Strategy: An introduction to Game Theory
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Lecture – 54

Hello, welcome to mooc lectures on Strategy, An Introduction to Game Theory.

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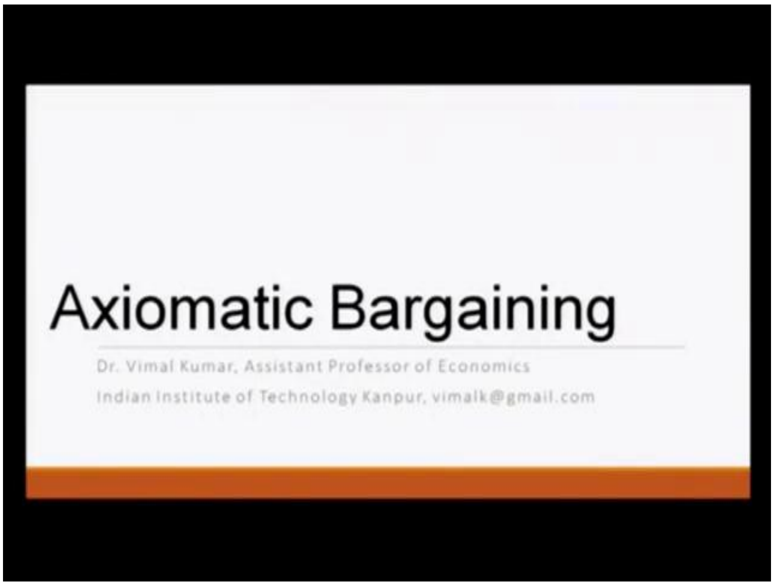

In this module I am going to talk about corporative bargaining or as it is called axiomatic bargaining.

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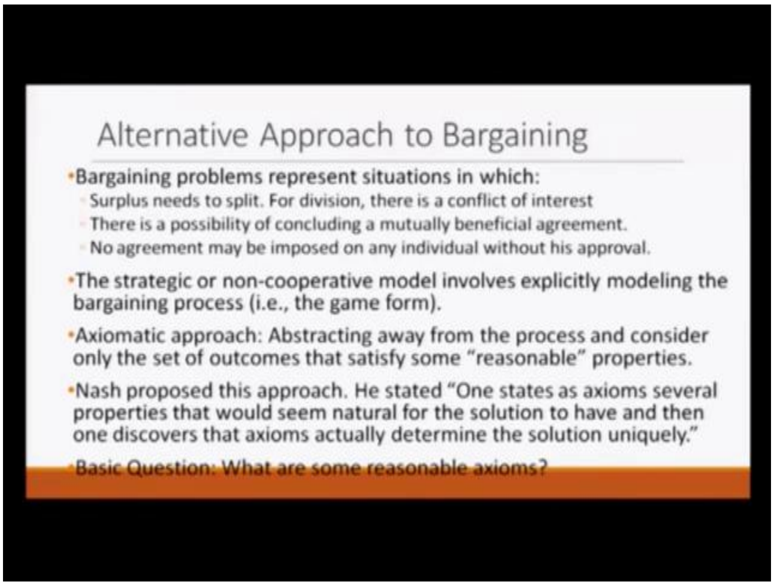

Bargaining problems represent situation in which we have to pay attention to three things, first a surplus needs to be split as we have learnt in the previous module. And for division there is a conflict of interest, in the sense that giving more for one person means automatically means that giving less to the other person. There is also a possibility of concluding a mutually beneficial agreement, if you remember the canonical example I gave in the previous module that in the buyer and seller case, that buyer is willing to pay as much as v and seller is willing to get at least c, so v minus c needs to be split.

So, anything between this if a prize is determined between d and c, then it would benefit both of them. No agreement may be imposed on any individual without his approval that is the third thing. So, for in the previous module what we did is called non corporative or a strategic model. What it did? That we explicitly modeled the process that, how this bargaining would take place. For example, we talked about one stage, two stage, three stage or infinite stage, alternative bargaining offer.

But, if we think about it in real life we face several situation in which the bargaining process cannot be pin pointed in a exact manner, that first player 1 will make an offer, then player 2 will get to accept or reject, then player 2 will get to make an offer. It is also possible that after player 2 rejects, the player 2 makes an offer and again, player 1 rejects player 2 again gets to make an offer. So, we do not know the exact process, so we are going to take a slightly different approach, this is called axiomatic approach in which we abstract away from the process and consider only the set of outcome that satisfy some reasonable properties.

