

Quantitative Finance
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Module – 01
Lecture – 02
Contd... Part-2

Given any n number of stocks, where n is a number, it can be 10 or 12 whatever it is, if you consider the concept to the variance of the expected value. So, they would be n number of variances and n number of expected value, but what is also required is basically, what is the inter relationship between two random variables, which we basically denote by the concept of either correlation or co variances. So, if we are able to draw the co variances between these n number of stocks, what we get as we are mentioning is that, we will basically have a n cross n matrix as shown here.

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Definition

$cov(X,Y) = E[(X-E(X))(Y-E(Y))]$

$\rho_{ij} = \frac{cov(X_i, X_j)}{\sigma_i \sigma_j}$

For 'n' assets denoted by X_i , we have their expected returns denoted by $\mu_i = E[X_i]$ and variances by $V[X] = \sigma^2_i$, respectively. Moreover if the correlation coefficient between the i^{th} and the j^{th} asset is denoted by ρ_{ij} , then the return and variance of the collection of assets (i.e., the portfolio) is given by the formulae given below, provided we have w_i as the percentage of money invested in asset i .

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And this n cross n matrix basically would have a principle diagonal, which would be listen to, if you listen it very carefully or understand is basically the variances of all these n stocks or the co variances of the n stock with itself. So, the first one would basically be the covariance of the first stock with itself and the last one would basically be the

covariance of the n asset stock to itself, so which are basically with the variances and the, of the diagonal element are basically with the co variances.

So, if we use the concept of the formula, the co variances; the covariance formula would be given by this, where this x is basically the first random variable, which can be r_1 minus this expected value, which is the first moment of the first random variable, which is r_1 expected value of r_1 multiplied by y , which is the center random variable, which is r_2 minus the expected value of y . And then, you basically find out the covariance of between x and y would be given by as we were writing down with the expected value of this.

So, at the end of the day when you basically solve, so what you actually get would be this ρ , which is basically the correlation coefficient existing between two different random variables or two different stocks would be given in the initial, numerator will basically have the covariance's and in the denominator we will basically have the standard deviation. It would be σ_x or σ_{r_1} for the first random variable and σ_y for the second random variable.

So, we will denote this correlation coefficient as $\rho_{x,y}$, technically what we want is this ρ_{r_1, r_2} , where r_1 and r_2 are the random variables. So, if we consider the principle diagram, what we have is ρ again as I am repeating it or the covariance of the random variables with itself or the covariance of the particular price of the stock with itself, which are the variances and of the diagonal element of the covariance's. So, remember, if we use this formula of the covariance's they are equal, in the sense they are just mirror image of each other.

So; that means, the first element, which is in the first row second column is exactly equal to the second row first column value, which is basically the covariance of first to second or the covariance of the second to the first. Now, remember here that we will be coming back to this correlation coefficient again. Now, if you have gone through the syllabus, at a later part of the course we will be discuss, what is copulos. Copulos are in a very simple sense, I just introduce not from the theoretical point of view, but I basically mention that the copulos are some sort of linkage functions or linkages will basically gives that, that characteristics between two random variables, how they vary among these either.

So, if we consider the linkage function has a correlation coefficient, so you will basically use correlation coefficient, in case we do not think correlation coefficient is a good measure between the linkages, we will use the concept of copulos and which is very heavily used in the area of finance. We will consider that later on, once we solve the first set of problems.

So, what actually we have? Again I am repeating n number of stocks, n number of expected values, n number of variances and their corresponding covariance's between them, which would be ((Refer Time: 04:01)) number. But; obviously, remember they are equal with respect to the principle diagonal.

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Definition

$E[P] = \sum_{i=1}^n \bar{X}_i w_i$

$V[P] = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \sigma_{i,j} = \sum_{i=1}^n \sum_{j=1}^n w_i w_j \rho_{i,j} \sigma_i \sigma_j$

Also remember that

$$\sum_{i=1}^n w_i = 1$$

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So, now, we have a portfolio, you want to formulate a portfolio. Now, for ever you are trying to formulate a portfolio, remember that as an initial amount you have some amount of money, consider that is capital W, which is the bell wealth if you remember. Now, what we consider in a very simple is that total amount of wealth is divided into some proportions and you have invest that amount of proportions in n, those n number of particular stock.

So, if you consider the total amount, which you are spend in the first stock is 10 rupees out of the 100 rupees, which you have in your pocket, hence the overall weight which you are investing in that particular stock would be 10 by 100, which is 0.1 or 10 percent.

If you are consider you are investing 20 percent, it would be that you are investing 20 percent of your total number of money in that particular stock.

So, we are using for the first time a variable, which are known as the weight. So, in such that given n number of stocks they would basically have r_i , where i basically changes from 1 to n and the random variables, they would basically have the expected value, which is basically the first moment as we have already discussed, then you will basically at the variances, the covariance's along with that you will basically have the weight.

So, what you actually need as that, given any particular portfolio you want to find out the optimum portfolio; such that you can find out the wage; such that the overall combination of the portfolio will give you the best benefit. And I am using the word best benefit, it means that depending on what your actual output is, whether you want to basically increase your expected value, whether you want to decrease your variances and so on and so forth.

So, the simple formula based on which you find out the expected value of that portfolio. So, this P word is basically being used for the portfolio remember that, would basically be the expected value would be found out using r_i bar, where bar is basically the expected value given, we have basically a sample and what is the sample, I am going to come direct to that within two minutes. Multiplied by the wages, so these are w_i , so it could be if there are n number of stock, the first term would be w_1 into r_1 bar, second term will be w_2 into r_2 bar, so on and so forth.

Now, why we use a sample is in a very simple sense that I will basically step aside in and basically give you a small definition, that given actually the overall information of that particular random variable, it is total population. Population means that the whole set of information which you have, in general it may not be able to find out the overall random variables values, which are the nearer value for the whole population. So, what we do is that we would basically take a chunk and that chunk is basically one of the samples; such that we choose the sample in such a way that basically it mimics all the properties of the population, which we have.

