Quantitative Finance

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Module – 01

Lecture – 01

So, now, with these words, I will start off that what we mean by a demand and supply; and, what are the relevance…say for example, demands and supply, and what we mean by the concept of a price of a particular product or a service.

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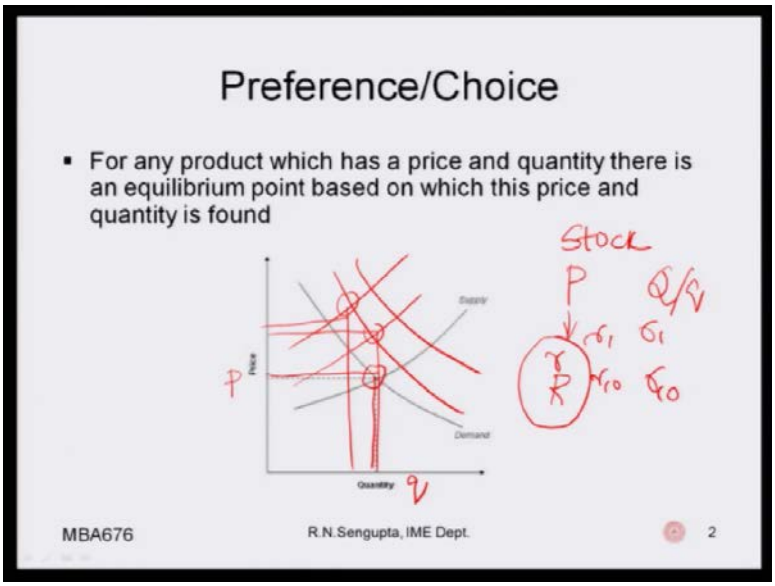

So, I am using a very general word of a product and a service. So, generally, product can be anything, which is being bought and sold in the market; and, it can be a stock, it can be an option. And, services can be different type of financial services which you are giving. But, we will only restrict our discussions later on with only related to financial one. But, generally, the first initial lectures would be very basic micro economics, which will give you that how demand and supply can be considered in order to find out the optimum price and optimum quantity based on which a product and service can be sold and bought in the market.

So, if you see this curve here; you basically have the price along the y-axis and you have basically quantity along the x-axis. So, when we… And, if you go to the market, buy

anything; you want to buy some goods whether it can be any vegetable, it can be any product and it can be financial instrument, whatever is being bought and sold; there would be a particular set of people, who are interested to buy and particular set of people, who are interested to sell. Now, another reason, which I think is very important to understand the concept of demand and supply is that, because say for example, if we are going to buy any product; we are buying – basically going to the market; basically gives us the very good feel or regulates the price in the demand based on which that product and the service can be sold.

Say for example, I go to the bank and want to basically take a loan. On the other hand, there can be people who want to go to the bank and basically deposit some money. So, obviously, the bank acts as an intermediary – basically a space or a meeting place, where different people with different demands and supplies for different products can go into the market and basically exchange such that apart and from perfect information, which may not be true in the actual sense, in the actual practically; perfect information being available to all the buyers and suppliers. The overall place where this supply and demand takes place helps to regulate the price and demand at which the value – the overall product can be bought and sold. So, we will come to those concepts later on also.

So, now, if you see the coordinate at where this price… So, at the price and the quantity which is sold. So, basically, what we know that is there is a quantity based on which that particular product is being bought and sold, and there is a price. So, now, consider that, if you have Bajaj share or say for example, you have Tata shares or say for example, you have some options. So, when you are trying to find how to investment returns, main concern comes is that, what is the price, based on which you can optimize your overall returns; what is the price at which you can basically go for a derivative; what is the price at which you can basically formulate your portfolio – point number 1. And, point number 2 would be that, what is the total quantum of such goods, which you can buy and sell in order to basically optimize your portfolio. So, I am using the word optimize in a very general sense. Say for example, if I am a human being, my main concern is only profit. So, I will basically try to initialize my overall portfolio, where I want to basically maximize or have the positive returns to the maximum sense.

Another person can be where he or she wants to basically minimize the overall loss. And, for him or her, profit is not important. So, for that person, it will be more of a concern in order to basically try to see that, the risk can be minimized to the maximum possible

extent. Again I am using the word – risk in a very general sense. There are different type of quantification of risk. And, what are the quantification? All of us generally know in a very general sense, it can be standard deviation, it can be beta. What is beta in the market sense, we all know. But, there are different techniques, which I come up, where used different terms in the concept of risk and try to analyze any different problems in order to mitigate the overall risk for persons who are more concerned about the losses.

So, if you see the overall – the risk return profile as I was discussing; so, obviously, the supplies can be of different values and the demands can be of different values. So, for different type of supplies and demands, you will basically have different type of prices. So, one will be this; this second one will be this, and third one will be this. So, when you are trying to basically analyze any stock; I am using the stock in a very general sense; it can be any financial instrument. What is more concerned is the price and what is the quantity, which is being bought and sold. This is basically small q. Now, this price basically gives you the concept of returns. So, returns I will use the word as a letter of small r. So, there are different type of books, where they use the – use of capital R also. But, those can be interchanged depending on how you are trying to analyze the problem.