Nash was the one same as same economist or mathematician, after home we have Nash equilibrium in non corporative setting. So, this is another work from Nash, Nash proposed this approach and he is stated to that one states as axioms several properties that would seem natural for the solution to have and then, one discover that axioms actually determine the solution uniquely. So, the most fundamental question would be that what are those reasonable axioms, what should we take as reasonable axioms.

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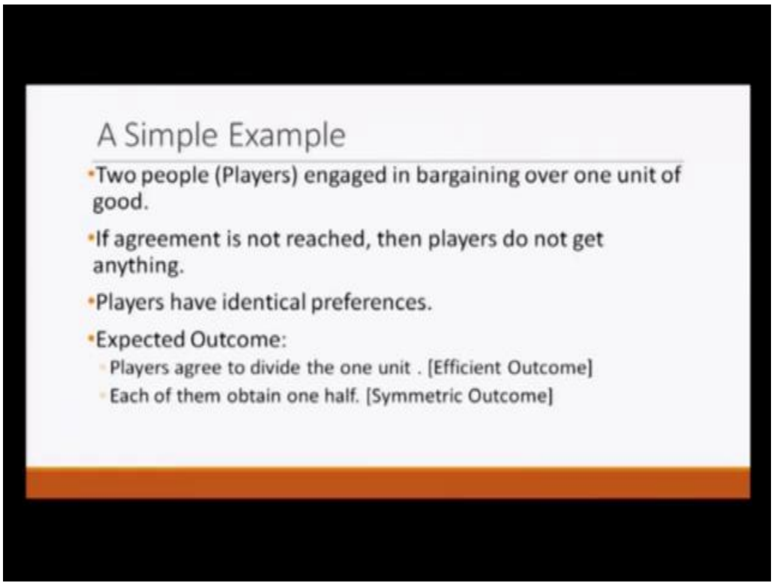

So, let us start with a very simple example without thinking about any game theory, without thinking about what we have learnt so far, that two players are engaged in bargaining over one unit of good. If agreement is not reached, then players do not get anything, both the players have identical preferences, they are in identical scenario. What do you except? What happened? What would happen in this case? We expect that players will agree to decide this one unit, because it will benefit both of them. So, this is basically efficient outcome.

And next that each of them obtain one half, this is the symmetric outcome of course, you can think that it is fear and all, but we will see how it is happening here without bringing fearness into pictures at this stage.

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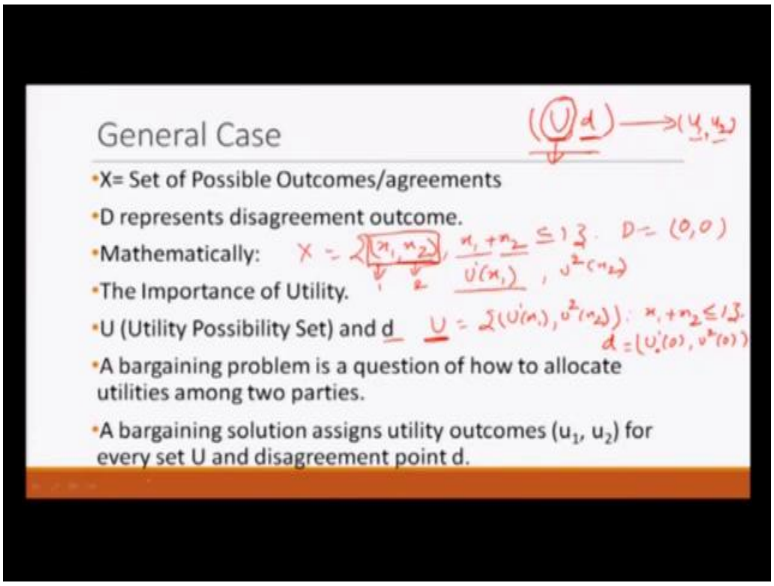

Let us take about a general case, what happens that let us say X is a set of possible outcome or agreement that can be reached and D represents the disagreement outcome. So, what we can write here mathematically is X is something like x 1 and x 2, where x 1 goes to player 1 and x 2 goes to player 2. Such that, x 1 plus x 2 is always less than or equal to 1 and D is of course, 0 0 in this case both of them get 0.

Now, it is also important that we understand the rule of utility, the same unit of money would not benefit or would not give the same pleasure to different people. Let us say that if you add, if you give 100 rupees to a person who has almost to nothing, he would be very, very happy he would be very, very satisfied that if you give 100 rupees to Bill Gates who has really high amount of money that would not make any difference.

