree that means draw it very clearly where the probabilities are given the actual values are given specifies the probabilities and the monetary outcome. So you have to make combination of the monetary values and outcome of the probabilities. Evaluate the various decision alternatives which are there and take a decisions accordingly based on whatever different criteria it is whether it is want to maximize the return minimize the risk other different combinations.

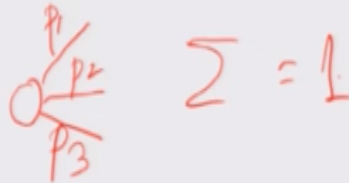
Remember the decision points are also called the decision folks so in the initial when we are discussion we said decision gates alternative gates. So they are something where you are on same line where you make a decision accordingly they are denoted by D and the alternative action available for experimentations and actions are possible at this point. So at a decision point you either take the right fork the left fork or if they are three outcomes you can take alternate one r decisions one or two or three.

Well the chance points are called the chance folks where there are chances of one alternative being two or the decision being true they are generated by C and these are the points with outcomes are dependent on a chance occurrence and the likely outcomes at this points are a conglomeration of the decision which is already been taken. Now you are trying to proceed for the decision remember that he would not basically go from start to the finish but backwards why it is that I will come back to that within few minutes.

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# Decision Tree Analysis

We discuss now a problem in all  
the technical details  
Project Management Decision Trees.d  
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So we now discuss a problem before going to the theory so we will discuss the problem because this will give you an idea that how decision and expected values and later on when we consider the different concepts of utility it will give you good picture and how it can be utilized. So we will just consider the problems in an holistic framework with probabilities and values are given.

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DATE: 14 Nov 2019, 10:11:20 AM, 1700 Issues

### Decision Tree Analysis

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- 3) Specify the probabilities and monetary outcomes.
- 4) Evaluate the various decision alternatives.

Remember the decision points, also called decision forks, denoted by D, and the alternative actions are available for experimentation and actions are possible at these points, while the chance points, called chance forks, denoted by C, are the points where the outcomes are dependent on a chance process and the likely outcomes at these points.

#### Example - 1

The scientists at Spectrum have come up with an electric roped. The firm is ready for pilot production and test marketing. This will cost Rs. 20 million and take six months. Management believes that there is a 70% chance that the pilot production and test marketing will be successful. In the case of success, Spectrum can build a plant costing Rs. 150 million. The plant will generate an annual cash inflow of Rs. 30 million for 20 years if the demand is high or an annual cash inflow of Rs. 20 million if the demand is low. High demand has a probability of 0.6, while the probability of low demand is 0.4. What is the optimal course of action using decision tree analysis.

So now I have shifted from PPT to word document because drawing the diagram a little bit complicated with all the decision modes and all these things was much easier involved even though there are other software which you can use. So consider this first set of points which is there we are already discussed so I will just bang on with the problem. The scientist in a

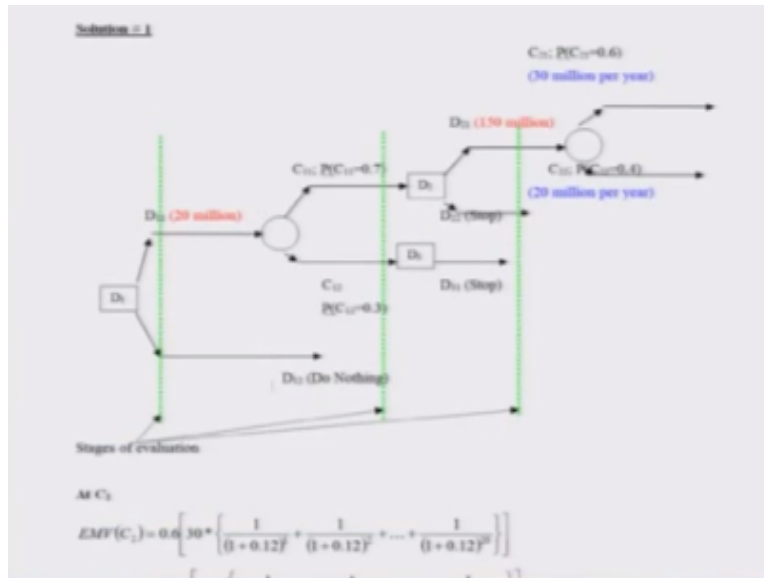
spectrum has come up with an electric moped the firm is ready for pilot production and test marketing this will cost about rupees twenty million and take six months.