Now, our main concern is to find out the risk and return based on a particular set of returns, which is small r. And, as we formulate the portfolio, how different types of stocks can be combined given their different type of values of r; like say for example, if there are 10 type of 10 different stocks, you would basically have r 1 till r 10, which will give you the corresponding returns of this 10 number of stocks; and, obviously, will have the corresponding risk. So, general terms – let me denote the risk as sigma 1 to sigma 10. And, all of you know basically sigma means the standard deviation and obliviously there are other different techniques or symbols or methodologies trying to quantify the risk.

So, our main concern is to see that, given a products price and quantity, I am using the product as a very general term – is a financial product; that how can you find out say for example, if the price of one particular stock is decreasing and you want to basically sell it; so, you want to find out that what is the other financial product, which can be substitute in your portfolio such that your overall risk mitigation is met at the maximum possible extent. So, how this relationship between risk and return can be done in the concept of financial instruments? I will come to that later on.

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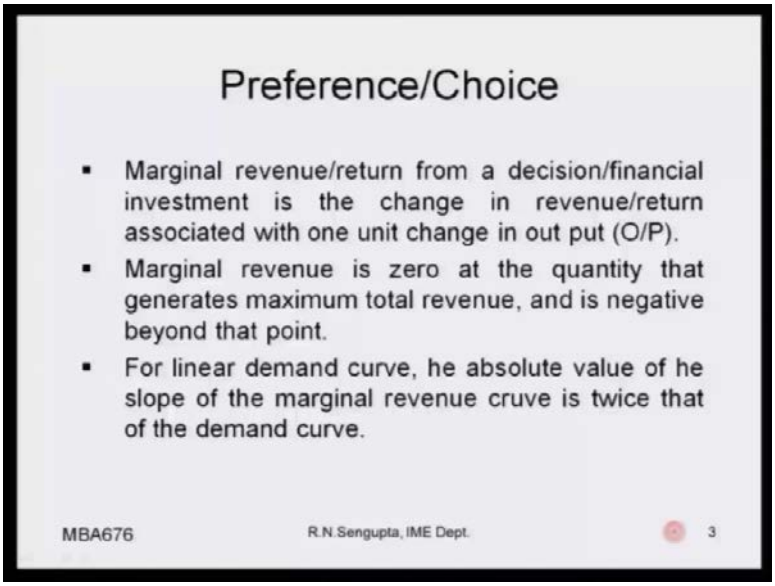

So, generally we know that for any investment or any concept of demand and supply, there is a concept of marginal revenues; so, marginal revenues in a very simple sense means the per unit rate of change of that particular revenue. Now, you may be thinking that, why we are going to consider marginal revenues. So, later on, we will see that, when you solve the concept of optimization problem, there would be a given a primal problem; I am using this word primal for the first time, but we will see that later on that, what that actual problem means. If you have a primal problem, consider that in a very general sense as the primary problem; there would be a counter part of that, which is known as the dual problem. So, dual problem is basically consider that, if you are standing in front of a mirror; so, if you have the primal problem or so-called primary objective, what you want to solve; and, the mirror image, which you see is basically the dual. So, there are techniques depending on what are the conditions based on which the primal problem have been formulated. You can convert that primary problem into a dual one and you will see that, the properties or the intrinsic properties of the primal and the dual problem as such – there is a one-to-one relationship between them.

And, later on also, we will see that, if you are able to solve the primal problem with some techniques, there would definitely be methods that how we can solve the dual problem also. And, there you will see the concept of marginal rates or subscription of products happening. So, how you can basically replace one product or one financial instrument with the other can happen. Then, we will also see that, what is the concept of shadow prices; so, what are the concept of shadow prices which I use in the optimization sense

can have very specific meaning in the area of finance and optimal portfolios. We will also see that later. Then, we will find out that as the prices or the quantities of the products change; product means again financial products. How does sensitive analysis help you to basically balance your portfolio, balance your risk, balance your returns would be; we will also consider that later on.

Now, marginal revenues or return for any decision or financial instruments is basically the change in the revenue or the return associated with the unit change in the output. So, say for example, your total output depending on the price change in the overall market falls by 100 rupees or falls by 200 rupees or increase by 200 rupees. So, based on that, you want to find out that, what are the readjustment you want to do in your portfolio such that you are able to mitigate the overall risk or you can basically try to maximize your return or whatever your concern is depending on how you have formatted the problem. We will see that, the marginal revenue is 0 at the quantity that generates the maximum total revenue when we consider the concept of maximization or minimization.

We will discuss now the concept of utility theory and why utility theory is generally utilized in finance and how it can be utilized in the area of decision making and how quantitative techniques as such is basically utilized in utility theory. So, what we mean by utility theory?