So, rather than dealing in terms of x 1 and x 2 we should be dealing in terms of u of x 1 that is utility that player 1 would derive from x 1 and utility that player 2 would derive from x 2. Now, in that sense we should talk about the utility set that would give the utility per u of x 1 for player 1 and u 2 of x 2 to player 2. Such that, x 1 plus x 2 is less than or equal to 1 and D is of course, the utility that player 1 will get from 0 and player 2 will get from 0, this is disagreement point and capital U is the utility possibility set.

So, the bargaining problem is a question of how to allocate utilities among two parties, this utility how we are getting, this utility is coming, because a surplus is getting divided between these two players. So, this is the bargaining problem, how to divide, how to locate utilities, how to divide, so that it will give some utilities, so how to locate utilities among two parties.

And what is the bargaining solution? Bargaining solution assigns utility outcome for every set, every utility possibility set and disagreement point. So, we are starting with this is a bargaining problem which gives all the possibilities, all the outcome in terms of utility, here all the outcomes are in terms of money. Here all the outcomes are, this is the set which gives all the outcomes in terms of utility and this is the utility at disagreement point, so this is the bargaining problem.

And what is the bargaining solution? Bargaining solution is a particular assignment that would be given to player 1 and player 2. This is small u, sorry for my poor hand writing, this is capital U and this is small u 1 and small u 2. So, bargaining solution assigns utility outcome for every set.

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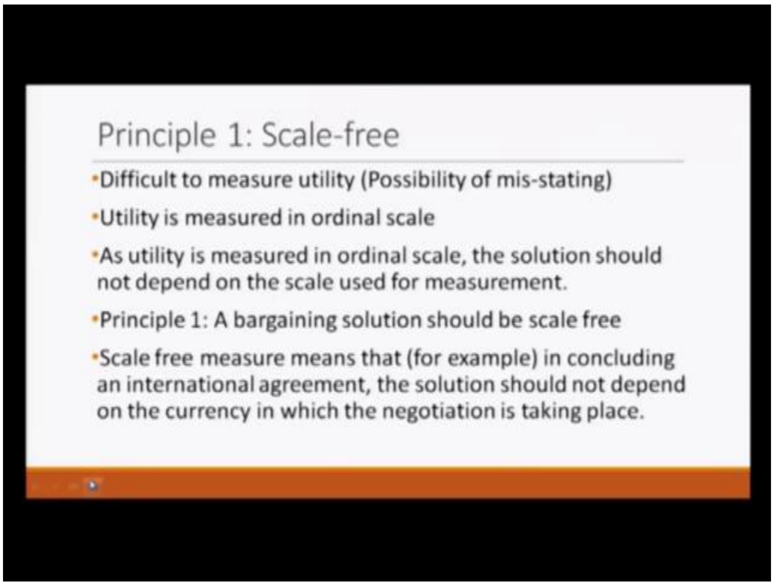

So, what are those principles? What are those axioms that we are talking about? The first important axiom is scale free. So, if you think of utility, how do we talk about utility, let us say when I say I prefer tea to coffee. What do I need? How much more I preferred tea to coffee that is very difficult to determine, I am just comparing between tea and coffee. If you remember one of the earlier modules, we are I talked about from preference to utility I talked about that whenever we have finite choices and we have complete and transitive preference, then we can completely rank all the outcomes and we can assign the number in a particular order, so utility in that sense is ordinal.

So, if utility is ordinal then our solution should not depend on the scale that is being used to measure the utility. So, first principle that we are talking about here is that bargaining solution should be scale free. What does it mean? Scale free measure means that in concluding an international agreement for example, the solution should not depend on the currency in which the negotiation is taking place.

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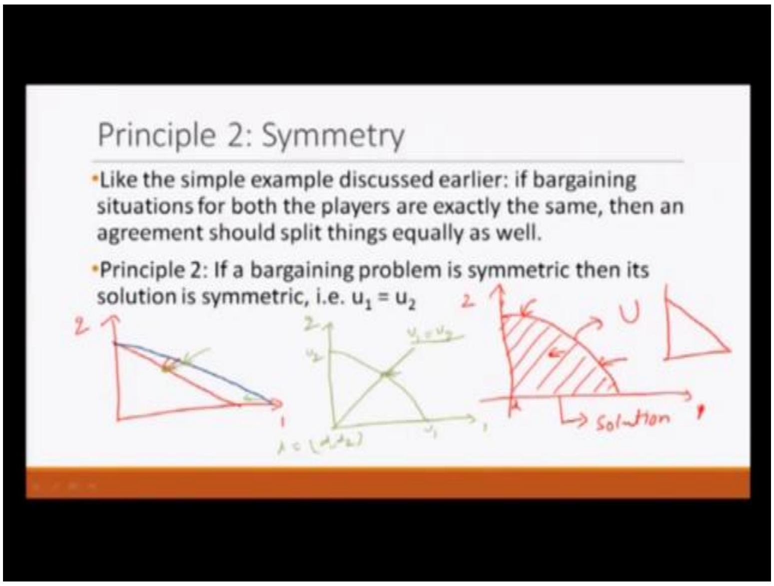

