**Project Management**
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Module No # 3
Lecture No # 11
Analytic Hierarchy Process for Project Selection

Welcome back to the eleventh lecture in this project management which is the twenty hour program Under NTPEL MOOC ambit. So as we discussion the AHP problem where I just mentioned in the last slide before I closed the last lecture.

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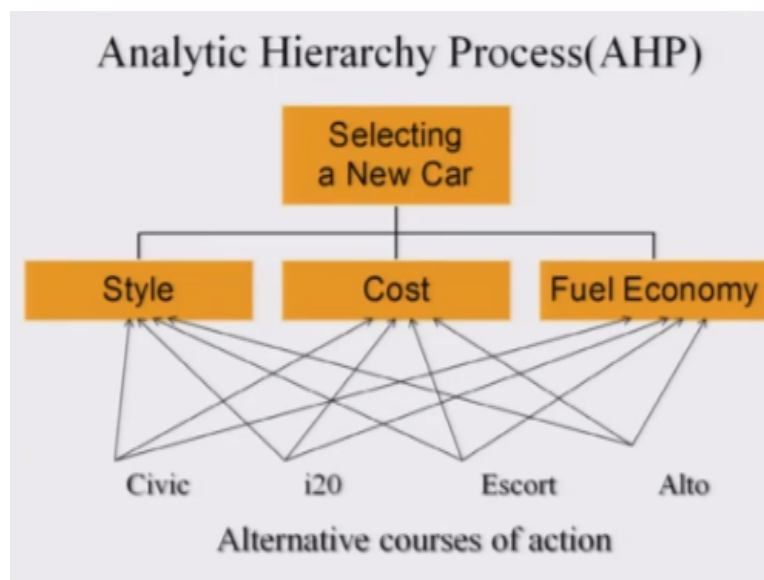
You are trying to select a car which is one half from this list Civic. I20, Escort and a Alto which is the last level which you see here. And if you remember I did mention about style cost and fuel economy. So they can be broken down into tertiary and more such level so this is the primary hierarchy I am not going to go into much so called complication but just simply consider the problem.

So that I will give a good flavor of how the AHP can be used now according to Satty the intensity of the importance when you are trying to compare two different criteria they are done on a one to one scale like one to one basis I would not use the scale basis means. Say for

example if there are three decisions you will try to compare decision one with decision two decision two decision three and again decision one with decision three.

So you will basically have a binary compression being made between the criteria whether at first level second level third level by the word level I mean the hierarchy the primary the tertiary the secondary. So when the level of importance between two criteria's or say for example when you come to the alternate is also consider the overall choice. If it is of equal importance you give a point one between them.

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Analytic Hierarchy Process(AHP)
Ranking Scale for Criteria and Alternatives

Intensity of importance	Definition	Explanation
1	Equal importance	Two factors contribute equally to the objective
3	Somewhat more important	Experience and judgement slightly favour one over the other.
5	Much more important	Experience and judgement strongly favour one over the other.
7	Very much more important	Experience and judgement very strongly favour one over the other. Its importance is demonstrated in practice.
9	Absolutely more important.	The evidence favouring one over the other is of the highest possible validity.
2,4,6,8	Intermediate values	When compromise is needed

Two factors contribute equally to the objective and which is buying the car in this case hence you give weightage of one. Then if you give one of this criteria point three point means not the decimal point of level three which means that in a in a sense the if you are forced to take the other criteria then the overall benefit which you will get by taking that criteria into your concentration would give you a score of 1 by 3 that means higher the score for taking 1 it would mean on the reverse scale it will be lowered the score for the other.

So similarly if I go 3 means somewhat more importance experience and judgment slightly favor one over the other. Similarly as I continue it is 5, 7, 9 which technically means much more important very much more important absolutely essential and the explanations are experienced in judgments strongly favored 1 which is for point of 5 for 7 it is experience and judgment very

strongly favored our while for 9 it is the evidence of the favor one over the other of the highest cost of the highest possible validity or our level.

Now if obviously you may think that why the given numbers were not taken even numbers as prescribed by Satty those 2, 4,6, 8 are intermediate values when compromise is needed between say for example criteria 1 and criteria 2 or criteria 3 and criteria 4 and so on.

So if are of equal importance or you are not able to give a decision you give the event points but the generally we will consider the odd points that means five one fifth three one third seven one seventh nine and one ninth based on which you will make a choice between two criteria at whatever level it is level means the hierarchy.