So, we consider that if the expected value for the population is not known, we basically we take a sample average; thus the expected value of the example in a very simple terms

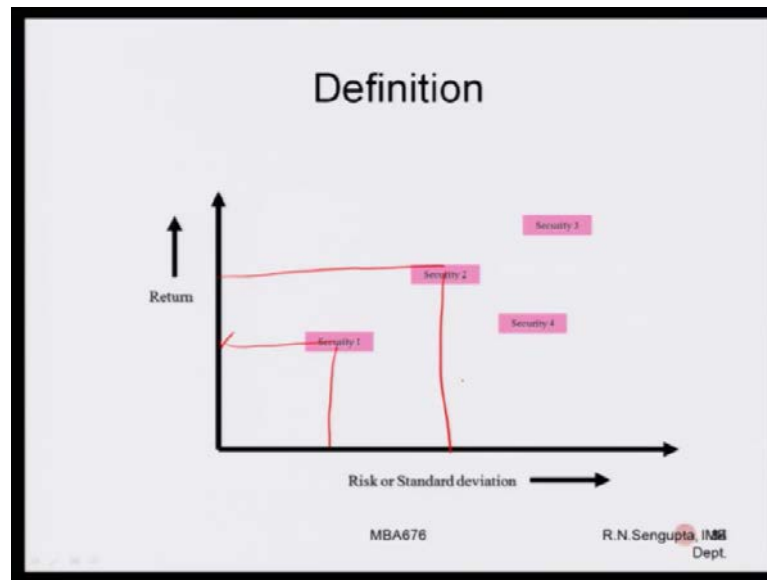
and that is basically denoted by \bar{r} or say for example \bar{x} depending on what about the random variable is. So, given the information of the sample average which is \bar{r} and the weight, which is w_1, w_2, w_3 whatever you have will basically find out the overall, the return of the portfolio or the expected value of that portfolio given by the summation of r_i into w_i .

Now, given the expected value of the portfolio will also being interested to found out, what is the variance of the portfolio. So, if you remember we had basically the variance, covariance matrix. So, what you do is that simply, you find out the double summation of w_i into w_j into the covariance correlation, which is existing between the random variables i and j , r_1, r_2, r_3 whatever it is.

So, if you consider the first element, which you have in the variance covariance matrix, it is basically the variance of the first random variable with itself. So, it will be basically multiplied by w_1 square, where w_1 is the weight for the first random variable. If you consider the last one it would be basically be the variance of the n th stock or the n th random variable, which you have with itself multiplied by w_n square or w_n suffix square basically and the after diagonal element, element will be multiplied by the corresponding weight for the first and second multiplied by the covariance's, which you have.

And also remember, which is technically true, that if we have basically n number of stocks we will consider that the weight are w_1, w_2 to w_n hence the summation should always be equal to 1. Irrespective of the fact whether you have I am using for the first on the concept of short selling, what is short selling we come to that later on as we solve the problems. If it is very simple case, that you have basically n number of assets or n number of stocks we basically try to utilize in such a way that you find out the weight, so that it basically gives you some benefit based on which you are trying to basically find out the best optimum solution.

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Now, what we do is that we want to basically find out that how does those particular stocks or those particular portfolio is look in your overall picture; that means, we are trying to basically draw for the first time that how they look on the Cartesian coordinates. So, in the Cartesian coordinate, what we have is along the x axis we have the risk or the standard deviation the variance, which is the second moment, which we are talking about. Later on will see that, how other different type of second moment, how other different type of risk can also utilize in order 2, basically draw this graph and along the x axis you basically have the returns, which is the first moment.

So, what we are trying to do is that on the y axis we are trying to basically draw our picture or upload the positive benefits which happens for any investment on the x axis, these are basically a set of negative benefits which happens. So, if you basically have different type of asset, so they are basically plotted by this pink color, so this a security would basically have a return, which is given here and the risk, which is the variance. Similarly, for the second stock will basically have this and so on and so forth.

So, what is now important for us to formulate is basically find out those weight based on which all these different type of security which are there can be combined; such that you can basically formulate a portfolio depending on, what your mean concern is. What is the concern and how you basically formulate the portfolio, before we solve that we give you a brief introduction, that what you basically mean by the combination of this and number

of research, so called financial assets or financial scripts which you have in front of you. Now, we consider for the first time the concept as I mentioned the concept of short selling.

So, short selling in a very simple way consider that, you want to basically formulate a portfolio, but you do not have the particular stock with you. So, what you do is that you basically worried from the second person and formulate that in new portfolio; such that after time expires you basically return that particular asset of the financial instrument back to the person from, where you have basically borrowed.

So, technically it means that you do not own that particular financial assets, still you are trying to formulate in such a way that it is utilized in such a way that you are able to maximize your return and minimize your risk or whatever you want to do. Considering that he had been consider in your portfolio as one of those single component which basically forms the overall portfolio. So, say for example, you have n number of assets, it may be possible whether the third, fifth and the sixth are short sold; in the sense they have been borrowed from the other person, they had been combine in, in order to basically formulate a portfolio.

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Investment Process

Let us now determine the return associated with SS. We receive X_0 initially and pay X_1 later. So the $R=(X_1/X_0)$. Generally the returns of any asset is random, in the sense we do not know what would be the amount of money we should pay (receive) for buying (selling) the asset. Hence rather than working with the random variable (r.v), we are more interested in finding the average/expected return of the asset and as well as the variance of the returns. This is the notion which we will apply for calculating the portfolio return, where by a portfolio we will always denote a conglomeration of 'n' number of assets, each of which has a certain random return.

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So, now, again without going to at the reputation will consider the returns to be basically small r and remember also, if you consider later on if you mentioned that I mention the word that summation the weight should be equal to 1. So, if there was no short selling,

all the weight should be basically between 0 and 1, but if there is short selling, then it would basically mean that some of the weight technically can be negative still will consider that the summation should be 1. But with the fact that the weights are not the actual weight with the other normalize weight. How the normalize weights are found out? I will come to that later, later I will solve the problem.

