Quantitative Finance

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Module – 03

Lecture – 02

contd…Part – 1

Welcome back to this course of quantitative finance. So, as we are discussing, we will wrap up the utility section and then start off with the investment and what we mean by rate of return and how rate of return is useful in portfolio analysis and where the concept of utility actually comes into the picture. And, later on, we will proceed depending on what are the different type of portfolios you can formulate and what are the different techniques of trying to formulate and find out the prices option. So, as we are discussing, the utility is basically the value or net value based on which a person invests. So, I am using the word investment, because it is basically related to finance, but utility is basically a value based on which a person takes a decision.

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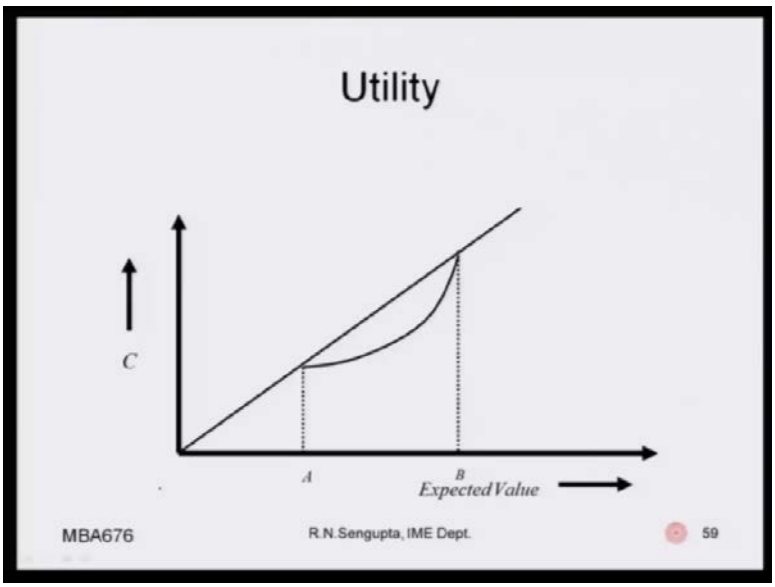

So, now, consider the utilities.

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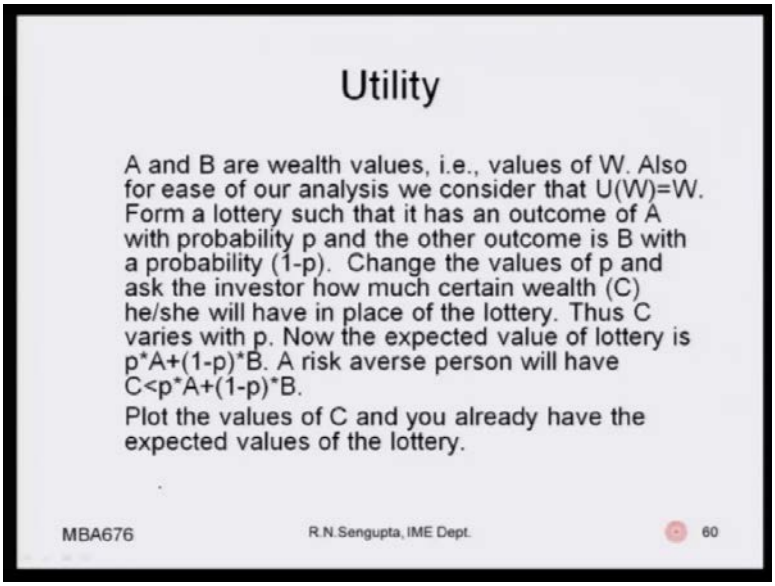

If they are not known, what we do is this simply as here. Now, consider there are A and B are two wealth's. And, consider for both A and B, you have some probabilities p and 1 minus p, because some of the probability should be 1. Now, with that scenario, you place in front of a person and you ask him or a her that whether that person will take that gamble or the non-deterministic scenario or the person will basically choose a deterministic case or a value of certain C; which if you remember, the value of C is basically the value based on which a certainty equivalent based on which a person is absolutely sure that he or she will take that deterministic amount. Now, if a person is indifferent; so, obviously, the person would be able to say a particular value of C and for a particular wealth A and B and certain probabilities p and 1 minus p or p and q.

Now, based on the first case, you keep changing the values of p and 1 minus p. So, definitely, there would be a dipping point at which the person would either change from the case from being an indifferent person to being a risk averse or being from being an indifferent person to the case where he is risk lover or any of one of these depending on where he or she is. Now, those… If you go back to this curve; so, if the probabilities are known, obviously, you know the case where the C value is. Now, if you consider the straight line, it gives you an utility function which is linear. Now, if the value of linearity does not hold; if it is basically a quadratic one or some other thing; and, depending on the characteristics of the person, the person will slowly give you different points on the Cartesian coordinate, which you seen in front of you. Based on this, where you can

basically mark the points to find an analytical set of points or practical set of points for that person such that you are able to find out what is the exact value of C or what is the exact nature of the person such that you can class him or her as a risk averse person, risk lover person or basically risk hater or an indifferent person. So, these are basically very simple analytical techniques. We would not be considering that in detail, but this is the methodology how you do that.

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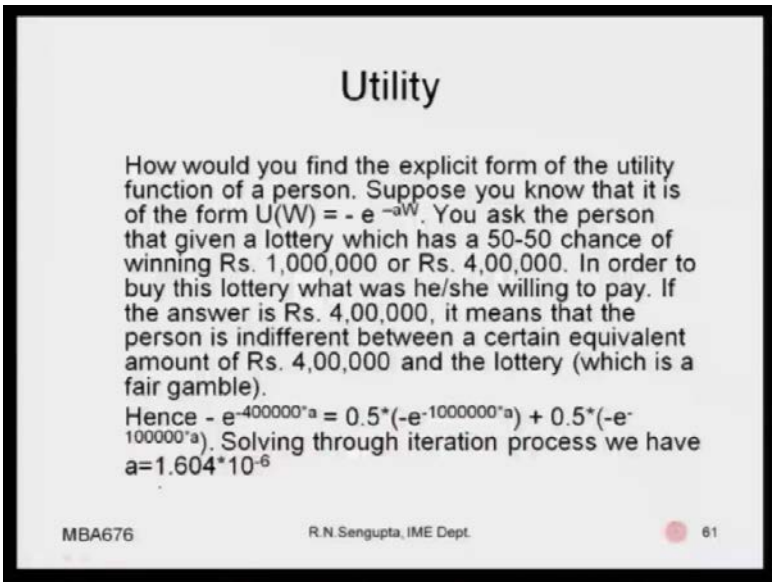

Now, how would you find out the explicit form of the utility function as status. So, consider that, you have an exponential utility function and you ask the person that, he has given a lottery with 50-50 percent chance and some values as they are stated; that is, one is 1 million which is 10 lakhs and another value is basically 4 lakhs. Now, in order to buy that lottery, the person would be either… If he or she is a risk lover person; so, obviously, the person would always be thinking that, the expected value of that lottery would be greater than and any concept of certainty equivalent. If the person is indifferent, that person would definitely be indecisive between these two scenarios. And, if the person is a risk hater, obviously, he or she would definitely refer a value which is deterministic. Now, either given the value of the utility function, which is the power function, you now keep changing the value of a – small a, which is the parameter which you do not know. And, as in the other example, which you had given, where you draw in a very simple geometrically fashion, draw state line which is linear utility and then find out that portion of the graph whether it is up of that or below that and classify that person. You can

basically do the same analysis to find out what is the estimated values of a and based on which you can basically classify that person that, given that exponential utility function, what is the parameter of a.