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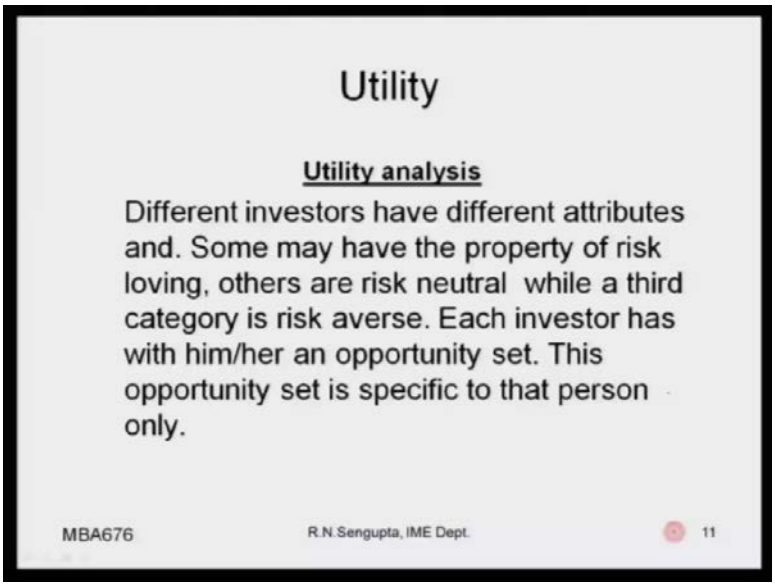

So, different investors have different type of demands. Somebody would like to basically increase his or her returns; somebody is interested to basically try to minimize the risk;

and, others may be interested in basically trying to make a compromise between risk and return. So, risk and return go hand in hand, but in opposite direction. So, we will see that later on also. So, each investor has in front of him or her a choice set; choice means a set of decisions based on which he or she can take that set of decisions. So, it is not single; there are many different type of choices. And, an investor or a person who is working in the area of finance would like to basically take up the decision in such a way that he or she choose the optimum one. And, what is the optimum one? We will discuss that later on also. So, to consider in a very simple sense, there are outcomes; and, outcomes are with one-third probability. So, there are 3.

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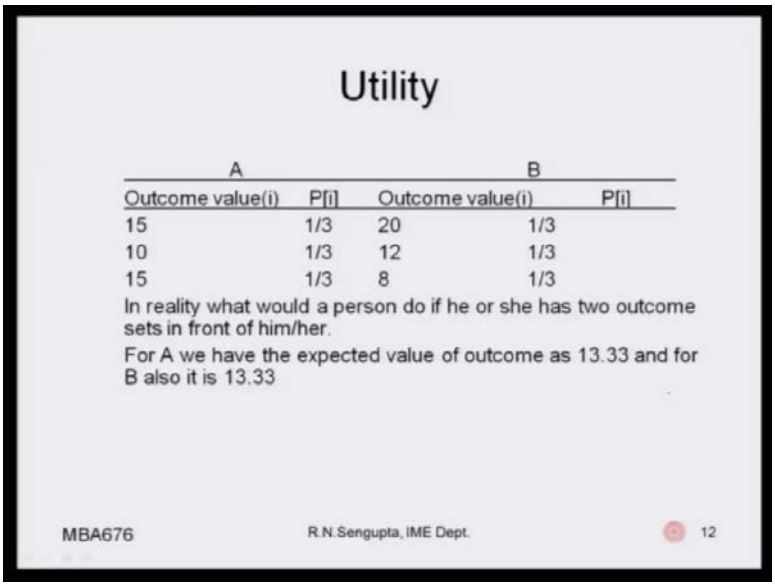

And, the outcome values are given by… In the first column, if you see is 15, 10, 15; and in other column, it is 20, 12 and 8. Now, consider the values of 20, 12, 8, which is in the second one and the first one, which we discussed is 15, 10, 15 as given. So, we will consider why those values are. Now, if I ask you a question that, which one decision would you take? So; obviously, all of you would basically see that on an average, what is the value of the outcome based on the probability of one-third for each an outcome which is there. So, if we basically analyze the two different situations, which is our A and B; your obvious answer would be that, if both the values are coming out to the same value, which is the expected value, which is 13.33; obviously, it means that, you are indifferent or you are basically not willing to take a decision whether A and B. But, there would be some different type of examples, which can be formulated or there can be different type of people, who are there in the market, who would be interested to take

either that A decision or the B decision. So, how that is done – I am going to come to that later.

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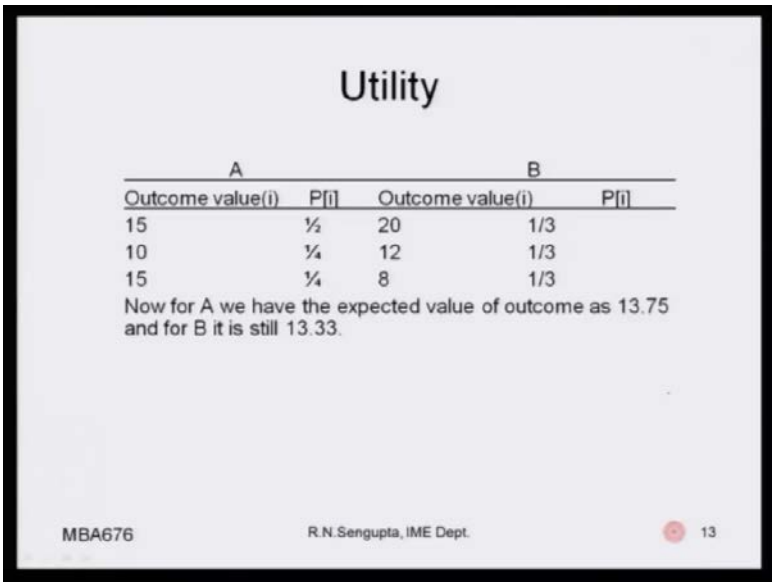

Now, consider the situation is changed. Now, you have the probabilities are half, one-fourth and one-fourth. So, obviously, the sum obviously always remains same for the second and the fourth column. But, now if you see the total expected value of the outcome for A is now 13.75 and for B, it is still remains 13.33. So, if again I ask you the question; now, people who were now indifferent between A and B depending on the so-called expected value, would rather like to take the decision, which is A because it is giving a higher value of 13.75.