Management believes that there is a seventy percent chance at the pilot production and test marketing will be successful in the case of success. Spectrum can build a plan costing about one fifty million rupees the plan will generate at annual cash flow of thirty millions for twenty years that means for a twenty year period do you expect that. If the demand is high on an annual basis a cash flow of twenty millions then the demand is low.

So there are two such occurrences it can be very high medium and low also in that case there must be three such occurrences but in this examples we will take two. High demand has the probability of point six low demand as a probability of point four. So if there are three different type of demand probability the remember the sum should always be one like here in this case point six point four is one in that case if there are three different folks means this one I am trying to basically consider there let me go back to the slide it would be easier for me to make you understand.

So say for example we have this is one and there are three alternatives so this probability which is given as  $P_1$ ,  $P_2$ ,  $P_3$  the sum is 1 in this case it is true because point 6, point 4, is 1. So again let me switch back to the decision tree problem. So now if you read the problem if you note down on a piece of paper based on that we will slowly proceed and draw the diagram.

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So the diagram is like this stages of evaluation if you remember are the green vertical lines so this say for example stage 1, stage 2, stage 3 and the decision is like this either you go or do not go that means do not do anything and this twenty which is read in color mean basically means that you are investing that means some amount of money is going out of your pocket and this blue one which is twenty million per year actually means the overall amount of money which is coming in your pocket on a yearly basis quarterly basis whatever it is.

So if I consider the problem from the mode extreme because what I will try to do is that try to basically analyze the problem from end result and slowly go into the actual source where the decision would be made.

So for the extreme right values thirty million means there is a probability was sixty percent of high demand and you would get on an annual basis thirty millions as a payback for a twenty year period in case if the demand was on the lower side which are probability of point four that means there are probability of arms then the value would to twenty million per year for again the same duration.

So this twenty year being same for both of them need not be so it can be different time frames. Now this is what I called a C2 2 that means there are two now alternative accuracy there is a chance. Now this one fifty means that you are investing if you read the problem let me go back

to the problem again spectrum can build a plant considering one fifty. So this one fifty is the actual investment and if I consider this two hundred actually this two hundred means it will cost and take the test from the pilot production to happen so this is basically the well the concept the firm is ready to invest so this is the twenty million.

So this time frame is six months so once it is done eight you go and go and make a decision accordingly. Now let us go one by one at C2 what you are trying to do you are trying to find out the expected value of that decision at C2. So at C2 how many folks are there are two so probabilities are 0.6, 0.4. What are the values? Thirty million, twenty million per annum now if you see the whole column inside the bracket it basically means the time value of money as of now.

So if I consider an interest rate of twelve percent which is constant if you remember the problem in the instances where we are discussing different type of financial concepts which can be utilized to make a decision I did mentioned that the interest rate can either be fluctuate hence you have to take an expected value here we are considering two the expected value to two to twelve percent for twenty year time period on an annual basis which is very simplistic it can change.

Now if I consider these for a twenty year time period for the thirty million which will be paid back to me for a very high demand one it means that after the first year if I get the money what is the money what is the present value of that. After the second year if I get back the money after two years gap what is the value of that so that means thirty multiplied by this term is after one year present value thirty million by the second term is the present value for the second term.

Similarly for the last value to be thirty multiplied by last term whether term as the denominator  $1 + 0.12$  which is the interest rate to the power twenty that means for a twenty year time period. So if you add up all of them I am able to add up thirty is a common factor I take it out I find out multiplying by sixty percentage of 0.60 is what is the value of that overall thirty million which would be paid back to the firm considering is a high demand.