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Analytic Hierarchy Process(AHP)
Ranking of criteria

	Style	Cost	Fuel Economy
Style	1	1/2	3
Cost	2	1	4
Fuel Economy	1/3	1/4	1

So now if I go primary hierarchy if you remember there was style cost fuel efficiency now consider arbitrarily person one is trying to make a decision without thinking the alternatives only concentrating on the criteria.

So he or she when he is trying to analyze style to style cost to cost fuel economy to economy which is the principal diagram or where it will be one that means you are indifferent but when I consider cost to style then you will see the level of cost is much more important to use who is the person who is making the decision hence you give us the score of two.

Now if you are force to take style into consideration with respect to cost then the overall score would be half that means you are now being the overall utility or the overall benefit which you are trying to get by taking the decision related to style only for that criteria with respect to cost giving you a benefit of half only. Similarly when I go into style and fuel economy the overall scores are three and one third if I go to cost then it is four and one forth between cost and fuel economy.

So this points which are given for this examples are arbitrarily but they do not definitely make sense if somebody asked on a quality scale that what are the different levels of importance one would like to place on different criteria at different levels. So if I am considering primary hierarchy I will compare them I means the only one person who is there will compare all the criteria and then do into the secondary level tertiary level I will have different matrices with these type of course.

Similarly when person 2 come she or she will again analyze the same set of criteria's primary, secondary and tertiary based on the scores but the scores would obviously be different and then what would be done if there are three different persons each of them give different scores the overall analysis is done and then the decision for all those three different persons are combined to give the best possible decision for the overall project which you are transferring.

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Analytic Hierarchy Process(AHP)
Ranking of priorities

- Consider $[Ax = \lambda_{max} x]$ where
  - ❖ A is the comparison matrix of size n×n, for n criteria, also called the priority matrix.
  - ❖ x is the Eigenvector of size n×1, also called the priority vector.
  - ❖ λ_{max} is the Eigenvalue.

- To find the ranking of priorities, namely the Eigen Vector X:
  1) Normalize the column entries by dividing each entry by the sum of the column.
  2) Take the overall row averages.

$$A = \begin{bmatrix} 1 & 0.5 & 3 \\ 2 & 1 & 4 \\ 0.33 & 0.25 & 1.0 \end{bmatrix} \xrightarrow{\text{Normalized Column Sums}} \begin{bmatrix} 0.30 & 0.28 & 0.37 \\ 0.60 & 0.57 & 0.51 \\ 0.10 & 0.15 & 0.12 \end{bmatrix} \xrightarrow{\text{Row averages}} X = \begin{bmatrix} 0.32 \\ 0.56 \\ 0.12 \end{bmatrix}$$

Column sums 3.33 1.75 8.00   1.00 1.00 1.00   Priority vector

So in analytical hierarchy process we use the concept of Eigenvector and the eigenvectors. So consider that very simple that that matrix a which is there which has to do something with the sports I will come to that within one minute in the same slide. So we want to find out the maximum value of eigenvalues so where is the comparison matrix of size N cross N depending on how many such criteria which you have at each level.

So it would be at each level different criteria's set of criteria are there I will definitely have different type of sizes of matrix a for N criteria. So if you see that this N criteria depending on which level you are so in this level for the car buying one we have 3 cross 3 is also called the priority matrix. So in the case if we in the in economy if we had 2 such tertiary or the secondary criteria sub criteria for economy.

So it will be a 2 cross 2 matrix if say for example style had 3 in the secondary level it will be a 3 cross 3 matrix corresponding to the sub criteria of style. So X is the Eigenvector of style also called the priority vector and lambda max is the Eigenvalues to find out the ranking on the priorities namely the Eigenvalues we have to use. So I am not going to the theoretical solution concept I just solve it using the problem as we are going ahead.

So normalize the column entries by dividing each entry by the sum of the column so if you see the scores which we have given initially person 1 the principal diagonal was all 1 1and the scores

for say for example the second row and the first column in the top point 2 was given the other opposite value to the principal diagonal and was half. So in hence it is point five so if you consider the first row and the third column the value is free the opposite value which is one third which is 0.33.

So see similarly you have matrix A as given now you normalize the score this normalization of the score is done in such a way such that if you see the column sums. So the column sums are given first column 3.33, 1.75, 8.What you do is that divide each individual value by column sums and you normalize them. So hence the sum along the columns are one and then what you do is that you find out the row averages.