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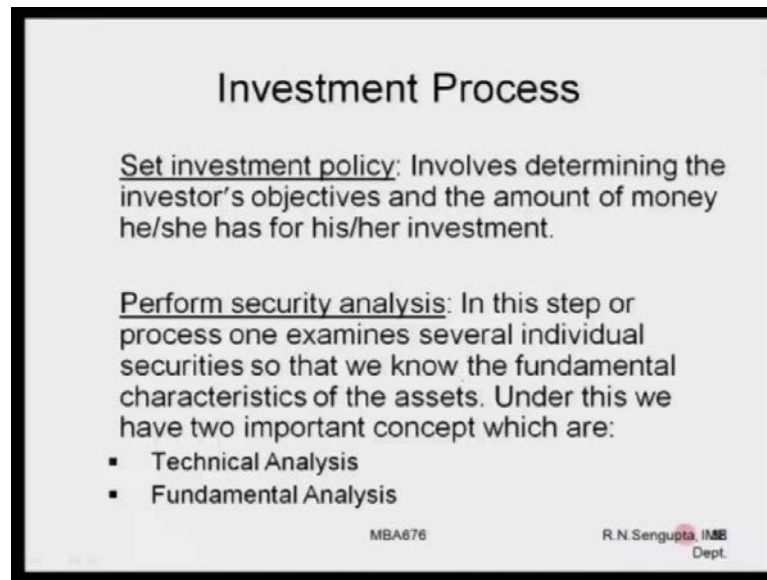
Investment Process

- 1) Set investment policy.
- 2) Perform security analysis.
- 3) Construct a portfolio.
- 4) Revise the portfolio.
- 5) Evaluate the performance of the portfolio.

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So, our main concern is basically to have set the investment policy and the last concern would basically to evaluate the performance of the portfolio. So, our main stress would be to formulate the portfolios that we are able to basically analyze the portfolio depending on the, what the human beings outcome is.

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Investment Process

Set investment policy: Involves determining the investor's objectives and the amount of money he/she has for his/her investment.

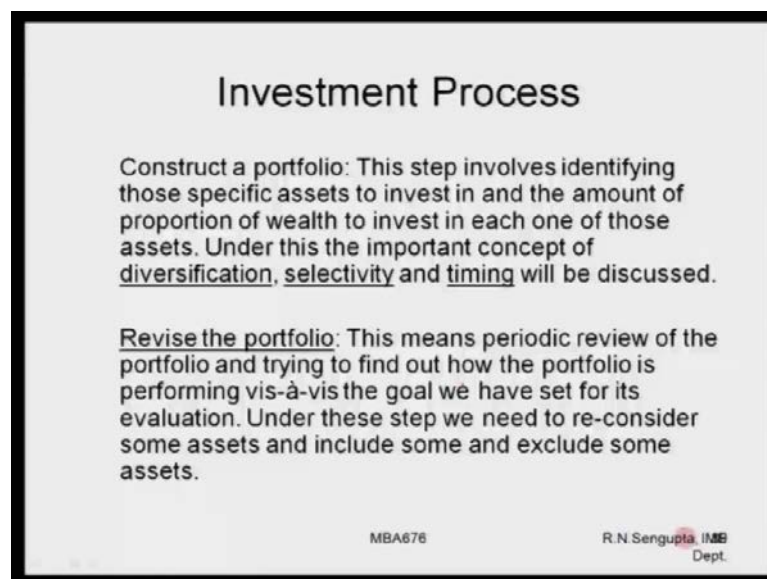
Perform security analysis: In this step or process one examines several individual securities so that we know the fundamental characteristics of the assets. Under this we have two important concept which are:

- Technical Analysis
- Fundamental Analysis

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So, as we discussed in the set in investment policy it will involve determining the investor's objectives. In perform security analysis, the main step would be basically to find out, how does security analysis can be done; such that we can basically study those particular financial asset; such that they can be included in the portfolio. We will very briefly go to the technical and fundamental analysis later on, where different type of chartings would be done in order to find out, what are the different type of performance index based on which our particular security analysis strategy is done.

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Investment Process

Construct a portfolio: This step involves identifying those specific assets to invest in and the amount of proportion of wealth to invest in each one of those assets. Under this the important concept of diversification, selectivity and timing will be discussed.

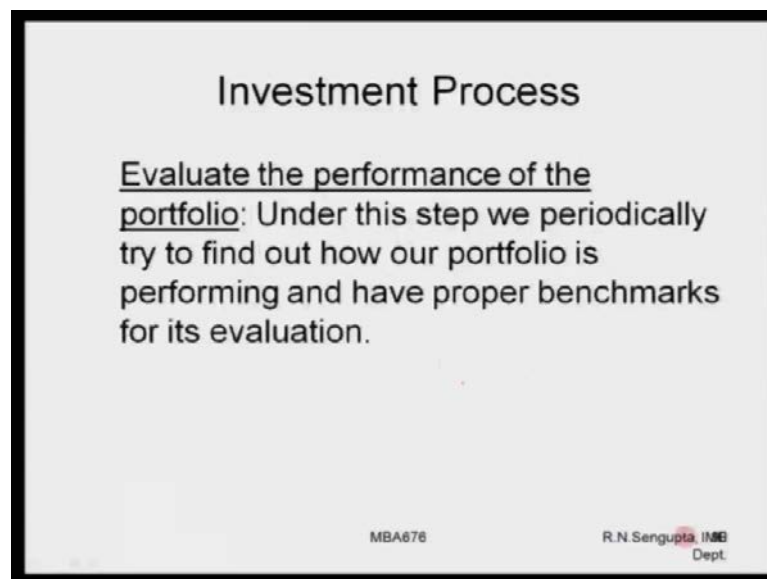
Revise the portfolio: This means periodic review of the portfolio and trying to find out how the portfolio is performing vis-à-vis the goal we have set for its evaluation. Under these step we need to re-consider some assets and include some and exclude some assets.

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Next will basically consider the how you construct the portfolio that what are the different type of timings where you want to do, how you want to basically formulate the portfolio, what are the weight you want to do. And based on the total outcome of the overall decision process, you will try to basically analyze your decision based on the fact that, based on what you basically went forward and basically made their investment process and once you basically or able to compare your actual output or form your portfolio analysis and then, basically are able to compare, what is the difference between exist, difference with exit between the portfolio which you formulated and the portfolio based on which you have actual started, you will basically leave as the portfolio.