Now, we would like to draw an attention to all the students or who are basically taking this course that, how you find out the utility function is more of a study, which is done in economics. We will only utilize the concept of utility in order to basically go into the depth of what is portfolio analysis and how optimization is done. So, rather than the techniques to find out, our main emphasis will be how will we use the utility function. So, before we wrap up, if you remember – for the utility case, if you remember we have mentioned that there were two important points. One was the point of non-satiation and one was the point where a person can be either risk averse, risk lover or risk indifferent person. So… And, there are other few – three important points, which are… Not all, there are three important points of utility function, which are known are axioms.

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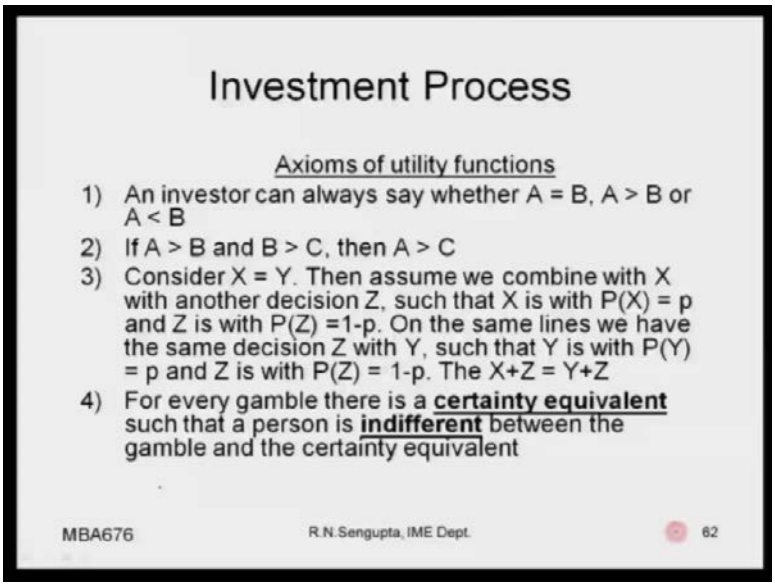

That if you are basically… There are two different decisions. So, obviously, a human being or a person – whenever there is any decision to be taken specifically in the finance area, the person would definitely have a decision whether he or she is basically indifferent between two decisions A and B; whether A is better; that means, the overall utility for A is better than B or it is basically vice versa. There is no confusion that how the grading would be done; that would definitely be clear with the human beings who is

going to take the decision. Next is that, if there are 3 or 4 – more than 3 different decisions; so, obliviously, the ranking system would definitely be applicable such that a person if he or she takes A with respect to B; A is preferred with respect to B and B is preferred with respect to the C; then, obviously, the relationship of A and C should be such that the person would definitely take the decision A and not C. So, there is a linearity or the hierarchy of the ranking based on which a person takes a decision. And finally that, if you consider the concept of certainty equivalent, that will give you a concept that, the person would be indifferent between two decisions if the sudden certainty value is same; which means that, if you have different type of gambles or different type of lotteries as shown later on in this graph.

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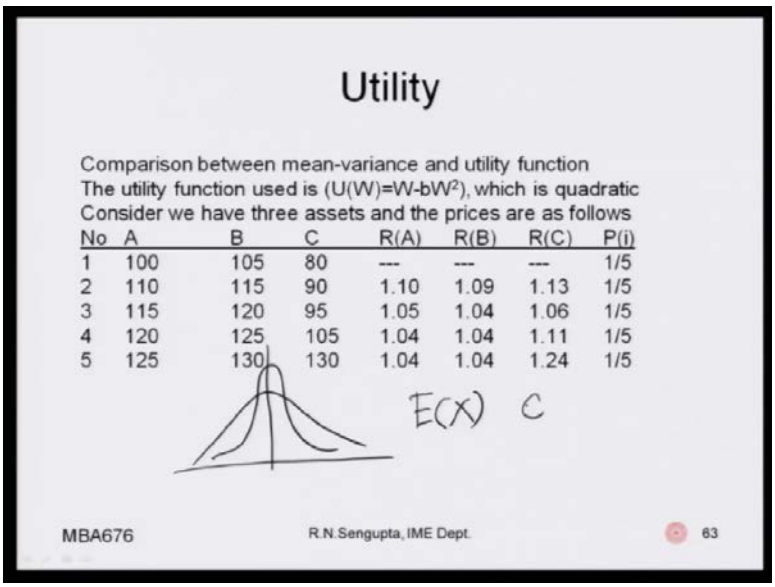

So, what I am basically… What I want to analyze is that, if you have basically different types of utility functions; so, these are just theoretical values which I am drawing. So, obviously, it would mean that, the person – if he or she is basically indifferent between two utilities based on their expected value; then, both the utilities would basically give you the same benefit for the person. Now, comparison between the mean variance of the utility; till now, I have never uttered the word of variance. Now, you may be thinking why variance is important. So, consider this very simple case. Let us not consider the utility function as such; let us consider a simple distribution. So, these are normal distribution, which you know that. And, there is another normal distribution, where both

the mean values are same. So, if you consider the mean values, you will basically have the expected values, which is same for both the cases.

Now, if I am trying to compare two different utilities; our main concern is not now expected value, because expected value are same; that means, the utility would give you the same certainty values. But, what is of more concern is that, what is that dispersion or the diversions between these two distributions based on the fact that, if for one case, if the diversion is very high; then, the variance which is the second moment of the distribution will be higher value. And, for the other case, the second moment, which is the variance would be a lower value. So, hence, we will first counter… This is the first time we are encountering that, given a utility function, given a decision, if both the expected values are same; there may be cases where you would be taking a decision based on the variance, which is the second moment. And, this is very important that, once we come to the case of portfolio analysis, we will see that how both expected value and variance would be utilized from the perspective of decision maker such that you are able to rank different portfolios in a very logical matter.

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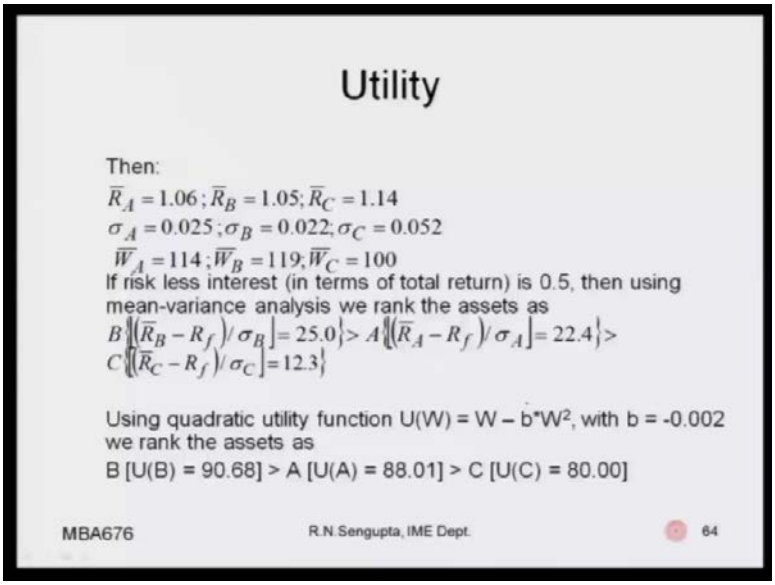

So, consider this. So, consider… Now, your question would be… Before I give an example, your question would be – what if somebody loves risk and somebody hates risk or somebody is basically willing to take a decision based on the expected value and somebody is willing to take the decision based on the variances, which is the second

moment? So, our main concern would be – rather than only concentrate on the expected value or rather than only concentrate on the value of the variance, we will try to basically take a ratio of the expected value to the variance and rank them from the values where the ratio is from the highest to the lowest if you want to basically take a decision which is always positive.