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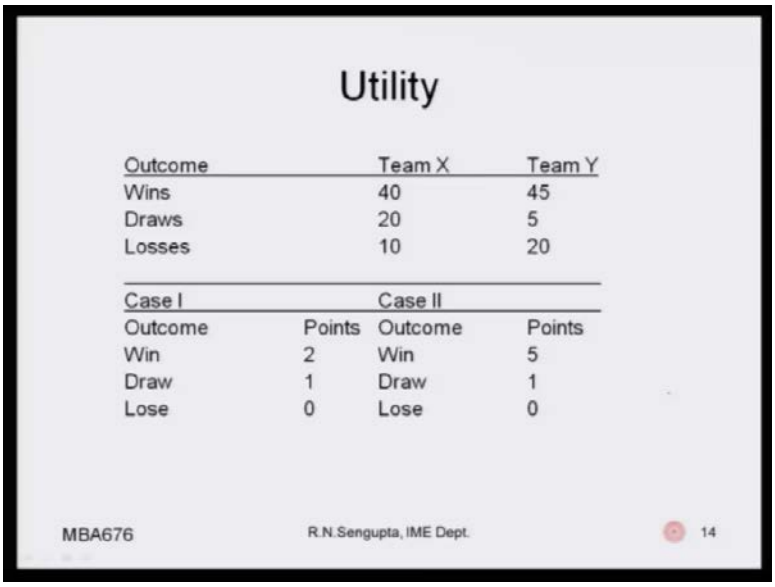

Now, consider that, there are two teams; and, basically you want to rank them. It is like there are two portfolios; you want to rank them. So, if you to see that; the wins, which you have in the first column for both the cases A and B, which is the first block and the second block; you will find out that, the overall points based on which the outcome would be won or loss; for the first case, the points of outcome are – if you win, you get 2 points; if you basically draw, you get 1 point; and, for loss, it is 0. But, for the other one, it is 5, 1, 0. So, again if you are trying to compare so-called portfolio A and B, which in our case is basically trying to rank team X and Y for a game, you will find that the system of ranking changes. So, my question would be… And, obviously, your question would immediately come out is that, which means the ranking changes as the outcome changes. So, outcome does not mean only the probability; it means the outcome in value sense.

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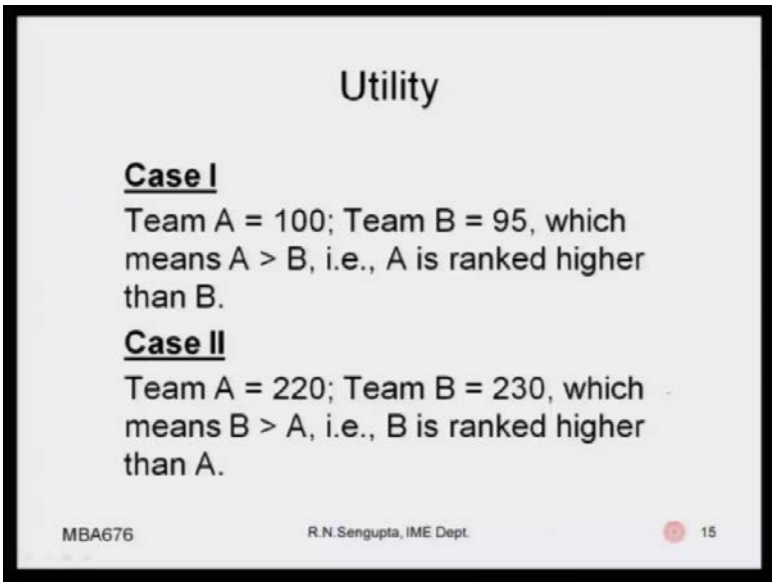

Now, in case 1, if you consider team A as 100, team B as 95; which means A is higher ranked. So, what… In this case is the points basis; like it is a round-robin league or whatever it is, a knock out one; you have the points based on which you take up decision – which team is higher, which team is lower, who is first, who is second, so on and so forth. In case 2, which you saw with the different points, it will be team A as 220, team B as 230. So, obviously, it means B is much better than A. So, A would… – it is ranked higher; which means that even when the situations are same, if you give different points for the outcomes, it means that, the ranking immediately changes; which means that if you change your actual criteria based on which you are going to take a decision, you will

have different results for different type of investment purposes or financial decisions.