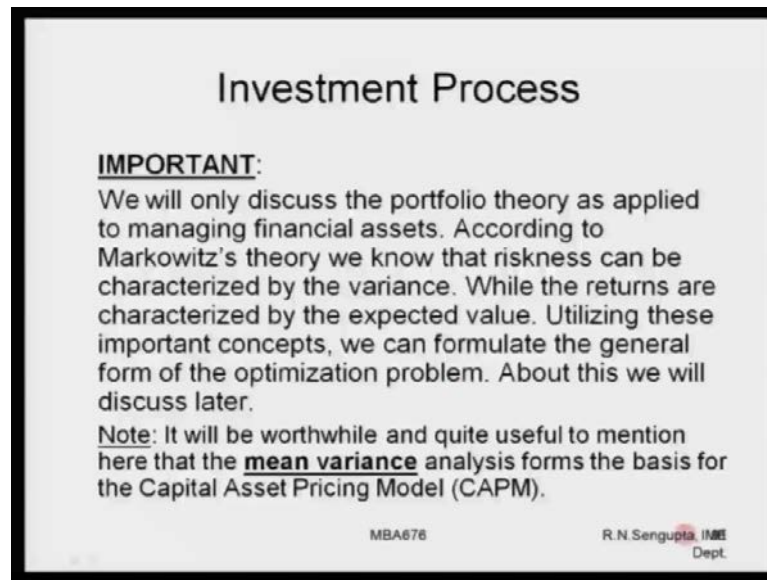
So; that means, either some stock would be brought in to the portfolio or some stock would be basically been taken out of the portfolio; such that the overall impact of the change would mix that it may be mix the actual values or actual sense based on which you have basically started your portfolio analysis at the beginning.

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And then, you will basically consider that, what are the different types of measures of how you analyze the portfolio. So, the measures can be either expected value, can be the variance or can be some other type of risk, measure which you consider if the returns are not normal. If you remember I am mention that, that technically or theoretically the returns are considered to be normal, but they are not normal at what are the different type of risk measures we will try to analyze in order to analyze the portfolio, which you have.

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Investment Process

IMPORTANT:
We will only discuss the portfolio theory as applied to managing financial assets. According to Markowitz's theory we know that riskness can be characterized by the variance. While the returns are characterized by the expected value. Utilizing these important concepts, we can formulate the general form of the optimization problem. About this we will discuss later.

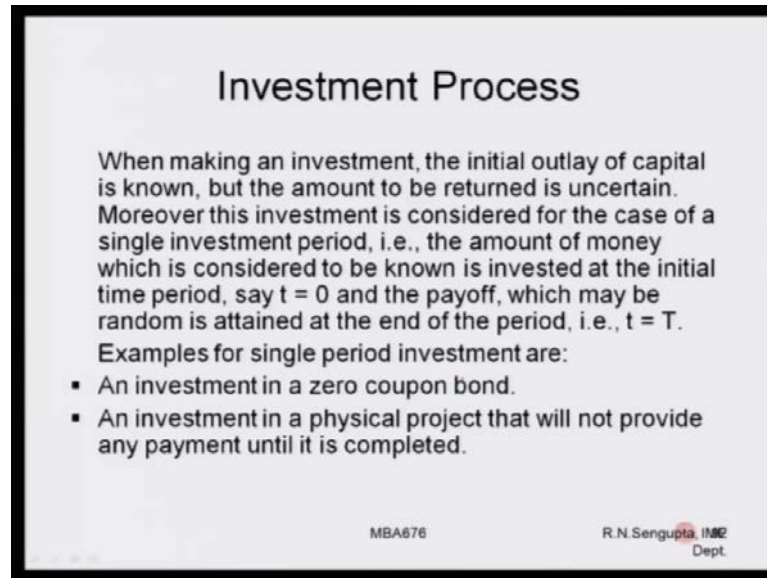
Note: It will be worthwhile and quite useful to mention here that the **mean variance** analysis forms the basis for the Capital Asset Pricing Model (CAPM).

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Now, what we want to analyze is that given those n number of on of as which you have or finesse, which you have the stocks, which you have and if you considered the diagram, which you are adjust consider there was a Cartesian coordinate along the y axis we have basically returns along the x axis whether the risk the variance of the standard division, what I whatever it is and we had with different points.

So, considered we are four different points we want to combine them in such a way that in give as the best benefit. So, what we want to find out is that given n number of assets how is it possible to find on the best combinations as that in means are demand and criteria based on which, a particular person is trying to invest

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Investment Process

When making an investment, the initial outlay of capital is known, but the amount to be returned is uncertain. Moreover this investment is considered for the case of a single investment period, i.e., the amount of money which is considered to be known is invested at the initial time period, say $t = 0$ and the payoff, which may be random is attained at the end of the period, i.e., $t = T$.

Examples for single period investment are:

- An investment in a zero coupon bond.
- An investment in a physical project that will not provide any payment until it is completed.

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So, now, will consider for the time being that there may be initially they may be only a number of assets, which are risky. So, risky means that, what basically means very simple that there is some return and there is some variance are there is some standard deviation. Later on will relax that assumption, where will consider both the risk as well as the return as an asset.

And then, basically will combine different types of coupon bonds; such that the portfolio becomes much more complicated in the sense you actually considered much more practical situations into the portfolio and able to analyze the problem from the point of view you have the theoretical sense as that you want to understand actual theoretical formulations you are able to basically analyze that portfolio which has all the realities into its overall structure.

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Investment Process

We already know an asset is an instrument or a financial investment that can be bought and sold frequently. Examples can shares, bonds, land, gold, cash, etc. Suppose that a person purchases an asset at time zero, i.e., $t = 0$ and $t = T$ period later he/she sells the asset. Then by definition the total return (R) for the investment is defined by:

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So, what will be considered is that the person as what a particular asset you sold that particular asset such that the prices are given the price the returns can be found out consider the returns are normal as we have been mentioning you find out the expected value you find out the variance you find out the covariance and based on the fact that you have formed of the covariance you want to basically proceed to find out that how you get formulate that the portfolio.