On the other hand, you can also do the same scenario unless in different way like rank the variance with respect to the expected value; that means, take the ratio of variance to the expected value and rank them from the minimum to the maximum such that if the ratio of variance to expected value is the least, that is the best one decision, which will be basically favored by the decision maker. So, we will come to that later on. And, if you see this – these equations which are given; so, given that, you want to find out the ratios of the expected value to the variance and so on and so forth. We will see that how given the distributions in normal, we can easily find out using the simple normal distribution tables of the z values. So, we will come to that in details later on.

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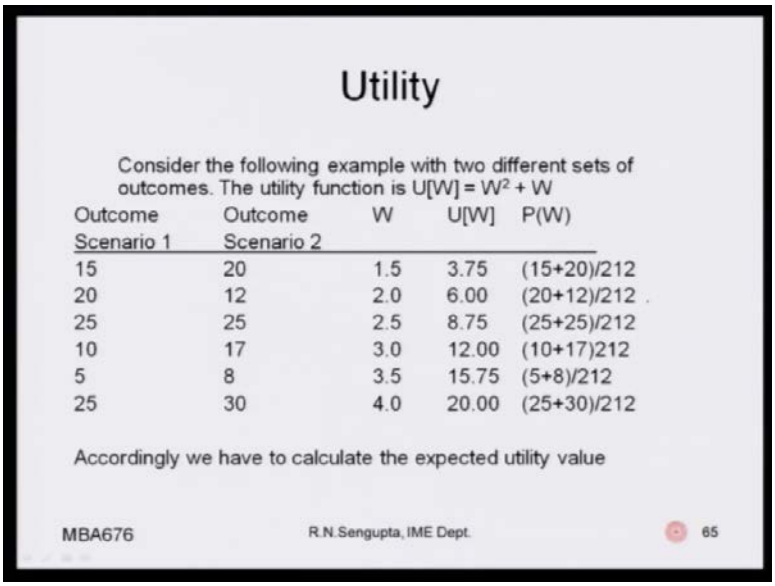

Now, consider finally, to wrap up ((Refer Slide Time: 10:08)) Consider you have a quadratic utility function. If you remember, I mention that very explicitly that, given two utility functions or two different decisions, any returns for the particular stock – for a particular financial tips, the returns are normal; then, definitely you know the utility function is quadratic and vice versa. So, consider there you have a quadratic utility

function and then you have basically the outcomes in scenario 1 and outcomes in scenario 2. So, what you do is that, you find out the probabilities and accordingly you can either rank them according to the expected value or rank them according to the minimum variance or rank them according to the case where expected value of two variances is the highest or variance to expected value is basically the lowest. So, as we were saying that, if you remember that, you will basically rank different decision based on expected value, which was basically certainty value.

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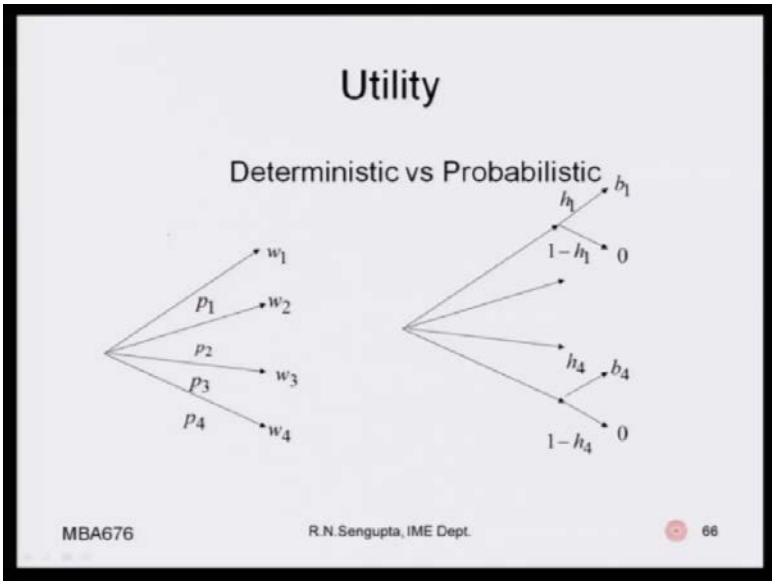

So, if we have on the left-hand side, there are different type of lotteries and the outcomes are w 1 to w 4; for these 4 scenarios, which you have and the probabilities are p 1 to p 4 such that the sum is always 1 or the probabilities; you can break up into basically lotteries such that at the end of the day, what actually matters is basically the expected value or the certainty equivalent.

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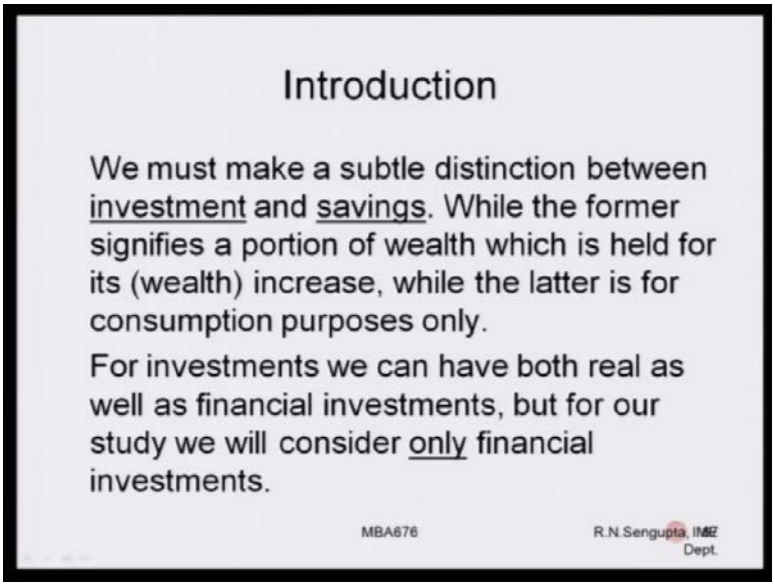

Now, with this decision, we will first that, we will come encounter in the case of portfolio analysis, which is an area of huge importance, where somebody tries to utilize the concepts or different type of mathematics and statistics in order to analyze the topics of financial investment. And, for that, we will try to basically go through different techniques as that the concept of optimizations and concepts of statistics are definitely used in a very simple manner in order to find out what are the different type of portfolios and how you find out the expected value and the variance of portfolios in order to basically find out the so-called optimal portfolio, which is best for one decision maker.

So, we will for the first time, we will basically consider that, there are some sets of financial assets. And, by the word financial assets, we will mean only some scripts; obviously, there can be gold; there can be different type of bonds; there can be different type of options. So, all these words which I am using for the first time in this lecture serious apart from the fact that we did mention about them when we were discussing in the syllabus; we will try to restrict our discussion in the initial part to the very simple examples such that, trying to analyze the problem later on becomes much more easier for us if we have understood the simple concept in a very nice manner.

So, consider that we have only financial assets. So, given financial assets for example, stock; so, generally we know that, if we have the stocks, at the beginning of the day, you have basically the opening price and at the ending of the day, you have the closed price.

Obviously, you have the maximum value, minimum value, amount of volume which are sold and what for these scripts; they are definitely important for us. But, to make our life simple, we will only consider the closing price of a particular stock or the adjusted closing price whatever it is. Now, it should be remembered that, as per the case, only prices are not important. What is important for us is what is the change or rate of change of the prices per day such that depending on the upward movement and the downward movement of the prices, we can make a decision whether that investment or that particular amount of money, which we are going to invest in a particular stock – whether is that positive benefit or negative benefit to us depending on what our outlook is.