Similarly if find it for the forty percent which is on the load demand so I multiply them by another expected value now here just note as I mentioned again I am mentioning twelve percent may change for both the cases timeframe twenty twenty may change for both the cases. So the expected value is one ninety four now if you consider the one ninety four here it means that if I conglomerate all the values here the overall net worth is one twenty four. So what I will try to consider is that whether one ninety four is greater than one fifty if it is yes I will invest if it is no obviously I would not go into that.

So let us go into the next stage at B2 where you will take a decision that means in D2 you would either stop or else you will ahead that means go ahead and invest one fifty to build up that that path such that production would be there with the probability of high demand sixty percent more demand forty percent. So add D2 you find out the expected value at D21 at this point is forty four point 2 hence as this decision with respect to the other one.

So here what you will do is there you will stop production if it is negative and you would not do anything so what you do is that we compare D2 and D3 this is positive means you will definitely take that which is shown here. So as D21 is greater than D22 hence D2 is same 21 is selected so these suffix is based on from which point to which point we are going the nodes and the arcs.

Even though it does not have any direct implication with the PERT and CPM just I wanted to mention that. Now similarly at the point of C1 so what I will do at C1 is like this this forty four point two is already the value which is positive here.

So what I will try to find out is that whether the probability multiplied by forty four point two and the value where I am not taking a decision whether that is positive or negative in the overall expected value sets. Similarly what I will do is that seventy percent of point seven multiplied by point four I am just skipping going up and down the slide but please bear with me. So it is point seven into forty four point two plus thirty percent into zero because Y the zero because there is no cost involved in case there was a cost involved if it was negative.

So obviously it can be true that seventy percent into forty four point two multiplied plus thirty percent into negative value if that was negative obviously you will stop that means even even if this value is giving you a positive one it will definitely mean that here there is not at all any logic in trying to proceed in the later stage are the right hand side of the file where the cursor is now hovering about.

So this means I am trying to go from left to the right and take a decision and each and every step so at C1 the value is thirty point nine now at D1 what I do is there this value I have already found is expected value is forty four into point seven plus zero into two point three and the trying to compare whether twenty which is the amount money which is going out of my pocket is more or less so again I find out this value is thirty point nine. Thirty point nine is greater than twenty which is ten point cents it would actually means I will definitely invest into that projects.

In case say for example these value is considered it was forty so in and in that case would have been thirty point nine – forty P which would be almost in the tune of -10 hence we would not go into the project. So this was the very simple example let us consider it and extend this concept for a second example.

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DSE14 Notes: 16-11-2019, 1700 hours

$D_{11} = EMV(D_{11}) = 30.9 - 20.0 = 10.9$   
 $D_{12} = EMV(D_{12}) = 0$   
As  $D_{11} > D_{12}$ , hence  $D_{11}$  is selected

**Example 2.2**

An oil company while evaluating the oil basin is considering three alternatives, which are (i) drill, (ii) conduct seismic test before at a cost of Rs. 20000 to find the nature of the underlying oil basin and (iii) do nothing. If the company drills then it is likely to find the oil basin as (i) dry, (ii) wet or (iii) soaking. A dry well yields nothing, while a wet well provides moderate quantity of oil and a soaking well generates substantial quantity of oil. If the oil company conducts seismic tests, then it can learn about the underlying structure of the oil basin before deciding whether to drill for oil or not. The underlying oil basin structure may be one of the following, which are (i) no structure, (ii) open structure or (iii) closed structure. If no structure is found then the prospect of finding oil is bleak. If an open structure is discovered then the prospect of finding oil is fair, while finally if the structure is closed then the prospect of finding oil is bright.

The oil company also knows the following which are



An oil company while evaluating the oil base in is considering few alternatives now you will see there are three alternatives is not two. So it is either drill conduct such missed test which is cost of twenty thousand and find the nature of underlying oil basing or do nothing. So doing nothing is also an alternative so if the company drills the oil basing can be of three types depending on the geophysical characteristics it can be try wet or soaking.