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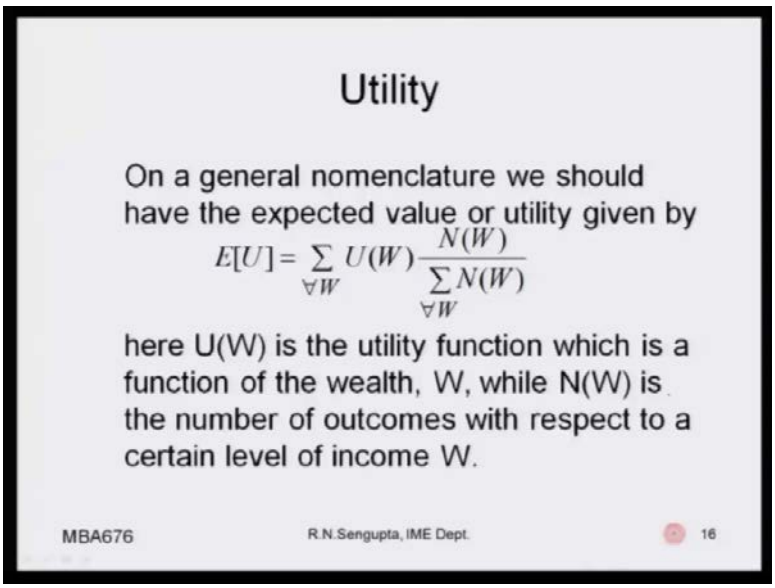

Now, in general, utility is given by expected value in very simple terms. So, when you have expected value, you have… We will see later. Two things to consider: one is the random variables and what are the values, which are known as the realized values; and, second one is what is the probability distribution. So, probability distribution means the relative frequency, which we all know in the general statistics sense. Now, if a random value, a variable is there, the probabilities distribution would basically be akin to the how frequently a certain outcome is happening. So, does any type of probability distribution hold for the finance? We will consider that later on. Yes, they do.

Now, here you have two things. One is the expected value of the utility is considered multiplying two terms: one is U W, which is the utility function. What is utility function? We will see that later; and, the second term, which is basically the ratio is the relative frequency, which is giving you with the probability. So, our main task would be to find out what is the utility and also to find out the distribution of the utility function based on which you are going to take a decision of the ranking.

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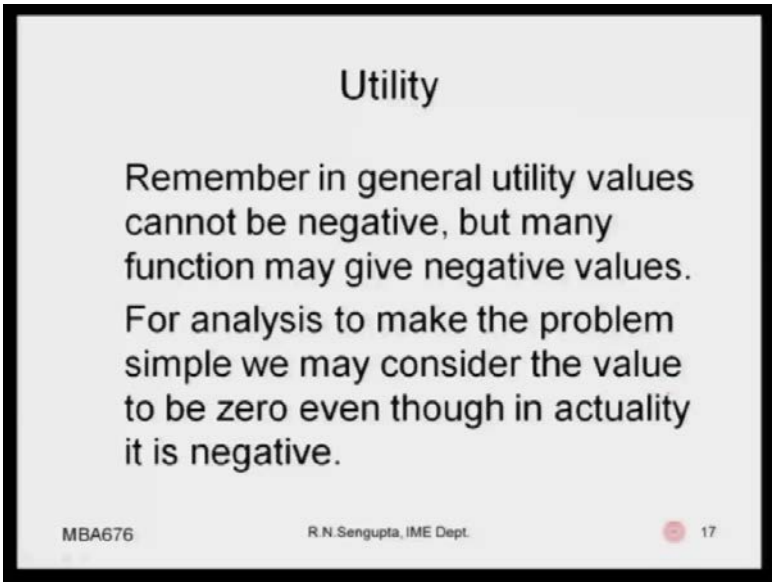

Remember in general, the utility values cannot be negative. So, obviously, if I am taking a decision; if it is negative value; obviously, I would never take that decision. But, later on, we will see that, in general economic sense, utility values – even if they are negative, some decisions are taken. But, in the finance concept, we will consider such decisions are not taken. And, to make our lives simple, we will basically have the rule that, if there is some negative returns, we would not be considering them; but, later on, we will consider also the cases – if there are different type of negative returns, how to basically choose the one, which gives you the least of the overall worst scenario. So, we will rank them accordingly. So, ranking would happen based on the value of the utility even if they are negative. Now, the question is that, if the probability of the outcome changes; as you saw in the gamed on the match between A and B; there was a change in the ranking. Now, the question comes that, what if the utility function also changes? That U W, which you are considering – if it changes also, obviously, there is a change in the ranking.

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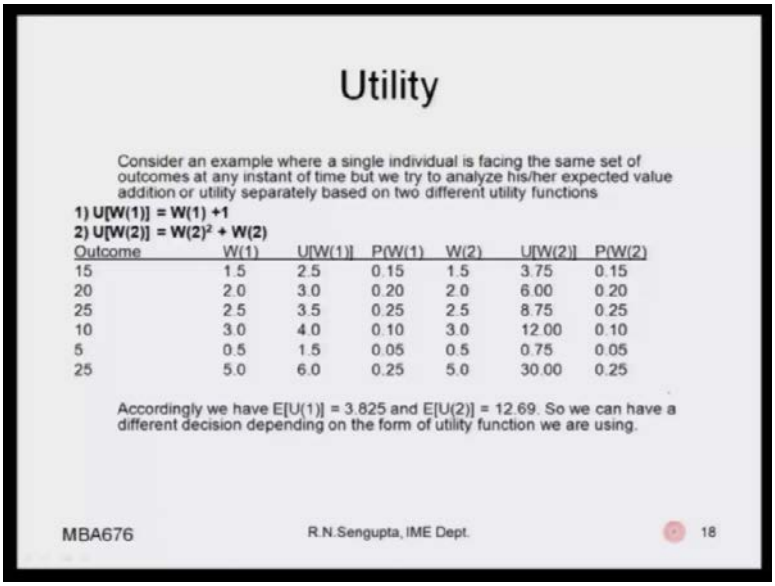

So, consider the first one, which is the highlighted one, which is U W is given by linear function. And, in the second case, you have basically U W, which is given a quadratic function. So, we are first encountering the concept of quadratic utility function; we will be considering later on in depth. So, if you have the outcomes of as given in the first column; so, these are just theoretical values, hypothetical values. And, if you see the W 1, which is the utility function based on W 1; and you have W 2, which is the utility function based on utility function 2; the corresponding utility values are given in the third column and the second last column. And, the corresponding probabilities are next to that. So, if I find out the expected value of the utility, using utility 1, you will have a value of 3.825 and for the second one, you will basically have a value of 12.69. So, obviously, it means that, your ranking of both the expected value would basically change depending on the utility which you are using.