Now, remember another thing – when I am using the word outlook; it means basically the person is of what type? Does he want to basically take a decision based on which he or she wants to minimize risk or is it that person – he or she basically wants to maximize the return or some different combinations depending on whatever the outlook of that person is? Now, I would request…

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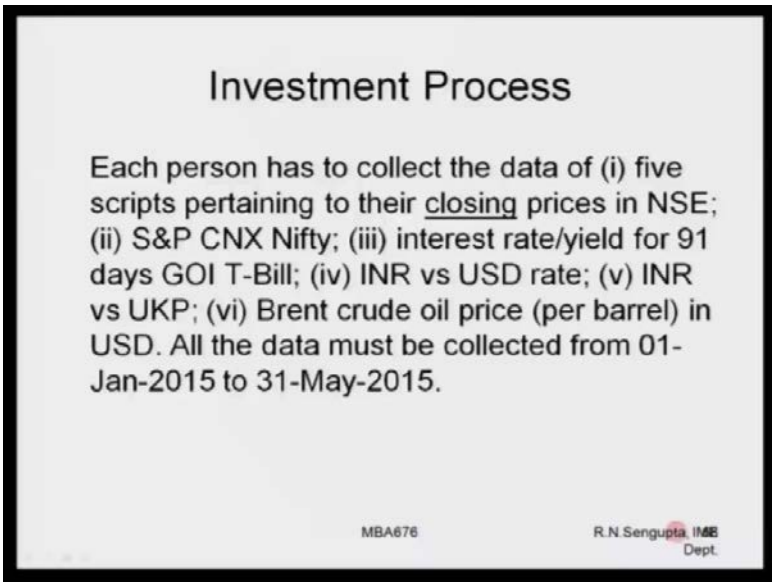

Later on, definitely we will encounter all these data, but I will request all the different persons who are taking this course to have a look at the closing price of different stocks like S&P and Nifty, which is the… You can find it from the NSE India; you can find out the interest rate or the yield rate, which is 91 days government bills; we will consider that how government bills are basically… The values are found out, which is the risk-free

interest rate. The word risk-free interest rate – we will come to that later on also. And, that can be found from the reserve bank site also. Then, we will also try to basically find out how the Indian rupee to the US dollar, Indian rupee to the UK pound; basically, they fluctuate. Or, what is the trainer? How do they look like? So, given that, the fact that you are able to basically plot those data points or plot those curves on an excel sheet; that will give you a good feel that, how does basically the price fluctuate. Based on the price fluctuation, you can find out the rate of returns; and, based on the rate of returns, you can find out what are the portfolio combinations are.

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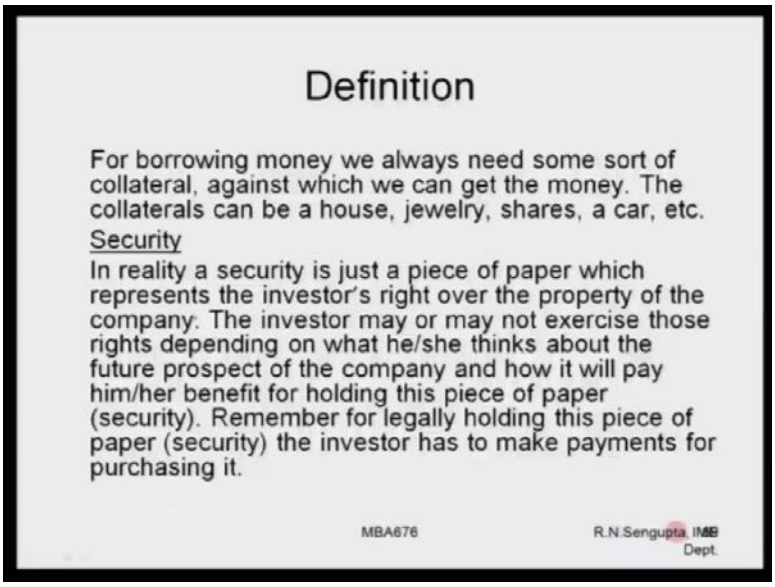

Now, for borrowing money, always we need some sort of collateral. So, we will consider that for… When you are basically investing any particular money for the initial case, we would not consider any collateral. Collateral means the amount of money, which you are giving as a security. Later on we will see that, when options are there; when forwards and futures or different type of combinations are there, there is a concept of margin money and margin account which is there, which is kept as the security for any positive and negative plus fluctuation of those particular options. We would not consider that. Number 2 – we would not consider here for the time being is what are the implication on the taxes. Number 3 – we will not initially consider what are the actual amount of money which has been paid to the broker like when I go to the stock market and basically either I buy or sell particular stocks; so, obviously, I have to give some money back to the broker in order to get his or her services. We are not going to consider that. We are not

also not going to consider any price fluctuation depending on any discount being given. So, all these things will be brought into the picture and our problems will be made much more realistic once we are able to appreciate the actual theoretical concept in all its implications.

So, we will consider very simply a security is a piece of paper, which is basically a some part of that stock, which is being bought and sold. Or, that company has floated a particular stock and does it being bought and sold. Also, we would not be considering any stock ((Refer Slide Time: 16:23)). So, stock ((Refer Slide time: 16:24)) is basically are stock was there depending on how the dividend and how the company is doing that, the stock would be split into 1 is to 2 or 1 is to 3. So, we would not be consider that, as I mentioned that, in order to make our lives simple for the first set of problems. So, consider that, the rate of returns.

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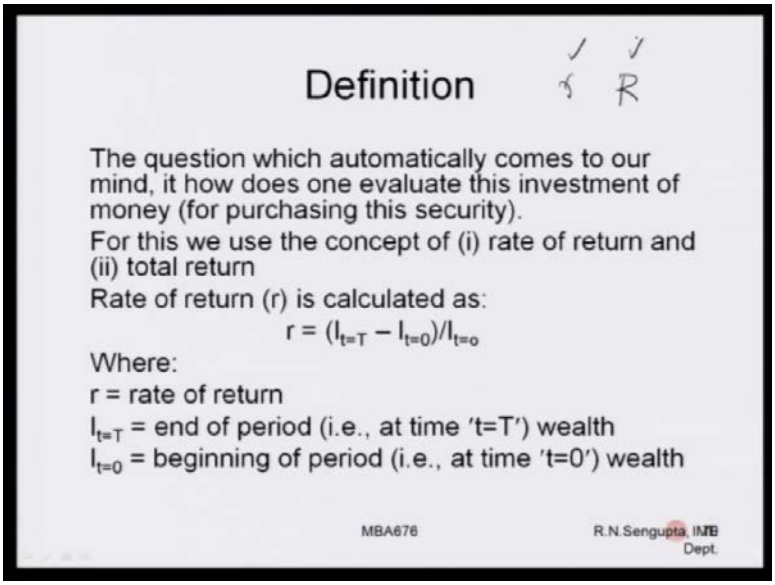

So, we have basically two types of rate of returns. One could be basically known as the rate of return, which is small r; and, remember the small r will be utilized for all the sets of problems. And then, would be another rate of return or R – capital R. We will consider both of them in our problem. But, the actual solution based on which we will try to solve the problem; whether we take small r or capital would be the same. So, as we were discussing that, what is important for is basically to find out the rate of return – whether small r or capital R. So, there is actually when we solve the problem, there would not be

much of a difference in the way you solve the problem considering whether it is small r or capital R. But, we will consider that all our problem, which you are going to solve later on would be based on the fact that you want to find out the rate of return, which is small – small r.