Now if I dry well yields nothing that means it would not give you any oil wet well will provide you moderate quality oil and soaking will definitely give you the maximum amount of oil. So now what will depend that underlying oil basin structure may can also be of different qualities or or variety one can be no structure one can be open structure and one can be closed structure these are just characteristic basic where you are trying to drill.

If no structure is found then the prospect of finding oil is very week almost zero if an open structure is found then finding out the probability of oil is fairly high and for if the structure is closed then the prospect of finding oil is very high that means probability is very high. So these are the qualities feel of the problem which is there now with that I just slowly tried to give you on the probability sense how this or situation makes sense that you have all the values and in the quantitative sense.

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$D_{12} = EMV(D_{12}) = 0$   
As  $D_{11} > D_{12}$ , hence  $D_{11}$  is selected

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An oil company while evaluating the oil basin is considering three alternatives, which are (i) drill, (ii) conduct seismic test before at a cost of Rs. 20000 to find the nature of the underlying oil basin and (iii) do nothing. If the company drills then it is likely to find the oil basin as (i) dry, (ii) wet or (iii) soaking. A dry well yields nothing, while a wet well provides moderate quantity of oil and a soaking well generates substantial quantity of oil. If the oil company conducts seismic tests, then it can learn about the underlying structure of the oil basin before deciding whether to drill for oil or not. The underlying oil basin structure may be one of the following, which are (i) no structure, (ii) open structure or (iii) closed structure. If no structure is found then the prospect of finding oil is bleak, if an open structure is discovered then the prospect of finding oil is fair, while finally if the structure is closed then the prospect of finding oil is bright.

The oil company also knows the following which are

Probabilities of various oil bearing states  
 $P(\text{state is dry}) = 0.50$   
 $P(\text{state is wet}) = 0.25$   
 $P(\text{state is soaking}) = 0.25$

Probabilities of various soil geological states  
 $P(\text{no structure}) = 0.40$   
 $P(\text{open structure}) = 0.30$   
 $P(\text{closed structure}) = 0.30$

So here are the values probability P means probability state is dried is fifty percent probability weight is twenty five percent probability is soaking is twenty five percent.

So if you add up all them then it is one which as should be as it is then I go to the probability of geophysical structure geological states no structure forty percent open structure thirty percent closed structure thirty percent again the sum is one as it should be also we have a joint probability a conditional probability based on different type of oil bearing states and juridical structure that means I am trying to how compare and find out the condition probabilities depending on one as occurred other may occur or vice versa.

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IEM 314 Notes: 16-11-2008, 1:50 hours

		Underlying geological structure			Marginal probability of the state
		No structure	Open structure	Closed structure	
Oil bearing state	Dry	0.32	0.15	0.03	0.50
	Wet	0.04	0.10	0.11	0.25
	Soaking	0.04	0.05	0.16	0.25
Marginal probability of the geological structure		0.40	0.30	0.30	1.00

Finally the oil company also has the following set of information regarding the net present value of the three states, which are: (i) NPV(Dry state)=0.6 million, (ii) NPV(Wet state)=0.8 million and (iii) NPV(Soaking state)=2.4 million

**Solution 2.2**  
M.C.

So this is where slowly complications will come but it will give you a very good idea how the problem can be solved so the pink one which you see are the oil bearing states though they are a drive at soaking as I have already mentioned and if we remember dry was fifty percent see the value on the right most column.

Wet was twenty five percent soaking was twenty five percent so these add up 2 1 if I consider the underlying geographical structure a geological structure which is in the blue color there is no structure open structure close structure or close structure has a probability of thirty percent open structure thirty percent no structure forty percent again probabilities 1.

So the value which you have are long or top most the bottom most row and the right most column are the marginal probability of the state considering that the geological and the oil bearing states are very nicely shown. Now if I want to find out these values which is given inside they are provided it means that provided is not structure and dry the probability which is give is the conditional value point three two or say for example with soaking and close it is point one six or sixteen percent.