Now having said that it is also possible to do the row sums they are one and do the normalization accordingly which means that rather than finding out sums along the column you can do it along the row and normalize it accordingly and then you get the values of the priority vectors what we will stick to whatever we are doing now.

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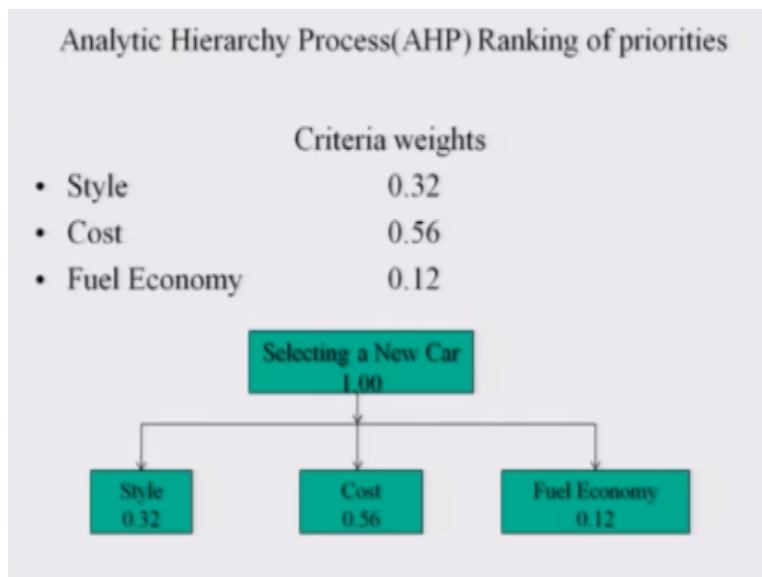
So once you do it the criteria weights are given as thirty percent, fifty six percent, twelve percent. So if you add them it comes to 1 which means that when person 1 is trying to analyze the first

level of criteria which is style cost and fuel economy for him or her the cost is the most important factor which the weightages of fifty six percent.

Now if person 2 has done it obviously the matrix A would have different values in the cells in that case the overall criteria weights for style cost and fuel economy would be different for percentage. Now similarly it will be different for percent three percent four and different type of people are there in the decision. Now if consider person 1 is doing the tertiary or the secondary ranking of the criteria of the sub criteria's then obviously he or she would get different such criteria weights for the second level.

Similarly for person two, person three, person four now selecting a car in the diagram shown in the green one which is this one. So there the person gets the weights as thirty two, fifty six and twelve.

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Analytic Hierarchy Process(AHP)
Checking for Consistency

- The next stage is to calculate a Consistency Ratio (CR) to measure how consistent the judgments have been relative to large samples of purely random judgments.
- AHP evaluations are based on the aasumption that the decision maker is rational, i.e., if A is preferred to B and B is preferred to C, then A is preferred to C.
- If the CR is greater than 0.1 the judgments are untrustworthy because they are too close for comfort to randomness and the exercise is valueless or must be repeated.

The next stage is to calculate the consistency ratio value so consistency basically means how consistent he decision making is. So obviously you are trying to give you means as one of the person who is trying to rank the criteria in order to make a choice among the alternatives. So in that case I want to find out or the decision maker wants to find out in a sense that how consistent are the ranking based on which the final decision can be taken so this CR is to measure consistent the judgment has been relative to large samples of purely random judgment because

they are just random. If I ask one person to compare between fuel efficiency and economy it may change from day 1 to day 2.

So I am trying to find out what is the best judgment AHP evaluations are based on the assumptions that the decision make it rational which may not be true. So if A prefer to B and B is preferred to C then A is always prefer to C which may not true in many of the practical sense. So if CR is greater than say for example then percentage or point one the judgment are untrustworthy because they are too close to comfort and bring into the into the realm that randomness really does play a part.