Let us talk about the second principle that we had talked earlier also. If bargaining situation for both the players are exactly the same, then an agreement should split things equally as well. Like, in the earlier example we were talking about how to divide one between two players. So, of course, there we were talking in terms of monitory outcome, here we are taking in terms of utility outcome.

But, the notion remains the same, if situation is the same, if a problem is symmetry, then solution should also be the symmetric while. Let me also do one thing to represent both the notion of symmetry and scale free in terms of a pictorial graph. Let us say let us take a bargaining problem, this is a bargaining problem, here this is the disagreement point, here I am describing thing in terms of utility, on x axis we have utility of player 1 and y axis you have utility of player 2.

So, let me first do what is bargaining problem, so let us say if surplus has to be divided, then all these possible outcome can be generated. So, this is the bargaining problem, because it has this capital U and this has this disagreement point. Now, what would be the bargaining solution? Bargaining solution would be a particular outcome, may be here or may be here or may be here, so a particular outcome is the bargaining solution, but we are concerned about not just this bargaining problem, our bargaining problem can be of this nature, what should be the outcome here in this case.

So, here we are talking about solution in terms of a function which takes a bargaining problem and assigns a particular solution, so particular outcome. So, bargaining solution is a function that assigns an outcome to a bargaining problem. Now, it is clear what is bargaining problem, now let us talk about scale free. Let us say that if our problem is like this, here we have utility of player 1, here we have utility of player 2.

So, we are taking let us say for some reasons using some axioms, we have obtained that this is the solution. Now, because we are talking about scale free, let us say that as we had talked about that utility or utilities are ordinal in nature. So, we can stretch it, shrink it, we just have to maintain the order, so let us say we have stretched only on axis 1. So, the new bargaining problem is this one.

So, what it says the scale free says that, if this is the outcome recommended by bargaining solution, if we stretch it back this point should coincide with this point, it should come back to this point, this is what a scale free means. Third point that we talked about symmetry, what we are talking about that let us take a symmetric problem, let us say here we have a value, here u 1, here is u 2, if u 1 is equal to u 2, here we have d 1 comma d 2 that is d, if d 1 is equal to d 2. So, the solution should be on the 45 degree line, here this solution we should get as at this point where both the values are equal, so that is symmetric.

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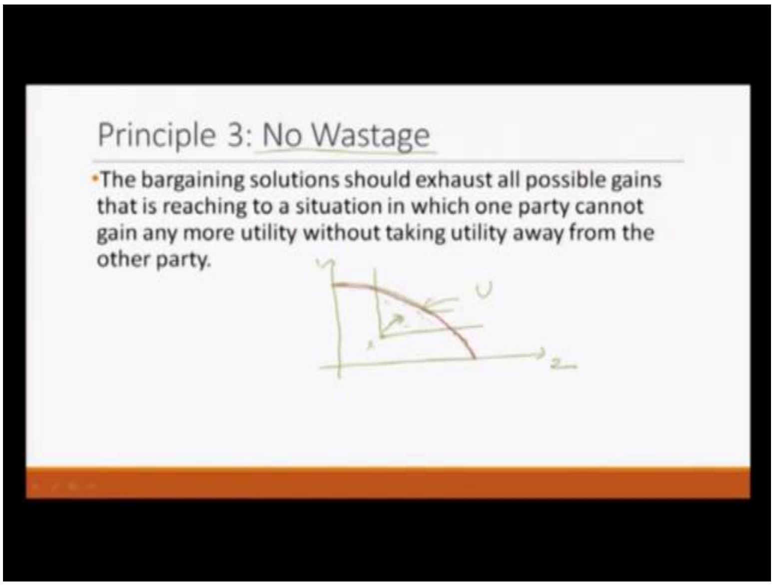

The third is that there should not be any wastage, we talked about the efficiency. The bargaining solution should exhaust all the possible games that is, reaching to a situation in which one party cannot gain any more utility without taking utility away from the other party. Notice, when I was talking about the bargaining problem and I gave a particular example of this bargaining problem, I said that the possibilities the outcome can be here, here or anywhere as long the bargaining outcome has to be belong to this capital U.