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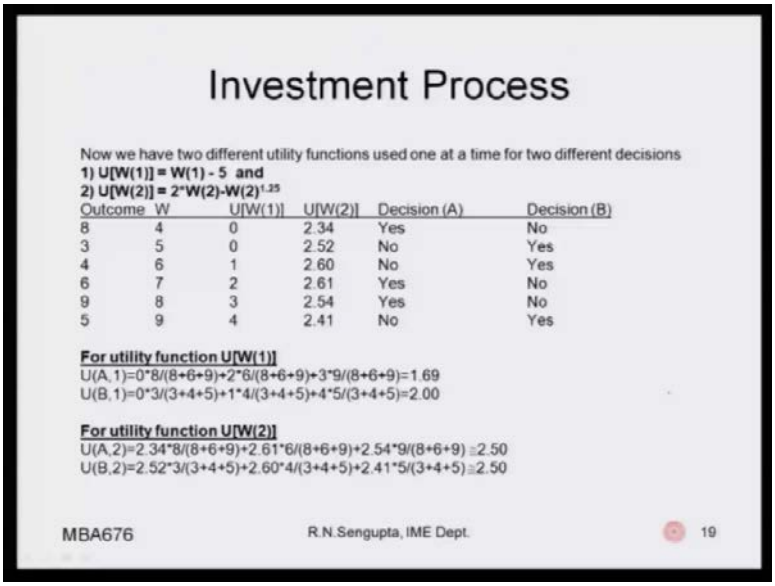

Now, consider we have two different utility functions. And, again the decisions are… Is like a go on no go; that means, if a decision is taken, obviously, you get some utility; if decision is not taken, obviously, you will consider your utility value is net for the 0. So, if you consider that for the utility function U W 1, final value comes out for the case of U A; given the utility function 1 is 1.69. And, for the case of U B, given the utility function is 1, it is 2. So, obviously, it means you will choose B because it is a value of 2, which is greater than 1.69. Now, what happens? Let us consider the utility function is now – again if you see, the value is basically a nonlinear one, which is not a linear one. Then, again if you see and if again considering the same outcomes, if you find out the expected value for both of them, the value comes out to be 1.69. So, now, you are indecisive between ranking of both the utilities. Utilities means the situations, which you have in front of you.

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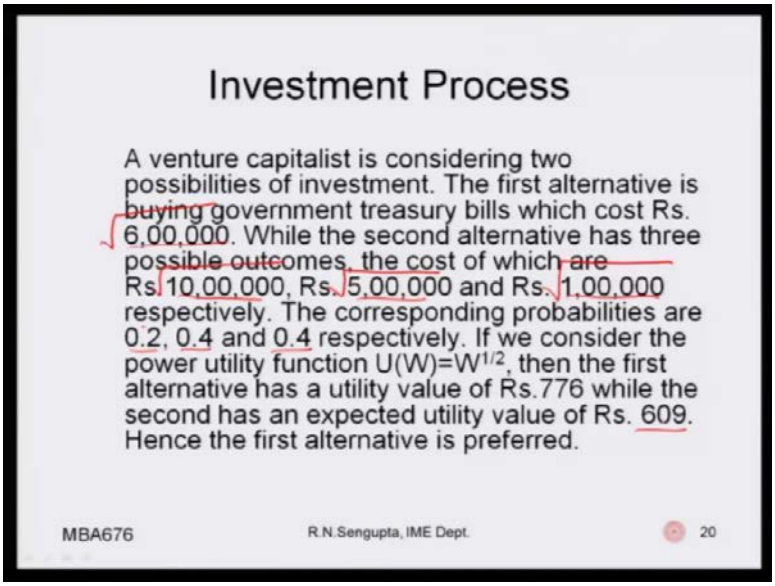

Consider a very simple case – a venture capitalist is considering two possibilities. The first alternative is buying government treasury bills. So, for the first time, you are trying to encounter government treasury bills and they are the cases where the actual risk for the government treasury bills as 0 in theoretical sense. In practical sense, obviously, we will see that, the values of risk-free interest rate, which is a characteristic of the government bills or T bills, which we say, do change; and, we will basically see what are the values of the T bills in the market; and, you can find out information of them in the RBI side, which is the reserve bank side. And, there is another outcome, which has 3 different nodes; nodes means the outcomes of particular security cases. One – with the value of 10 lakhs, another is 5 lakhs, and the third one is 1 lakh. And, the corresponding utility function, which you are seeing is given by U W, which is W to the power half, which is basically a square root of that.