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Investment Process

$$R = AR/AI$$

where:
R = total return
AR = amount received
AI = amount investment
Similarly rate of return (r) is defined as

$$r = (AR - AI)/AI$$

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So, as just a recap we can find out is a if there using their capital R, which is the total return or you can basically find out the small r, which is the total weight of return.

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Investment Process

If we denote AI = X_0 and AR = X_1 then:

$R = 1 + r$

and

$X_1 = (1 + r) X_0$

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So, given x as, which are the investment amount the relationship between amount investment can be found or that, that x and small r are given by the last formula will come to that later on all so. So, considered that if you have 100 rupees in your pocket and if you invest in a bank or I interested simple interested of r to the overall amount of money after one year would be given by this; that means, if the interest is 10 percent.

So, you get interest of 10 rupees out of the 100 rupees you are investing as that the end of the day you will get the actual initial amount, which was 100 was the interest of 10. So, we will basically considered r as an a very simple case on return based on which, we are able to analyze r, r investment.

So, considered that now, you want to formulate of portfolio with n number of assets, where all expected value the variances of the correlations are given and you have the formula that how you can find out the expected value or the portfolio also, which is if you recap, which is basically summation of the weight multiplied by the corresponding sample average or the expected value are the sample, which is w_i into \bar{r}_i some them of for all the n number of assets.

And when you have the variances you can find out the variances of that portfolio by multiplying the weight square, which is w_1 square into σ_1 square, where w_1 square is the weight square of the first one σ_1 square is basically the variance of the first one and if follow the principle diagonal, which you already discuss the last element in that particular calculation would be w_n square into σ_n square and of the diagonal element will be considered twice, which basic is the multiplication of the weight of one to each other the means w_1 and w_2 r w_3 into w_4 and so on and so forth multiplied by the covariance's.

And you can use the formula, where the covariance is given by the formula, where which is basically the multiplication of three term, which is the coordination coefficient, which is the standard deviation at the first and the standard deviation of the second.

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Investment Process


Note, both the total return and the rate of return of a portfolio are equal to the weighted sum of the corresponding individual asset's total return and rate of return respectively. The weight of an asset being its relative weight.

Thus:

$$R_p = \sum_{i=1}^n w_i R_i \quad \bar{r}_p = \sum_{i=1}^n w_i \bar{r}_i \quad \checkmark$$

and

$$\bar{R}_p = \sum_{i=1}^n w_i \bar{R}_i \quad \bar{r}_p = \sum_{i=1}^n w_i \bar{r}_i$$



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Now, as all if we have if you remember given the returns of the portfolio will given by this formula, where just I put the tick mark and the expected value wants to find out would basically result in \bar{r} , which is the expected value of the sample of the portfolio; such that you have utilize the sample corresponding to the fact that they there are such expected value of the n number of portfolio, which you have, which is basically \bar{r}_i and given that you can basically find out the expected value of the portfolio given by this \bar{r}_p .

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Investment Process

$\alpha_1^2 \sigma_1^2 + \alpha_2^2 \sigma_2^2 + 2\alpha_1\alpha_2\rho\sigma_1\sigma_2$
 $r_p(2) = \alpha_1 \bar{r}_1 + \alpha_2 \bar{r}_2$

Now for the portfolio with 'n' number of assets, each has a certain random return and let it be denoted by $R_i, \forall i = 1, 2, \dots, n$. Also suppose that out of the total money we have (be it our own or borrowed), we invest that total amount of money in the proportion of w_i in the asset i , i.e., w_1 in asset 1, w_2 in asset 2 and so on till w_n proportion is invested in asset n

α_1 α_2

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Now, consider that for the portfolio if we have two different portfolios, so; obviously the formula would give you if the overall weight in the first one is alpha 1 and the weight for the other one is alpha 2, which; such that alpha 1 plus alpha 2 is basically 1. So; obviously, the return would 1 given by alpha 1 into \bar{r}_1 plus alpha 2 into \bar{r}_2 and the new find out the overall portfolio, which is r_p and I am putting a number two it means that there are definitely two number of such as result.

Similarly, you can find out the variances, which would be σ_1^2 into \bar{r}_1 into the, so it will be would be like this σ_1^2 is basically the variance multiply t multiplied by alpha 1 square, which is the weight. Similarly, this second term in the 2 by 2 matrix, now it is n is basically two will basically have alpha 2 square into σ_2^2 square and of the diagonal element would basically be alpha 1 into alpha 2 into the covariance of first and second such that it could be row, which is the coordination coefficient multiplied σ_1 into σ_2 , so there two elements obviously twice.

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Investment Process

So we have $\sum_{i=1}^n w_i = 1$ ✓ $0 \leq w_i \leq 1$
 $\sum w_i = 1$ $\sum w_i, \text{normal}$

This means that we have to invest all the money we have with us. Also remember that $1 \geq w_i \geq 0$, $\forall i = 1, 2, \dots, n$. This is only possible iff we do not take the recourse of SS. If there is SS, w_i 's can be negative also. Remember that w_i 's are also called the weights in the respective assets.

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Remember, also one thing the weight always equal to 1 and also remember if this no short selling this would be true; that means all the weight are between 0 and 1; obviously, the short selling this may not be true. So, such that sum is equal to 1 if there is no short selling where w_i or the weight in the general sense non normalize case would also resolve the and in case if their short selling, so rather than finding of the weight is equal to 1, what we need is basically into find out the normalize ways. So, I am putting the concept as normal weight how you find out the concept on normalize ways with respect to the w_i or come to that later on as we solve the problem.