Now, if you see to the formula, small r basically means that, given the prices or given the investment, let us consider for the initial time being that, the investment which you are doing at time t is equal to 0 – consider it is today and you get a return based on whether the prices are increasing or decreasing, you get some output, which is given as I suffix 1 or suffix 2 depending on whether it is day 1 and day 2. So, considering it is day 1; so, obviously, you have I suffix 1 is for day 1; ending investment which you get back, which you want to get back, if you basically close your position. And, as you get the actual amount of investment, which you do at the beginning of today, which is zero ((Refer Slide Time: 18:05)) So, what is the actual positive and negative return in amount terms would be I 1 minus I 0, which is in the numerator. And, if you want to find of the rate of return, it will basically be a person in the sense that, given the amount of investment, which you are doing at I 0, what is your actual percentage of return, which is given by I 1 minus I 0 in a numerator divided by I 0? So, that will basically give you the value of r.

So, if you consider the price of the stock; if the price of the stock is 100 and tomorrow the prices of the stock is 110; so, obviously, in numerator, you will basically have 110 minus 100, which is plus 10 divided by 100. So, obviously, r would be positive. Now, if I mention that, the prices have decreased from 100 to 90, it will be basically minus 10 in the numerator divided by 100. Now, if you consider that these are the r's; remember one very important thing that, if you mention the rate of return having normal distribution; which means that though values of r – if you brought them and if you consider them to be normal distribution; then… And, if r is a random variable; then, plotting the distribution of r would give you basically a normal distribution with a certain means, certain variance and so on and so forth. And, which would mean. Again, if you go back, we have mentioned that, the utility function was quadratic such that, it means that investment decision, which a person is making is based on the fact that, the person has a quadratic utility function. We will come back to that later on as we proceed. But, in actual theoretical sense, this may be true; but, in a practical sense, that is not true,

because there are different type of non-normal distribution which are utilized; and in practical sense, they give you much better results.

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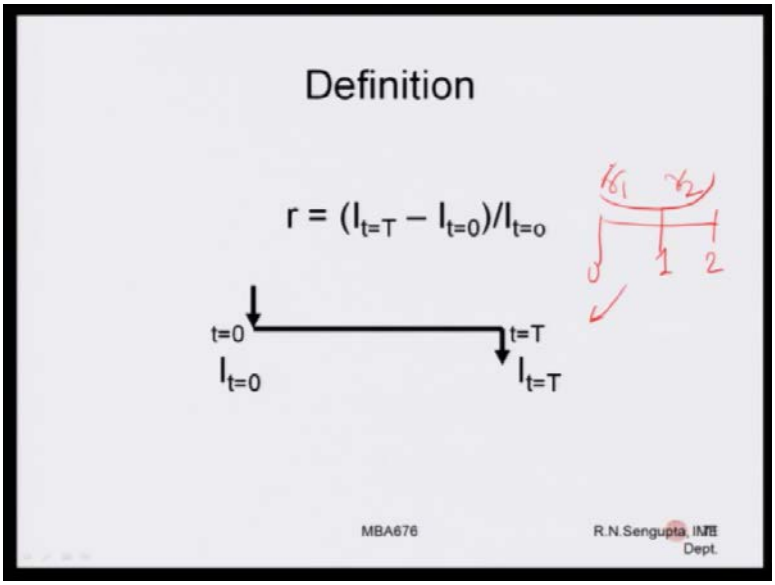

Now, if you see this simple line diagram; so, now, consider the line… Why I am highlighting on the fact there is a line diagram? Because if you consider a static model in the sense that, only one time period is there; so, the time period is basically from I suffix 0 to I suffix 1 or 2 or whatever it is for the time period. Here if we consider the I suffix capital T; that means, if we invest the money at time t 0, in between you are not going to invest anything; you are not going to take out any money. And, at the end of the day or end of the time period, which is capital T, your total investment which is worth for the initial investment of I 0 would be I suffix capital T. So, hence, you can find out the value of r. r would be basically the r 0 or r 1 depending on how you are trying to basically define it. So, if…

Now, there are two different time frames; that means, I naught for day 0; I 1 for day 1 and I 2 for day 2. So, obviously, the returns for the first case would be I 1 minus I 0 divided by I 0. So, what you have is that, there are two different time frames; let me go back. So, basically, there would be two time frames: one – which is 0; another is 1; and, another is 2. So, here we have only the first instant. So, if you have basically I 0, I 1 and I 2; so, if this value would be given by r 1, which is I 1 minus I 0 in a numerator divided by I 0. For the second one, which is r 2, it will be I 2 minus I 1 divided by I 1. So, what I

was talking about the distribution being – this distribution would basically have a normal distribution. So, we will come to that later on.

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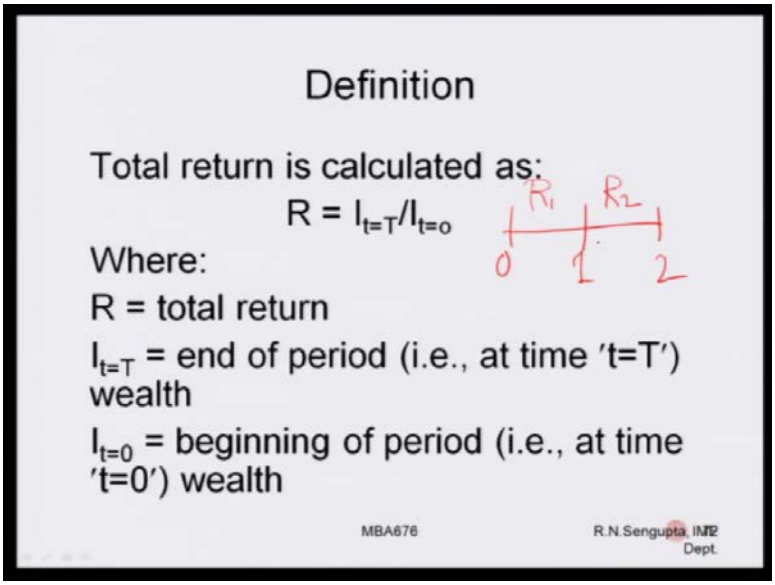

Now, consider that, if we have only capital R, which is the total return; so, obviously, it will be given not by the minus value of the difference, it will be given the value of returns or the total value, which you have in your pocket for any investment if you close your position; that means, you either sell it or buy it – divided by the actually initial investment or amount of money, which you had which is basically I 0. So, for the same diagram, if you have two different things; so, this is time 0, time 1 and time 2. So, the capital R would be I 1 divided by I 0 for the first instant. So, this would be R 1. And, for the second instant, it will be R 2 – I 2 divided by I 1. So, this would basically be the ending period and the by beginning period wealth. Wealth means amount of money which you have.

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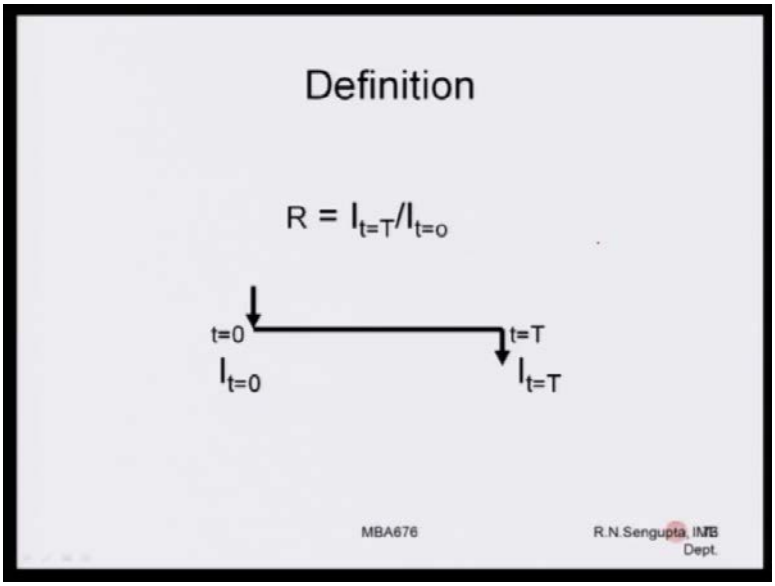

So, this on a time scale of only the single time period, you have then initial input and the initial output of money, which had gone into the decision or making the decision. Now, as… If you remember, we have mentioned that the risk-free interest rates – small r suffix f or capital R suffix f; f means risk-free. So, consider very simply that, this is the government bond for 90 days based on which you can find out what is risk-free interest. In a very theoretical sense, the word – risk-free means that, if you invest in those type of investment, your actual variance is 0, because there is no risk, still there is some expected value for the returns. So, this would be very helpful for us to analyze different type of decisions, based on which a person will either invest in some decisions – some decisions which are probabilistic, which are non-deterministic such that their expected value or the average value is more than r f. And, if in case some expected value are less than r f for some non deterministic cases; obviously, the person would not be willing to invest in those decisions.