Now, if you find out the value of the utility based on the expected one, then the first alternative would basically have a net value of 776. While the second one, which has 3 outcomes would basically have a value of 609; you can find it out very easily calculating – the simple calculations. Like in the first case, if it is 6 lakhs, we will find out the square root of that. And, the second case, if you basically have these 3 values and the corresponding expected returns value probabilities are given, what you will do is that, you will multiply the corresponding utility, which is U W; that means, square root of this multiplied by half, square root of this multiplied by 0.4, square root of this multiplied by 0.4; and, you can find out the value of 608. So, obviously, your first government bond

which was the first decision gives you higher expected value, you will take that.

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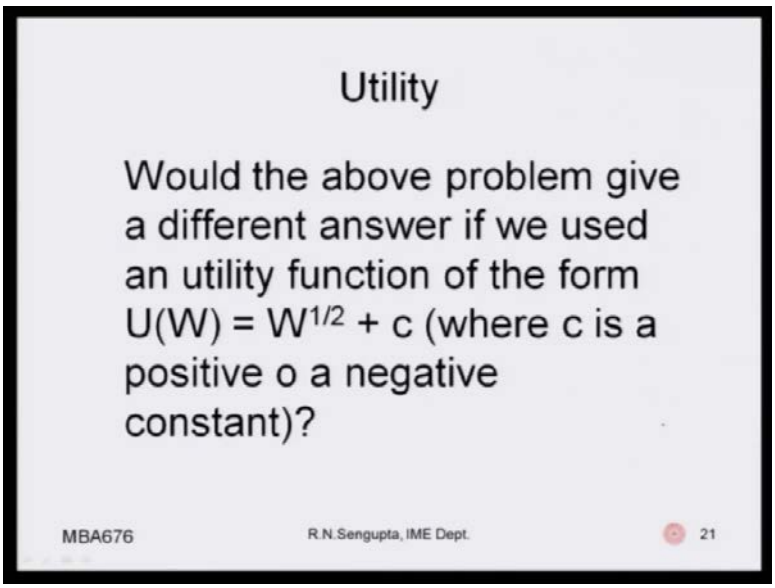

Now, would the above problem give you the same result if you basically add any constant term? Answer is yes. Without going to the actual conceptual framework, we will consider there would be some decisions, where if a I constant is added, how the value does not change.

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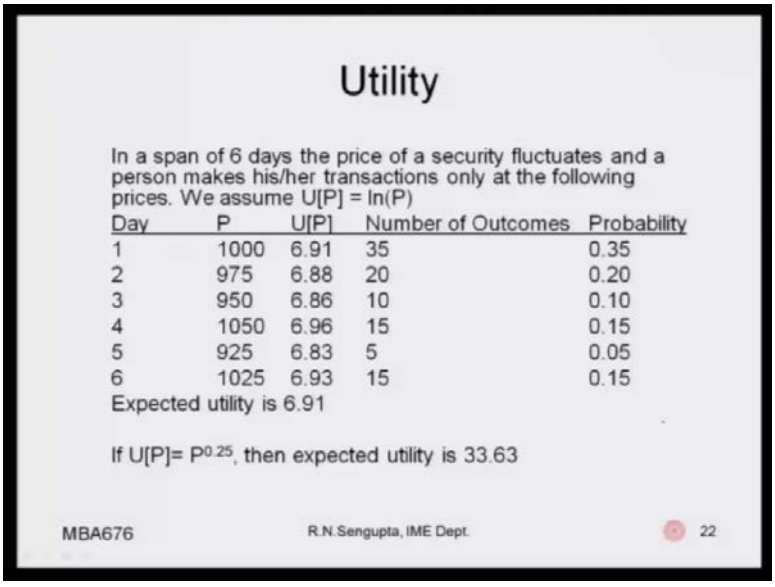

Now, consider a very simple case – a theoretical one; but, for the first time, we will try to basically bring a different type of utility, which is basically based on the property of log. And, why log is coming for the first time? We will consider that very slowly within

another 2 or 3 minutes. The first column are the days, second column are the prices, third column are the log of the prices, and the any of the number of outcomes and the corresponding probabilities are given. So, if you find of the expected value, expected value comes out to be 6.91. If you use a different utility function, which is given by p to the power one-fourth, then the value comes out to be 33.63.

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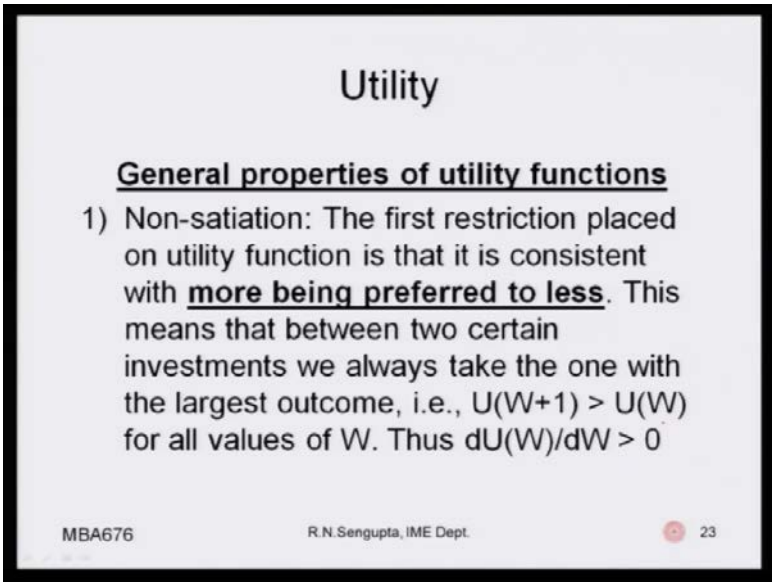

Now, the general properties of utility, which would be important for us later on are only two important characteristics: one is that, more I give to a person, more he wants, which is known as basically the property of non-satiation. So, the more wealth a person has, more he or she wants in order to basically increase the overall expected value of the outcome. That is point number 1. And, if you see the last line of this slide, it means the rate of change of the utility function has to be positive, which means the marginal rate is always positive. So, that has some significance later on.