So you have the different values which are given finding the oil company also has the following set of information which means the net present value of the dice dry state is this is minus value which is negative net present value for the positive set is positive point eight and for the soaking it is two point four. So this is minus zero point five plus zero point eight plus two point four. So if I go try to basically find out the overall problem the problem in diagrammatic sense in the decision analysis since looks like this.

So the first I will show you the diagram which I will try to basically reduce size of the word document and try to basically show you at one go rather than going up and down in word document.

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So let me go first to the diagram so let me reduce and zoom in so if you zoom in just concentrate on the diagram as such. So this is the overall diagram which I have when I am just moving my

cursor so it means that these values which are there dry wet and soaking are the structure which I have discussed so this is dry so quit soaking dry wet soaking and so and hence so forth and this minus and plus probabilities have been very nicely shown in color in order for the students to understand.

If you remember I did mention negative point six so this is zero point six in the red color then we mentioned positive point eight which is point zero eight point eight in the blue color and then I also mentioned for the soaking part the values returned was plus two point four which is again in the blue color. So these are given for all the states now if it is given for the all the states now let us go through all the arts in little bit detail it will give you a very good picture how the problem has been found out.

So if you drill you have to invest which is the upper most part if you do not drill do not do nothing this is the which is there which is D13 do nothing. Now if you drill without considering any test so immediately you have either dry, wet and soaking. Now if you conduct a test for which you remember there was twenty thousand cost overlay for that because that is the cost which you have to incur for doing the test the doing the test would give you the structure has not structure which is the some open structure which is C22 arm and this value basically give you the closed structure.

So closed open and no structure would always lead to the fact which it is dry, wet or soaking depending on the first part consider no drill or not test and this whole half this whole three set of decision which are there depending on whether you drill or do not drill considered that you have already conducted or test and found out the properties. So these are the values which are very sure clone clearly this green one are the decision making stage and as already discussed the circular part and deep part basically gives you the decision and the chance for folks.

Now again let me go back to the problem and solve it one by one so if you remember note down these values on a piece of paper these are the probabilities. Note down the overall problem which is given with the probabilities and let us slowly solve the problem. So at seat one between that is the value for which is there for C1 is this part the probabilities would be now just I did skip but

let me again come back to this diagrammatic and discuss the probabilities if you note these probabilities fifty percent twenty five percent twenty five percent then four by eight which is eighty percent one tenth one tenth.

These are the values which are already there so either you utilize this this matrix which is there considering the marginal value or conditional values based on that you have basically seen on all the probabilities values which are there. So if you consider probability C2 and four by ten if you consider the probability C2 3 is 3 by 10. So all these values are considering the fact that you have conducted a test for this one conducted the test and then you have found out that there was basically closed structure here which is 3 by 10.

Considering you conducted the test then his open structure that the corresponding probabilities are given. So all these values are coming from the table which is already provided so if you go one step at a time at C1 there would be three values negative point six positive point four positive two point four but they would be multiplied by the corresponding other probabilities what are the probabilities the probabilities are fifty percent twenty five percent twenty five percent.

You find a value of point five which is positive now at C3 again the values of minus point six plus eight two point four remain the same as this blue and red values show but the corresponding probabilities are changing depending on the conditional values. So now if you have C3 so let me go through C3 is this portion it will be minus point six into four by fifth plus point eight into one by ten plus two point four into one by ten which is give here.

So you find the value of minus point one six which means the value over net value which you have at this point is negative. So negative with no drill definitely would mean that you would not take any action technically means if there is conducted a suspect test there is no structure which means you would never invest in that one that means because if it is negative if you have drilled the value would be negative.

Similarly let us combine to this set of arc's and this set of arc's one at a time at C4 again minus point six point eight two point four. So point A and two point four are positive and what are the probabilities half, one third, one six. So let us go so these are the probabilities half, one third, one six again I am repeating these values which are there the probabilities can be found out from the matrix.