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Investment Process

For the portfolio the total return (which is also random) is given by:

$$R_p = \sum_{i=1}^n w_i R_i$$

But we are interested in only the mean or expected return, which his given by:

$$E[R_p] = \bar{R}_p = \sum_{i=1}^n w_i \bar{R}_i \quad \checkmark$$

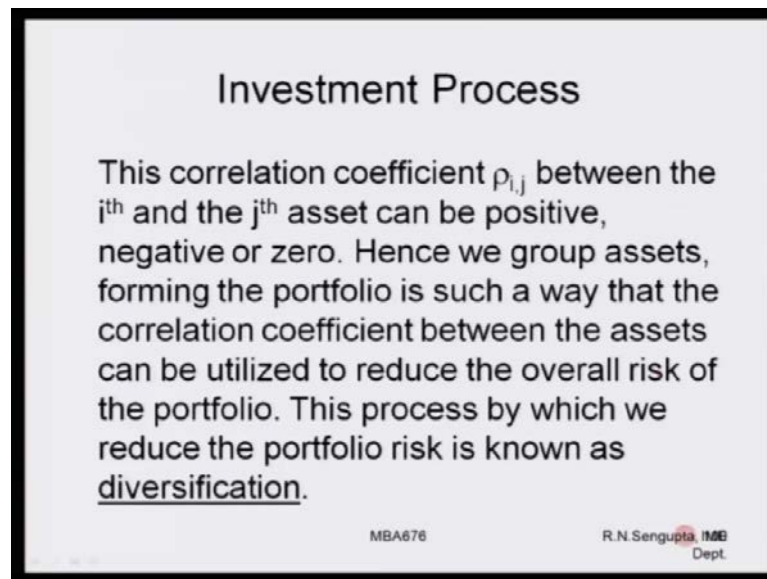
While the variance of the portfolio is given by:

$$V[R_p] = \sigma_p^2 = \sum \{R_p - \bar{R}_p\}^2 = \sum_{i,j=1}^n w_i w_j \text{cov}(R_i, R_j) = \sum_{i,j=1}^n w_i w_j \rho_{i,j} \sigma_i \sigma_j$$

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So, main concern is again I am repeating there are expected values are given by this formula, where capital R or small r would be in material whatever use basically you just find as long as you understand the concept. And then, the variance could be find out using this formula, where this last few terms if they combined to whether basically the covariance's existing between the ith and jth term.

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The slide is titled "Investment Process" and contains the following text: "This correlation coefficient $\rho_{i,j}$ between the i^{th} and the j^{th} asset can be positive, negative or zero. Hence we group assets, forming the portfolio is such a way that the correlation coefficient between the assets can be utilized to reduce the overall risk of the portfolio. This process by which we reduce the portfolio risk is known as diversification." At the bottom left, it says "MBA676" and at the bottom right, it says "R.N.Sengupta, IMB Dept." with a small logo.

Now, our main concern is basically to formulate the portfolio in such a way that you are able to find out the wage; such that the overall risk in the long term for the portfolio is minimize. So, basically; that means, diverse verification I am able to put all my amount of money in different type of stocks as the overall risk or the overall variance or overall standard deviation of that particular portfolio is minimized to the maximum possible extend depending on what mind scenario is that, what is the total amount of money, which I have a if the total number of different type of stock of which, I have I want to do.

So, basically given the fact that I want to diversify plus the fact that if you remember I mentioned about two properties of in utility, one was basically concept of ((Refer Time: 22.17)), where they means more I give you more you want and other one mean that I human means can be classified as risk lover risk hater and risk in different processor. So, if you able to combine these two characterizes will be able to finalize and a very good look that how that diversity happen and how diversification along with these two utility

concepts and basically give us the best picture, that how the portfolio and this is can be done.

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Investment Process

Let us consider an example. Suppose there are two assets with $\bar{R}_1=0.12$, $\bar{R}_2=0.15$, $\sigma_1=0.20$, $\sigma_2=0.18$ and $\rho_{1,2}=0.01$. A portfolio is formed with weights $w_1=0.25$ and $w_2=0.75$.

Then
$$\bar{R}_p = w_1\bar{R}_1 + w_2\bar{R}_2$$

$$\bar{R}_p = 0.25 \cdot 0.12 + 0.75 \cdot 0.15$$

$$\sigma_p^2 = w_1^2 \sigma_1^2 + 2w_1w_2\rho_{12}\sigma_1\sigma_2 + w_2^2\sigma_2^2$$

$$\sigma_p^2 = 0.25^2 \cdot 0.20^2 + 2 \cdot 0.25 \cdot 0.75 \cdot 0.01 \cdot 0.20 \cdot 0.18 + 0.75^2 \cdot 0.18^2$$

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So, consider in a very simple case you have to different stocks and I am considering a capital R again I am repeating it does not matter where you consider as a small r or capital R. So, then the if the capital R values are given at 12 percent and 15 percent and standard deviation for the first is given as twenty or 20 percent whatever it is. So, you can similarly the second one is given as point one and the correlation coefficient is given by 0.01. So, if I assume a portfolio, which I read formed with 75 percent from the second and the 25 percent from the first.

So, you can find out basically there will be the expected value and the returns from that. So, as that you can basically find out what the expecting value or returns about r portfolio r based on the fact that the weight of they my is 25 percent of the, now remember this may not give us the portfolio for this it gives you the best characteristics or main concern would be if you want to basically increase the returns and find out the best returns what you will do is that you basically trying to find out the expected value being maximum value.

So, if you look at this formula what you have is this basically the risk and if you look at this formula is basically whether it come. So, actually if I have rather, then 25 percent if I basically alpha 1 here and 1 minus alpha 1 here, which means r bar p or small r bar p,

which is the return the portfolio is now, basically a function of only one variable, which is basically alpha. Because, if you consider if the rate of returns of already given apiary from the past data. Obviously, you will find out there are expected value variances and try to basically find out the expected value from the portfolio in the variance on the portfolio.

So, in this formula what you will have is basically alpha 1 multiplied by its return, which is r_1 , where r_1 is known plus $1 - \alpha_1$ is basically the investment which you are doing for the second stock multiplied by its r_2 . So, if you want to find out the portfolio for, which you get the best term simply what you would do is basically differentiate the value or the return of the portfolio with respect to alpha and put it to 0. And then, again you double differentiate this function with respect to alpha it was alpha 2 again and check whether is the greater than 0 or less than 0 and based on the rule.

Therefore, the maximum relation and minimization will basically take that if it is less than 0 you get the maximum point if you greater than 0 get the minimum point. And then, you can find out that value of alpha, which will give you the maximum return on the other hand if might main concept is basically due to minimize the risk. So, what I will do in this equation what we have or again they would alpha.