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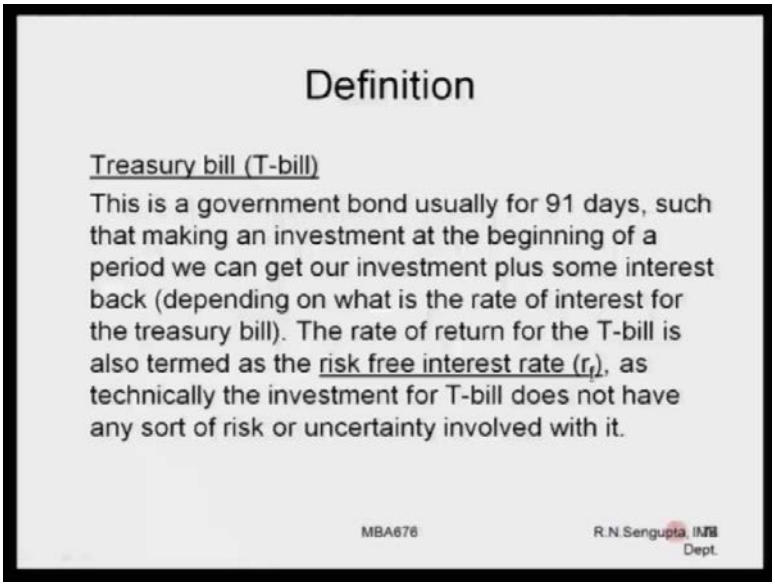

So, we will come back to the r f value later on either in the portfolio case or in that drawing in the efficient front or whatever we encounter as we proceed with the course.

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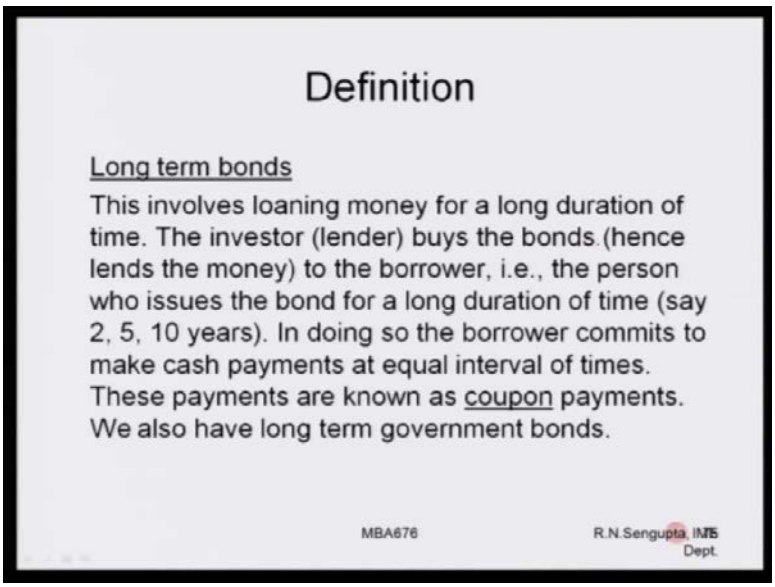

Now, also later on, we will consider long term bonds. For the reason being for the long term bonds, the risk based on which you are trying to basically analyze long term bonds or the interest rate of… Or, you consider the overall variance or you consider the expected value, time would have a great effect on the interest rate or all what we call a basically the volatility or the risk or the variance. So, we will consider later on also that,

if there is a huge time frame; that means, say for example, for one year or two years, then the long term… And, if your long term launch in your portfolio or in your kitty or the box based on which you are trying to basically make your portfolio; so, there the interest rate would basically be a big role in how you want to find out the overall effect or the long term bond, because those interest rate, which will be calculated as fluctuating; if they are fluctuating, you have to basically take the optimum value of the interest rate, which is basically in its simple sense the expected value of the interest rate, not the actual r, is basically the expected value of r which you find out for a whole time period.

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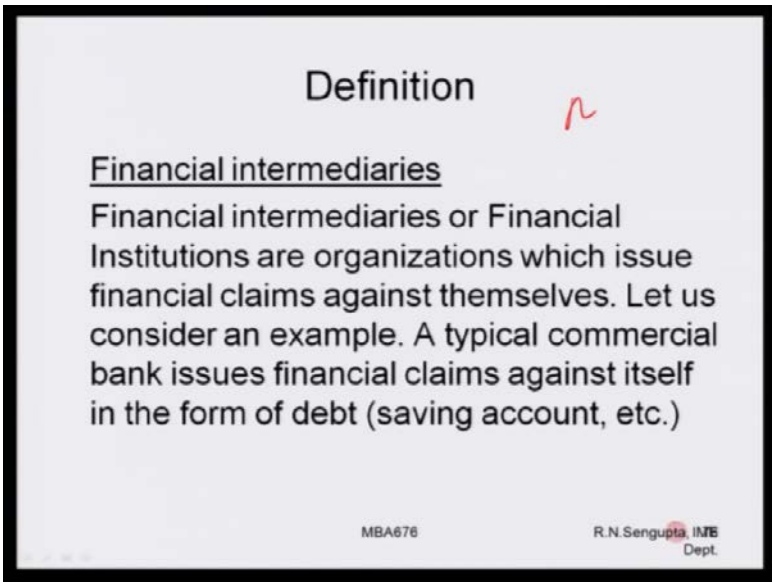

Now, we will also consider later on that, financial intermediaries are basically the houses, the banks, the cleaning houses, the stock market, where people come and sell and buy any particular financial instruments. So, if you remember my first initial lecture, when we discussed, we had basically a demand and supply. So, those demands and supplies are based on the fact that there is some number of buyers and sellers. Consider that is small n and small n is not small; small in the sense there are number of players in the market. Small n I mean this. So, this number of small n is not small in number. It is basically… It is such that any price fluctuations and the demand and the supply would be immediately affected in the market such that all the players who are there in the market have the exact information of the prices such that any fluctuation of the demand, any fluctuation of the price p – all that particulars stock would be available to all of them such that everybody can take a rational decision will be based on the fact that what that persons characteristic

is from the point of view of whether he or she is a risk lover whether he or she is risk hater or that the person is basically a risk indifferent person.