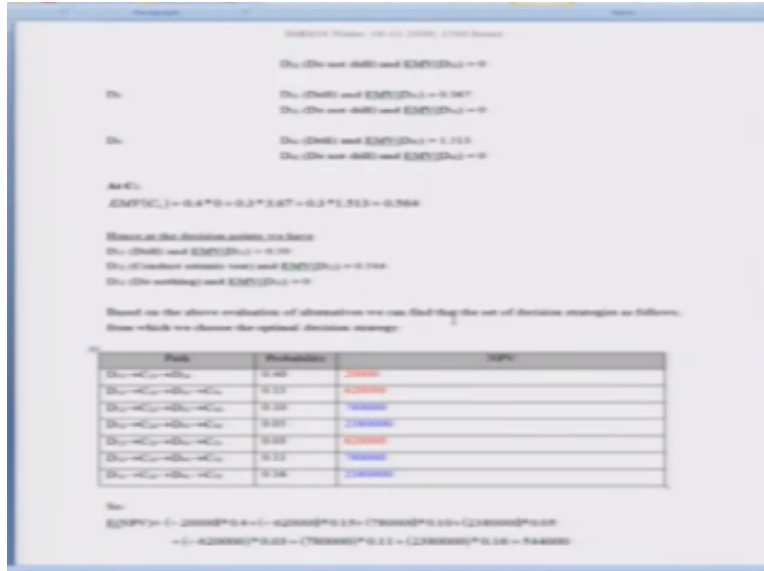
Similarly if I go into those last so called node it is minus point six into one by ten here where the cursor is then again plus point eight into eleven by thirty and this value of two point four into sixteen by thirty. So now sixteen by thirty actually means that what how you are getting is basically you have conducted the test then you found out that there was basically closed structure and based on that when you had drilled there was a soaking based on which you find out the maximum probability of getting the oil. So this is one point five three one three. So hence the decision you have to make is that at this point when you have basically this was negative.

So obviously you would not do anything now if I consider this value with do not drill this value with now drill obviously it will mean that in all the instances they would be corresponding no action some action some action means the drill concept you can say. So at D2 it is negative hence do nothing at B3 positive drill D4 positive drill what are the values at D2 it was negative which is so hence compare with zero at zero and D3 it was positive zero point three six seven.

Hence the value is zero point three six seven and D4 it is one point five one three hence it is one point four five one three now if I want to find out the values at this point what do I have this whole value is zero. Zero multiplied by the probability four by ten here it is four by ten this arm had a positive value what was the positive value zero point three six seven here it is again point three six seven multiplied the probability thirty percent and one point five one three multiplied by point six five point five six four.

Hence again I compare zero five six four and considering this so this is zero point five six four you had a value already found out here and this was zero. So you will basically try to find compare them and then find out the values accordingly.

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So based on the above evaluation the alternatives can be found out there is set of decision strategies are as follows. So this basically give you a probabilities and the values in the negative and the positive sense are given in the red and the blue color. So the expected value if I find out so if you follow up any arm D 12 C21 D22 the probability are given from the tables we can find out and the net present values if which are already there.

So once the net present values have all been found out considering the calculations which you have already done the expected value can be found out as five four four triple zero that means positive and in case if you want to find out the variance you can use the simple concept of the variance to try out the value as given in the formula.

So we can make our own decisions accordingly to the where there that we want to maximize the return as highlighted minimize the risk or make a balance between them where you take basically a ratio of trying to maximize return to risk or minimize the risk return or different type of multi-criteria decisions can be made in order to make the decision.

So these two problems which I discussed one was there was no conditional probabilities in the second case there was conditional probabilities which is simple values and the concept of present value was also there. So in assignments we will also try to tackle such simple problems and there and the assignment solutions would already be which will also be provided to the students to

understand both the concept of decision trees which is the second set of problems we did it as well as in the concept of AHP in order to make you understand how these two different techniques are used in a big way have a nice day thank you very much.