So, in this you are basically alpha square in this case you basically alpha into $1 - \alpha$ and in the last case you are basically one minus alphas whole square if you see the whole equation again it basically a function of a new one variable, which is on known to you. So, again you have differentiate the this would respect to alpha and again by signified out the second differential of that that function which were the function is not now, the return is basically the risk.

So, if you different double differentiation is that and then, find out that the value is greater than 0, which is for the minimization 1 then, you find out some value of alpha. So, that that will give you the minimum, so called risk of the portfolio now, if you consider any problems where you want to maximize the return on minimize the risk the could not be or the same alpha value, so they would be different. So, if I am a risk lover or basically I am basically want to find out from let me could it simply before I get go with the concept of a basically we consider that I am I want person there were i; obviously, hate risk; that means, I want to minimize.

So; obviously, I will use the second principles as that finding out the differential of that variance with respect to alpha putting it to 0. And then, finding out the double differentiation and checking is greater than to greater than 0 would basically give me the value, where I minimize by overall risk for the portfolio if I want to act increase my return to the maximum possible extend, what are I will basically differentiate this function with respect to alpha and find out; that means, where it is maximum value and basically formulate my portfolio accordingly.

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Investment Process

For ease of understanding the concept of diversification let us consider a hypothetical example, where we have 'n' number of assets, denoted by $i = 1, 2, \dots, n$, such that

- 1) The prices of assets are moving in such a way that their respective prices are almost uncorrelated or the correlation is very low.
- 2) The return of each asset has an average value (mean) of 'm'.
- 3) The variance (risk) of each asset is σ^2 .
- 4) Weights of each asset considered in the portfolio is assumed to be of equal proportion, i.e., $w_i = 1/n$ for each i .

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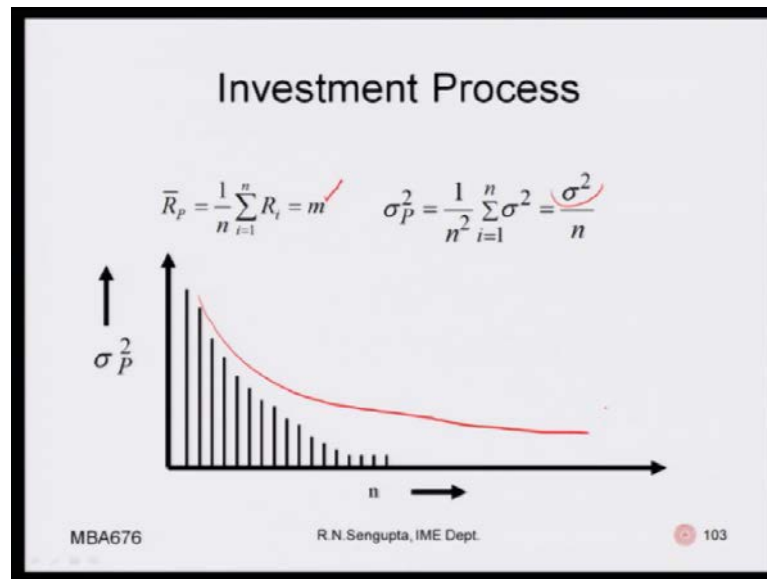
So, for these are understandings to concept of diversification, so always there will be different values in between. So, consider here n number of stocks the price of the assets is moving the, such a way that the respective prices of almost always core uncorrelated. So, the uncorrelated if we consider the correlation of 0. So, if you consider the variance covariance matrix along the principle diagram you have the variances along the covariance is the value would be 0.

So; that means, rather than having and we have a variances plus the n c to the values of the co variances, which you have; that means, they are count twice if you remember what we have is basically the after diagonal n means are 0. So, you have basically n number of times the return of each portfolio are considered to this same, which is m the variance of each other aha ha stocks is again considered to be the same, which is sigma square and

also you considered that you are investing equal proposition of your total amount of money this enhances.

So, with this sub multiplication, what you will have all the ways are equal to one by one if you have that and if you basically try to utilize that in the formula.

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To find out the expecting kind of the portfolio the variance of the portfolio actual formula, which results is return on the portfolio is given by m or the average return, which you have for each stock and the overall variance to the portfolio is give by sigma square by n where n in the number of squares. Now, if for were if I am able to draw the variance of the portfolio with respect to the n value you will see very simply the numerate is constant as you increase the value n it.

Basically, be an curves as that is asynchronous you will become 0, which means technically and theoretically they may be cases, where you formulate a portfolio in such a way with the condition as I have just discussed that you can make the overall variance of the portfolio in the long run 0 depending on the fact that you have taken infinite number of as basically assets in your portfolio. So, this is the essence of diversification where you combine the, this is such a way depending on some weight and in such a way that the overall risk for the portfolio can be made as go as possible. But. that is technically not possible why it is not possible we come to that picture within another two minutes.

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Investment Process

Thus if we consider diversification, portfolios with only a few assets may be subject to a high degree of risk, represented by a relatively large variance. So the variance of the return of a portfolio can be reduced by including additional assets in the portfolio.

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So, thus we considered diversification with the portfolio with the only assets subject to idea of risk that risk basically you see there it can be divided into two parts one with the risk which can definitely be diversified another set of risk which cannot be diversified will use some technical terms whatever, but still you still, let us follow with the simple example and try to basically analyze the our problem, how we are able to basically eliminate or draw the portfolio curves for depending on different type of weight that we are able to draw the efficient fund draw try to find out the portfolio based on which, you can make a distance accordingly.

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Investment Process

Consider another example but now with correlated assets. The return and variance of the assets are as in the above example with mean m and variance σ^2 , but now each pair of assets has a covariance of (for $i \neq j$) $0.3 \cdot \sigma_i \cdot \sigma_j = 0.3 \cdot \sigma^2$.

Hence $\sigma_p^2 = \frac{0.7 \cdot \sigma^2}{n} + 0.3 \cdot \sigma^2$

so as $n \rightarrow \infty$, we have the second term only. Thus $n \rightarrow \infty$, or diversification implies reduction in portfolio risk.