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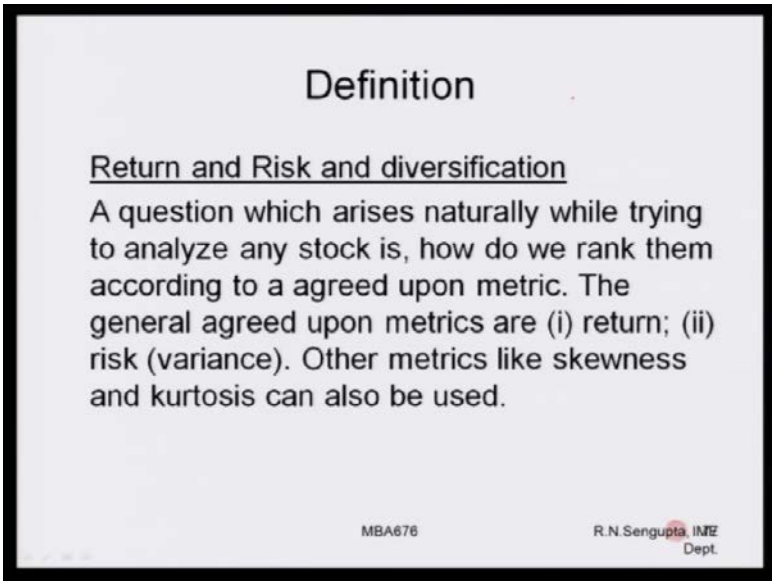

So, now, we will for the first time encounter the concept of risk return and how risk and return is utilized in order to basically analyze any investment or analyze any decision. So, now, in 1952, Markowitz, who was the Nobel price winner in economics – he basically proposed a very interesting concept, where for the first time, the concept of a particular investment from the financial perspective was basically clubbed into two important points: one is that the expected value; one was the variance. So, given any particular financial instrument, you can basically club that in set of information into only two different bullet points: one is if the expected value of a particular stock is higher; obviously, you will try to invest more in that. If the variance of the particular stock is lower; obviously, you will try to invest in that particular stock also. So, our main concern would be – given the expected value and given the variance of particular stock, we will try to basically find out that how you can combine this different type of stocks to form of a portfolio such that you are able to optimize your overall output. What is that output? You will see that changes from person to person such that you can basically form the optimal portfolio for a human being, who is willing to make a decision.

So, we will consider for the first time the concept of return and risk. Risk – initially, I will use as a term of variance. We will use that formally later on. And, later on we will

see also that how the higher movements, which is skewness and kurtosis are also used in a very big way though in a theoretical sense such that you can formulate different type of portfolio base on not only the first moment, which is the expected value; not only on the second moment with the variance, but also on the fact that, there are the information related to the third moment or the fourth moment, which is skewness and kurtosis can be utilized accordingly.

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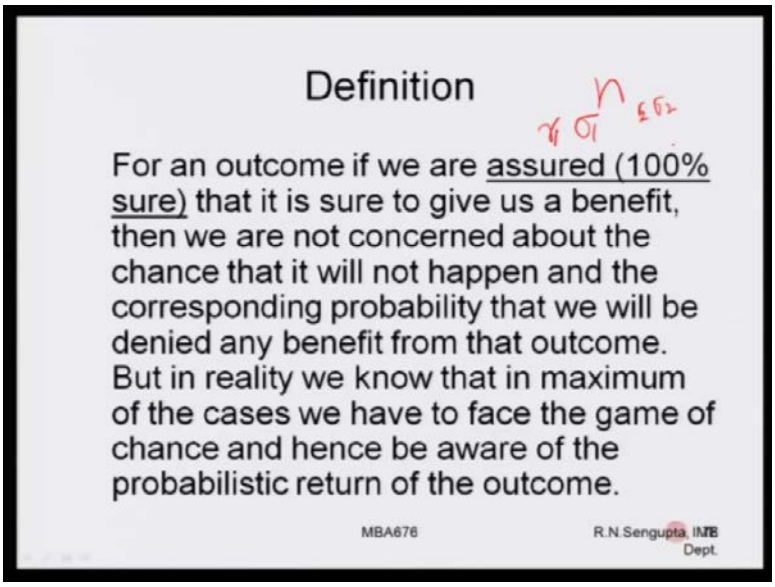

Now, for any outcome, if we are assured that there is 100 percent return; obviously, it is a deterministic case. But, if you consider the prices of the stock or the properties of the stock, they have some distribution. We are not going to consider the distribution per se as such; we will only consider the distribution is there for the random variable r – small r, which we just discussed. And, we will consider that random variable has some characteristics such that given n number of different type of investments, which you have – they have the characteristic such that r 1 will be the actual random variable, which will denote the return of the first stock and sigma. I am using the variable for the first time. Sigma 1 would be the standard deviation of the particular first investment which you are doing. Similarly, you will basically have r 2 sigma 2. Correspondingly, you will basically have r n sigma n depending on how many step of different type of such financial skips are there.

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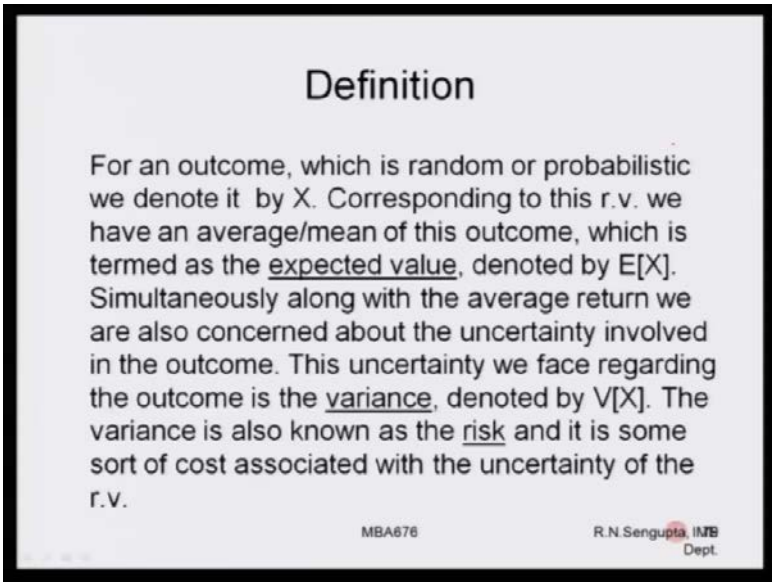

For an outcome which is random or probablistic, you will basically have the first and second moment, which is the expected length of variance, which will give us whole amount of set of information about that particular stock or a particular financial instrument which we have.

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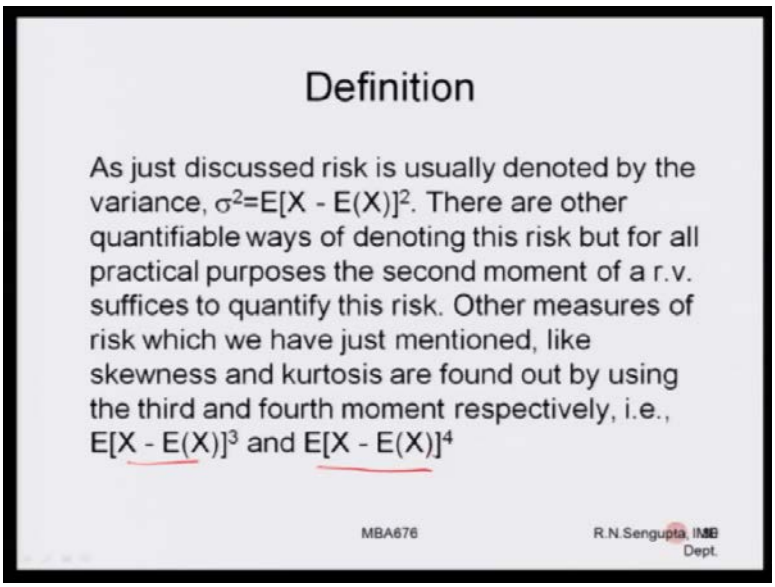

Now, also as we discussed that expected value if you consider, it is basically given an average value. What we mean by an average value? We will come to that within another two minutes. And, what we mean by the dispersion on the variance? Also, we will

discuss that in little bit depth in the sense that, how they can be calculated given different type of datasets, which you have. We will later on also find out that how skewness and kurtosis – given these formulas can be utilized for different type of theoretical portfolios, which one wants to basically formulate given that there are different type of stocks that, the first moment and the second moment does not give all the information of the portfolio such that they will be tempted to utilize the higher moments to find out the characteristics of this portfolio of the particular stock.