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Analytic Hierarchy Process(AHP)
Calculation of Consistency Ratio

- The next stage is to calculate λ_{max} so as to lead to the Consistency Index and the Consistency Ratio.
- Consider $[Ax = \lambda_{max} x]$ where x is the Eigenvector.

$$
\overset{A}{\begin{bmatrix} 1 & 0.5 & 3 \\ 2 & 1 & 4 \\ 0.333 & 0.25 & 1.0 \end{bmatrix}} \overset{x}{\begin{bmatrix} 0.32 \\ 0.56 \\ 0.12 \end{bmatrix}} = \overset{Ax}{\begin{bmatrix} 0.98 \\ 1.68 \\ 0.36 \end{bmatrix}} = \lambda_{max} \overset{x}{\begin{bmatrix} 0.32 \\ 0.56 \\ 0.12 \end{bmatrix}}
$$

- λ_{max}=average{0.98/0.32, 1.68/0.56, 0.36/0.12}=3.04
- Consistency index , CI is found by CI=$(\lambda_{max}$-n)/(n-1)=(3.04-3)/(3-1)= 0.02

So the next stage is to calculate lambda max so as to lead to the consistent index and the consistency ratios consider as I had already mentioned that you are basically trying to consider A lambda A = AX sorry AX is good to lambda max into X. So this A was that matrix of the weights X is the Eigen matrix and Eigen values where X is the Eigenvector as I mentioned.

So and these A which I have found out I want to find out X now X I have found out using the normalization. So what I have is basically value of lambda max so once I multiply A into X I would basically have the value as lambda max. So that lambda max as on average would give me the consistency ratios and the CI index. So I do it very simply using that lambda max is the average of these values.

So average means 0.98 which is by multiplying this into this gives you that means1 into 0.32 + 0.5 into .56 + 32.12 would give you a value of 0.98 and I find it out thought these values are 0.98, 1.98 and 0.36. So I try to find out the lambda max value such that is the ratios of 0.98 to 0.32. three two is the thirty percent fifty sex percent and twelve what I have already defined out found out.

There the value comes out to be 3.04 now here the sample size is there are three sample size means how many such criteria's are there based on that I find out the values of the constancy ratio index as 0.02.

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Analytic Hierarchy Process(AHP)

C.R. = C.I./R.I. where R.I. is the random index

n	1	2	3	4	5	6	7
R.I.	0	0	.52	.88	1.11	1.25	1.35

C.I. = 0.02
  n = 3
  R.I. = 0.50(from table)
    So, C.R. = C.I./R.I. = 0.02/0.52 = 0.04
C.R. ≤ 0.1 indicates sufficient consistency for decision.

Now consider the ratio is basically the ratio of CI index by RI so RI values are the random index which are given in the literature. So N means the number of observations I have 1, 2, 3, 4 depending on a number of such criteria's I have in any level am not combining different levels by primary, secondary, tertiary separate. So once I find out four three it is 0.02 I found out and as per the calculations given in the table it is 0.52 then I will find out that the value of CR comes out to be 0.04 which indicates sufficient consistency in the decision because the cutoff value as you mentioned was ten percent or 0.1 percent.

So it can be done so if you are satisfied with first level of the decision making for the criteria's by person 1 you will go into similarly the decision making for person two in the same criteria set of criteria for the same level and continue doing it for the third person, fourth person, fifteen person for the first hierarchy.

Then you shift to the second hierarchy do it for first person, second person, third person and continue doing that that there is not inconsistency in the decision making process for each and every individual considering conglomeration of criteria's at different levels.

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So what I do is that I now compare the alternatives if you consider the alternatives based on the criteria this is where things get are generally very interesting how what I have done is that for each individual in the first sense they had compared the criteria amongst themselves. Now what I am going to do is that I am going to compare the alternatives based on each and every criteria at one go.

So point one will be comparing the alternatives for criteria one point two would be comparing with alternatives with respect to criteria two and so on and so hence so forth for all the different type of criteria's I have. So it will be for level 1, level 2, level 3 but as we have only one primary level will only do it accordingly. So with respective style if I compare the alternatives this this principle diagonal again is 1.

Now if I compare civic to I20 it means generally the overall point which I am trying to give between civic and I20 are in the ratio of 4 is to 1 by 4. So this four and these value I have just utilized the values of even and odd in order to make comparison that how the values of even and odd when you are trying to utilize consider the decision maker is not aware it would not affect your result much.

Because it will give you a very nice manner that how you can conglomerate all the decision making processes for different individual. So first set of value which is there where I however my pen is for the style for the four alternatives next one is cost for all the alternatives and these are the priorities vectors which I have. So technically it means that if on qualitative sense if I am only considering style I would give my maximum point to all two which is fifty six percent.

If I go to the cost factor it would mean that I will give my maximum amount of points would of thirty eight percent for the cost factor only for all the four alternatives. So obviously they are different to each other this priority matrix and this priority matrix consider that only in the first case only style is the factor. In the second case cost is the factor so there you are trying to analyze each individual or criteria for each alternative on a case by case basis.