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A consider another example now, with correlated asset, so initial case you uncorrelated asset now consider the correlated assets and with this sets of information you can find out now they would be a principled angle which is the variance of each stock plus they would be of the diagonal elements. So; obviously, if we consider and expand the equation now you have to terms the first term is basically dependent on n says that as any increase system can be made zero technically as you can seen in the last slide.

But, this second term which we have is basically always demand which is basically the level of which, cannot diversifies. So, what we are seeing for the first time is that they will two sets of rest for any portfolio handle with the set of which can be diversified in the longer then, can be technically made zero and another set of this would be which cannot be make technically zero. So, in the theoretical sense actually I am again repeating this can be made zero if you are able in a theoretical sense to formulate a portfolio certain assets.

But, in general see that you can minimize the list to maximum possible extend where the overall risk or overall variables can be basically made it to two parts one is the diversifiable risk and other one is a non diversifiable risk and will see that later on as we discussion.

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Investment Process

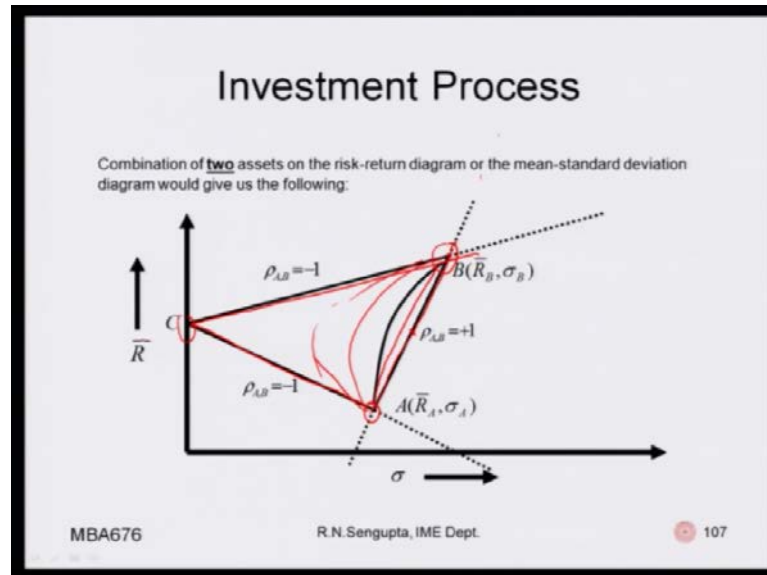
For assets A and B the respective returns and variances are given. Let the corresponding weights be $w_1 = (1 - \alpha)$ and $w_2 = \alpha$, i.e., $w_1 + w_2 = 1$. As α varies from 0 to 1, the portfolio goes from one that contains only asset A to one that contains mixture of asset A and B and then to one that contains asset B. Remember values of α outside the range $0 \leq \alpha \leq 1$ make one or other of the weight as negative, corresponding to SS. The curve looks like that shown in the diagram above but what exact shape we get will depend on the value of correlation coefficient ($\rho_{A,B}$). The solid portion of the curve corresponds to positive combination of two assets, while the hashed portion corresponds to the shorting of assets. Remember one important thing. The curve must always lie inside the triangular region, denoted by ABC.

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Now, consider how we are able to draw, so again going back to the same example consider two stocks with the ways are alpha 1 minus alpha 1 the variances given by

sigma one square and sigma two square returns of given by r_1 bar r_2 bar if you are able if we have that particular portfolio which consists of two stocks were actual values of the portfolio.

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We now depend on one important fact, which is consider these the points, which is the first asset these basically the second asset. So, if you combine them and if you have kept the value of alpha as staying whatever it is and the rate of the returns in this and the and the standard deviations are different. But, now should bring in the correlation coefficient in the picture they can meet to extremes one extreme is that when the correlation coefficient is minus 1 vary of this graph and other case is basically in the correlation of positive one which is this graph.

So, any values it for example, zero will be this minus one nine point nine point five you can be this plus point nine five can be this and so on so forth. So, what we are doing is that we are trying to basically formulate a portfolio as that the value of the value of the portfolio depending on alpha depending on r depending on sigma would be such and depending; obviously, on the correlation always be some point inside the triangle now this is a very important fair once we are assumed and we and we convince in a conceptual sense that the point would always be a triangle inside the triangle we can basically proceed to the next.

But, before we go to the do the next slide on the next step remember the dotted lines which are given here or for the case where there is sort selling. So, you would be asking that how is it possible if we consider the straight line running A and B. So, in middle point would be what very investing 50 percent in a 50 percent in b if consider point b on it means that your investing all of your money in b and not in a and vice versa. But, if you if you extend this the graph along this line or go around basically means that you are hot selling one with respect to the others.

So, if you go along this one direction along to infinity; that means, you are basically trying to be the by one of the stock, which is do not have and utilize that money into, but other particular stock depending on how you want to be basically performed you to portfolio. Similarly, for a correlation coefficient in a minus 1 this dotted lines along this direction along this directions other case and short selling is the...

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Investment Process

The curve in the risk-return diagram defined by non-negative mixtures of two assets A and B lies within the triangular region defined by the two original assets and the point on the vertical axis of height

$$\frac{\bar{R}_A \sigma_A + \bar{R}_B \sigma_B}{\sigma_A + \sigma_B} \quad \sigma_p^2 = \sum \sum w_i w_j \sigma_{ij}$$

Now if we denote the portfolio formed by A and B as P, then we have the return and variance for the portfolio about which we will now discuss.

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So, now, the curve which you have if you are able to solve it how would be solve with the question should be, but because for the correlation coefficient being existing between minus 1 and plus one you will have the equation, which is like this is sigma square which is for the portfolio is given by double summation of w i w j, which are the weight if there are two number of assets. And you have the correlation coefficient existing between covariance existing these two stocks and if you have two stocks, what you do is that if you see the last slide what you have basically the y axis.

So; obviously, y axis would mean that that the risk is 0, so obviously, we report the risk as 0 you can of find out the value of the of that particular stocks as that the overall risk can be made 0 in spite of the fact they would be some return. So, before I go and proceed for that this would give you some in how risks feel interested would be utilize later on.

Thank you.