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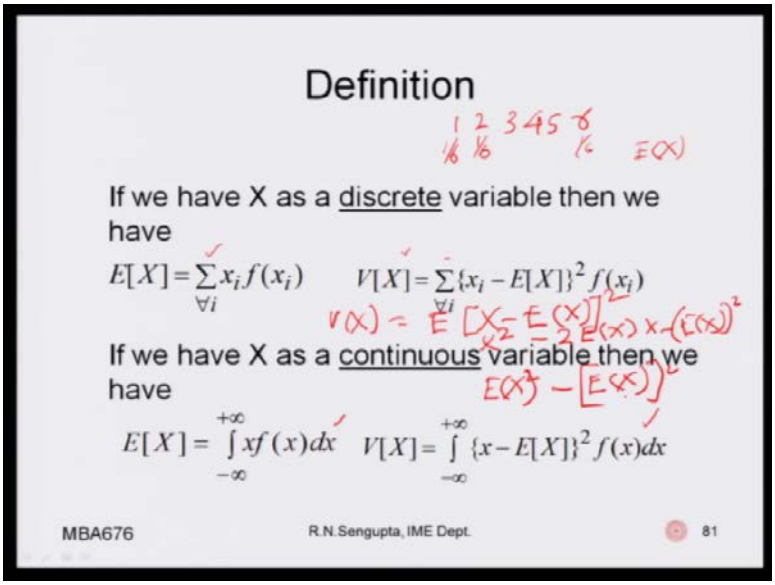

Now, consider very simply that, if you have two discrete variables; one is a discrete variable; one is the continuous variable. Discrete variable in the very simple sense means that it can take only discrete value. Consider you roll a dice; so, the values of the face of the dice when you are playing a game of Ludo is either 1 or 2 or 3 or 4 or 5 or 6. So, obviously, the corresponding value or the probability of getting any face is 1-6 depending on how many outcomes are. And, we will consider that is an unbiased dice. Say for example, when you toss a coin, the corresponding probabilities – it is half and half for an unbiased dice, which is a head and a tail. So, we will consider either a discrete case; and, for the continuous case, where the random variable can take any value – we will consider the probability is to be continuous or the random variable can take continuous values. So, if you consider the random variable x with a suffix i depending on how many such values which you have and f of x as is noted as the probability

distribution function or rather probability mass function based on which you can take a decision, whether it is basically a discrete or continuous.

The random variable – the expected value would be given by the formula as given here; which is summation of the multiplication of all the x's and f and f of x; which means that, if you have basically the dice which you are rolling. So, all the random variables, which have the outcomes are either 1 to 6. So, obviously, you have 1, 2, 3, 4, 5, which are the different random variables. So, when r is 1, you have r 1; when r is 2, you have 2. For the case when I am using r as a random variable; in this case, if x 1 is 1, then the corresponding probability is one-sixth; if x 2 is 2, then the corresponding probability is one-sixth and so on and so forth. So, if you want to find out the expected value, what I will do is that, I multiply the random variable, which realize the value, that is, realized value which you see multiplied by its corresponding probability; add it up. So, obviously, it will be 1 into one-sixth plus 2 into two-sixth, so on and so forth till the last value, which is 6 into one-sixth and find out the expected value.

And, if I want to find out basically the variance, the variance would be in this case would be... So, once you find out the expected value and consider that the expected value comes out be say for example, E of X, whatever the value is depending on what type of distribution, which you have; you want to find out the variance. So, variance would be in this case, when you are basically rolling the dice, it will be the random variable ((Refer Slide Time: 31:29)) value, which is 1 minus is expected value. So, it will be 1 minus E of X whole square multiplied by the corresponding probability is one-sixth; 2 minus E X, which is the expected value whole square into 1 by 6. Once you find out that value, sum it up; you will basically have the variance. So, remember the variance is the square of that value. And; obviously, it is positive.

Now, if you have continuous variable, then this summation sign gets replaced by the case, where you have basically the integration. So, hence, for the case when it is continuous distribution given the random variables can take any value between minus infinity to plus infinity, expected value will be found out using this formula, which is integration and the variance would be found out using this integration. Just a note before we proceed that, given this expected value of the variance, a very simplified formula, which will see later on would be this. So, say for example, you have variance is equal to expected value of X minus E to the bar X. So, this is the exactly thing which is… So,

here E and the summation, which you are using the same thing. So, what you can do is that, if you keep expand this equation, the first term will be X square. The second term would be minus of 2 x into the E X; that means, you are basically multiplying the first term would be X square; second term would be E X into X. And, the third term would be basically E X the whole square. So, now, this is the term, which is inside the bracket.

If you take again the expected value of this; so, obviously, at the end of the day, what you have is basically E to the power X square; it means the expected value of the square of the random variable minus… So, if you basically find out these two values; so; obviously, it will become minus of E X whole square; that means, the first term is finding out the expected value of the square of the random variable and the second term is basically finding out the expected value and then squaring it up. So, we will see that later how it can be utilized in order to find out the variance for different type of random variables depending on the problems which we solve.

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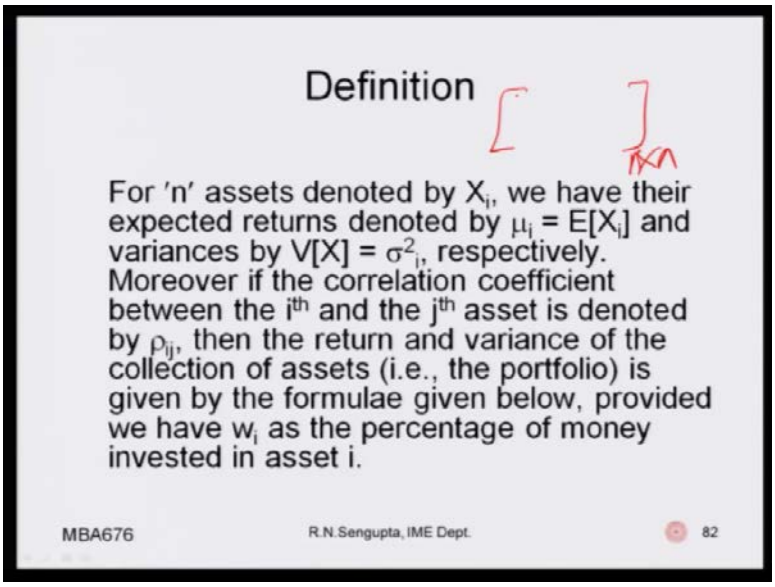

Now consider for any different n number of assets; if there are n number of assets, what you have is basically n different random variables; if there are n different random variables, each will basically have expected value; each will have basically the variance considering that we have utilized those formulas. So, what is also required is basically the correlation coefficient and the covariance is existing between these n number of random variables. So, say for example, if we consider a matrix; so, matrix is basically for

the simple case if you have n number of assets, we consider n cross n matrix. So, there are basically n number of rows and n number of columns. Now, what is important for us to understand is that, if you basically write the variance and the covariance matrix of all these n number of matrices, we will be able to get the covariance of the portfolio based on which we are able to find out actually the variance of the portfolio in the later on. So, we will basically continue that discussion in a later sessions.

Thank you.