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Analytic Hierarchy Process(AHP)
Ranking alternatives

		Miles/gallon	Priority Vector
Fuel Economy	Civic	34	.30
	i20	27	.24
	Escort	24	.21
	Alto	28	.25
		113	1.0

Note: Since fuel economy is a quantitative measure, fuel consumption ratios can be used to determine the relative ranking of alternatives.

If I consider the fuel economy so here the rankings are given so here are the miles per gallon which I have 34, 27, 24, 28 are the miles one gallon would be utilized to drive this car civic 120, Alto and Escort and Escort and Alto. And the priority value is given remember double check the concept which you are trying to utilize as I mentioned whether you are trying to use the column normalization of the road normalization and the end of the day the sum should be one.

So not since fuel economy is a quant quantitative measure well consumption ratio can be used to determine the relative trying to know the alternative So what we have done is based on that only now once the and the overall the points are given for each and every alternative for each and every criteria what I do is that I have in this level again going back to the same set of diagram which I had your main aim is to select the car and the three primary criteria was style cost and fuel economy if you remember they were at the level of thirty two percent, fifty six percent and twelve percent.

Now when I try to compare all the four cost for style all the forecast for cost all the forecast for fuel economy there the second level of point sharing are accordingly so what we have just found out.

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Analytic Hierarchy Process(AHP)
Including Cost as a Decision Criteria

❑Adding "cost" as a a new criterion is very difficult in AHP.
❑A new column and a new row will be added in the evaluation matrix.
❑However, whole evaluation should be repeated since addition of a new criterion might affect the relative importance of other criteria as well!
❑Instead one may think of normalizing the costs directly and calculate the cost/benefit ratio for comparing alternatives!

	Cost	Normalized Cost	Benefits	Cost/Benefits Ratio
• CIVIC	$12k	0.22	0.28	0.78
• i20	$15K	0.28	0.25	1.12
• ESCORT	$ 9K	0.17	0.07	2.42
• Alto	$18K	0.33	0.34	0.97

Now as I proceed in the next slide it will be its shown that what I want to find out is the overall score based on the fact that you have been able to compare the criteria's among themselves and

have been able to compare the alternatives based on each and every criteria single criteria each and every stage. So once I do that there for the first time I would have basically the matrix where in the left most line you have the different type of alternatives and the top post part you have the different type of criteria based on which you are trying to take a decision.

So the priority matrix is now a conglomeration of all the criteria's for each and every individual combined together. So the criteria weights are given which you have already found out when you compare them and when you multiply in them you basically have the overall score based on the fact then the alternatives have been compared for each and every criteria. So analytical hierarchy process including cost as a decision criteria.

So if I consider adding a cost as a new criteria becomes very difficult because if you add the cost factor it would mean that you have to add a column do your calculation and repeat it for all the facts because now costs is a criteria. So you have to basically bring that in the picture and include the cost as a criteria and compare the alternatives is based on the new set of criteria's consider they were already four criteria's cost came to the picture.

So and they were ten different alternatives so each alternative would now basically be compared on the overall set of criteria where now cost is also a part and parcel of one of the criteria. So we will again do the calculation accordingly. However the source whole evaluation should be repeated since the addition of the new criteria might affect the relative importance instead one may think of normalizing the cost directly and calculated accordingly.

So once you have cost factor cost means not the fuel factor the cost of each and every car you normalize it you find out the benefit with respect to the cars which you have and the unifying of the cost benefit ratios which is given on the last column.

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Analytic Hierarchy Process(AHP)

- The "ESCORT" is the winner with the highest benefit to **cost ratio** and we rank it 1st,
- Then at 2nd position i20,
- At 3rd is Alto,
- While at 4th position is Civic.

So if your name is basically now try to find out the overall cost ratio based on which you will take the decision then obviously it would mean that Escort is the first one second one is I20 third one is Alto and the fourth one is Civic. So what you are trying to do is that you are trying to add up the weights for each and every criteria in each and every alternatives and them in such a way that the overall value for the alternative considering all the criteria's are take into consideration is the maximum.

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Analytic Hierarchy Process(AHP)
More about AHP: Pros and Cons

Pros
- It allows multi criteria decision making.
- It is applicable when it is difficult to formulate criteria evaluations, i.e., it allows qualitative evaluation as well as quantitative evaluation.
- It is applicable for group decision making environments

Cons
- There are hidden assumptions like consistency. Repeating evaluations is cumbersome.
- Difficult to ue when the number of criteria or alternatives is high, i.e., more than 7.
- Difficult to add a new criterion or alternative
- Difficult to take out an existing criterion or alternative, since the best alternative might differ if the worst one is excluded.