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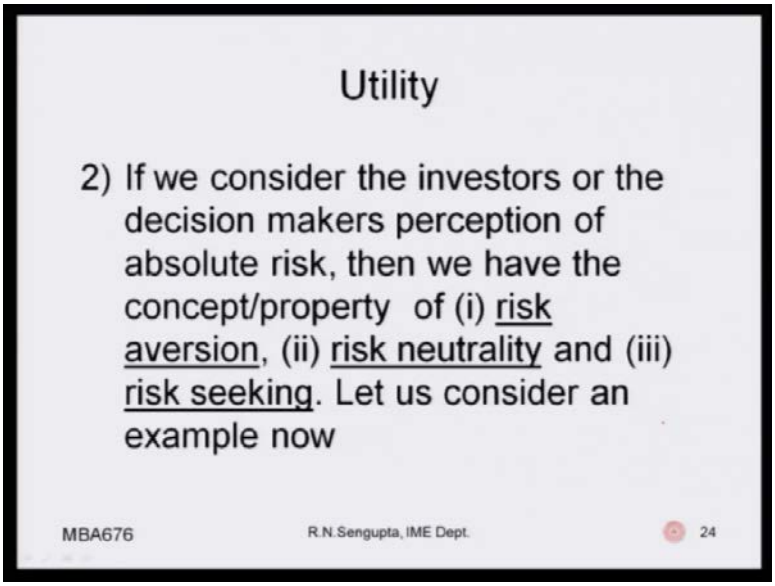

Next property we will consider is that, any human being can be classified under three categories; that means he or she is a risk aversion person; that means he or she is not interested to take a risk. The second characteristics – sub characteristics is basically risk neutral; that means, given two situations, where the one – you we have the deterministic outcome and another one is probabilistic outcome; the person will always try to analyze the expected value of both the cases and take a decision such that he or she is basically indifferent between them. And, the third case is basically a person who seeks risk; that means, in a way that, he or she is taking the risk in such a way that, he or she thinks that in the long run, the outcome would be positive in a sense that, that would basically mitigate the risk, which is being taken by that person. And, we will consider these properties when somebody takes a decision in the area of finance.

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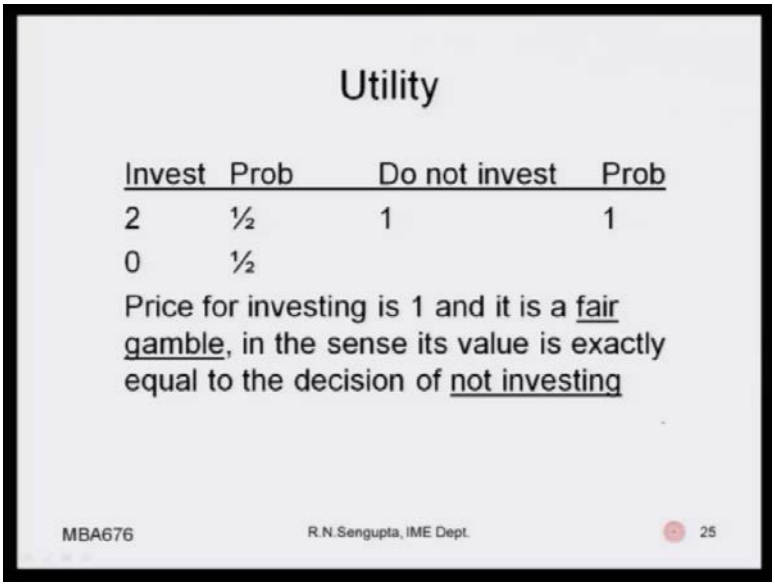

So, now, we will consider a very simple case, which is known as a lottery. So, consider on the right-hand side, you have the Sholay coin, where if we toss it, both sides are heads and your actual probabilities is 1 and outcomes are also given as 1. Other case – you invest 1 and your probabilities are simple coin; you get a probability of half, outcome is 2; probability of half, your outcome is 0. If you analyze both of them, the expected value in both the cases are 1 and 1. Now, if I ask you a question without going to the details that which one would you choose? Would you choose the right one, where it is a deterministic case or would you choose the left one, which is the probabilistic case. So, we will see that there would be in general sense, three categories of person: person 1 who takes the probabilistic case, person 2 who is indifferent between them, and person 3 who basically takes the deterministic case.

So, if I change the probabilities, if I change the outcomes, you will find out a human being can be categorized in any one of the category depending on what the outcome is. So, I may be a person. So, if the value, which is given as 2 and 0 and 1; if those values changes in quantums; so, I may be tempted to change my decision in order to basically take such a case; that means, I am basically being benefited by the overall decision.