So analytical hierarchy process has many pros and cons so I will discuss that and I will also the would try to tell to my students that even though the problem was very simple they would be different type of books or in where the people can refer the concept of AHP and understand that

how AHP use and obviously they would be simple assignment also which will help them to clear the doubts and we are there to help them the students in order to clear the doubt and proceed with the concept of AHP and other problems which will consider.

So the pros are it allows multi criteria decision making that means you are trying to basically take different type of criteria in under to make a decision it is applicable when it is difficult to formulate criteria evolution that mean where qualitative feel has also to be brought into the picture that means it allows quality evaluation as well as qualitative evaluation and also due the fact that rationality means may not be there in any of the decision making process.

Because if you consider the concept of consistency ratios and the index it will give you a good picture that whether the decision based on the criteria or all the alternatives or rational or not. It is applicable for group decision making environments and gives you a collective decision that how the different sets of people do make a decision while on the other hand there are hidden assumptions like consistency repeating evaluations are comparison because if you remember in the problem where we just consider if cost is brought in the factor.

It may as a new item it may so happen that you would be tempted to ignore the other alternatives or the criteria on the alternatives and go ahead in trying to do the calculations. But obviously it would mean that you would have to do all the calculations based on the fact one or two or three different criteria's whether in that level 1 or level 2 or level 3 that means primary secondary and tertiary are into the picture such that it will give you a good feel that how in an overall sense you are able to dream the adjustment for all the criteria's in order to make the best possible decision.

It is difficult to use to when the number od criteria's are alternatives are very high size say for example they are they are more than seven then trying to do the calculations seven cross seven matrix repeatedly because there would be sub criteria tertiary criteria's it may become difficult. It is difficult to add a new criteria because as I said that it would basically take some time to do the adjustment but generally there are different type of packages which help in trying to make the decision much more much more easier that mean to do thing calculation much more easier.

This is difficult to take out an existing criteria so now adding an alternative or adding an criteria becomes easy but if you want to take out then then trying to compare and trying to do that the comparison at one go may become difficult the reason is very simple I will try to give you a qualitative feel. Whenever you are comparing the alternatives on the criteria remember they are done on a one to one basis. So if there are four such alternative or three different type of criteria you will try to compare the first to the second, second to the third, first to the third and continue considering the criteria.

If in the secondary level if there are five different criteria it will be first to the second, second to the first to the third first to the fourth first to the fifth. Similarly you will do with the second to the third, second to the third, second to the fourth, second to the fifth and continue doing that. Whenever you are trying to do a decision between two distinct criteria or even between to distinct alternative based on any one criteria.

The fact always remains that you are trying to ignore the other sets of criteria where which are available in a sense that if you are trying to compare person A and person B, decision A and decision B, project A and project B you tend to ignore the other existing projects such that the decision making which you are trying to make between A and B who is on a standalone basis that means you are not basically consider all the other alternatives which may affect your decision between A and B only.

Point one point number two is that so this is a big to be noted for AHP even though on a very simplistic sense that actual calculation which you do for AHP does give you good results. Another important factor is that when you are trying to compare two different alternatives or two different criteria it may so happen there is a grey region such that you are indifferent between criteria one and criteria two.

If you remember when we are basically mentioning the scores 1 1 if they are of the same level two half three one third and points accordingly you were always able to take a decision that whether you like or dislike one but it may so happen that in many of the cases that trying to take

a decision whether you are forced to take A or you are forced to take B does not give you a maximum benefit which you want actually.

So in those cases there would be some indifferent region or neutral region where you would not be tempted to take either decision A or alternative A such that trying to analyze those type of problem using AHP may not give you best result that means now your decision making is basically added into three such final output one I definitely like A one I definitely like B and another is that I am indifferent having said that also remembered that if I am forced to take decision A with respect to B.

My liking to take A or my liking to take not take B would basically be of a different consequence if I am being forced to take decision B. So if am for A my liking for A would be much higher than if I am able to take a decision B where I do not like B.

So all this nuances would definitely be not possible in the simple AHP concept but I am sure the students would be able to appreciate AHP once they solve the problem and understand the assignment which is given with this I will end the this lecture and continue the discussion on the project management for other different type of topics or decision analysis and other related issues thank you very much.