Now, no wastage see look at this point, if this is the outcome what would happen, if we move in this direction both players would be better half and it is possible, because these points are in utility possibility set. So, it is possible to move in this direction which would make both of them better half. So, this cannot be the bargaining solution, if we follow the principle of no wastage.

The only outcome in this particular case which are possible, if we follow the concept of no wastage, then what we will have, only outcome on the boundaries would be possible. In this case what would happen? If we want to increase the utility of any person, it will in variably mean the decreasing the utility of other person, so no wastage is should be cleared.

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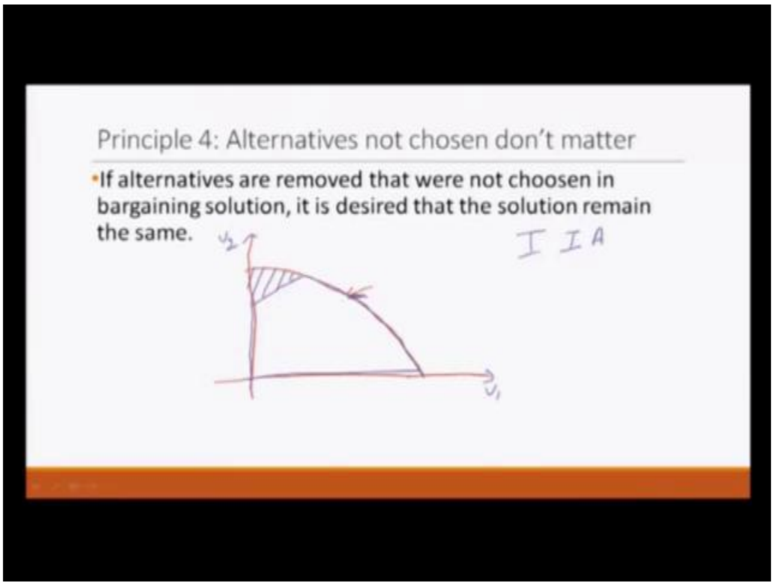

The forth one is alternatives not chosen do not matter, what I mean here again let say that if we removes some of the alternatives that were not chosen, then it is decide in the new bargaining problem, the solution should remain the same, let us look at graphically. So, here is the bargaining problem and the bargaining solution recommends this particular outcome. What it says that let us take a new bargaining problem, in which we have all the utility a possible utilities except these.

In that case, in the new bargaining problem which has the boundary given by this purple color, in this case what happens, solution should remain the same, this is also called IIA Independence of Irrelevant Alternatives. What is the logic, that two people are bargaining over something and they consider all the choices and they eventually reached to this particular outcome. So, if we take out some of the possibilities which were considered in the earlier case, but now in the new problem discarded they were any way not selected earlier. So, even in the new problem new situation they would not be selected, because the outcome that was selected earlier is still present, so this is called IIA.

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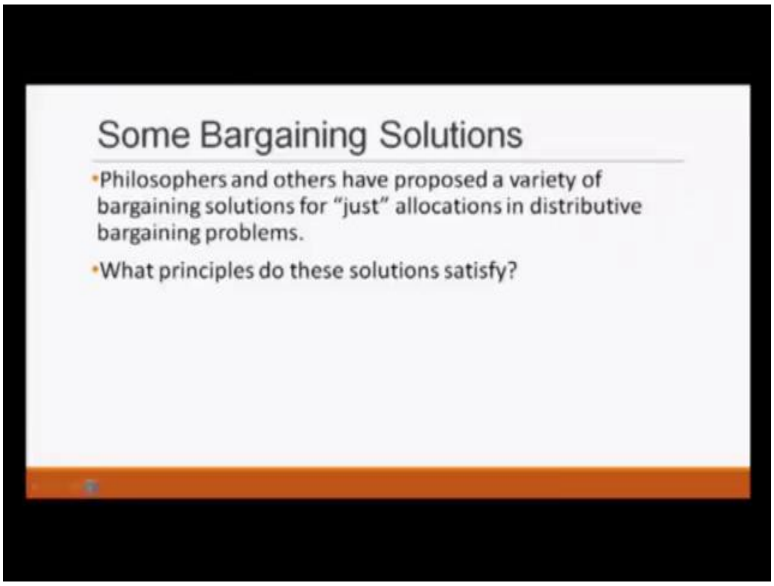

So, let us take about these are the four decide properties that we talked about, let us talk about some of the bargaining solution proposed by philosophers and in the social scientist in the earlier ages and we should check what principle do these solution satisfy.

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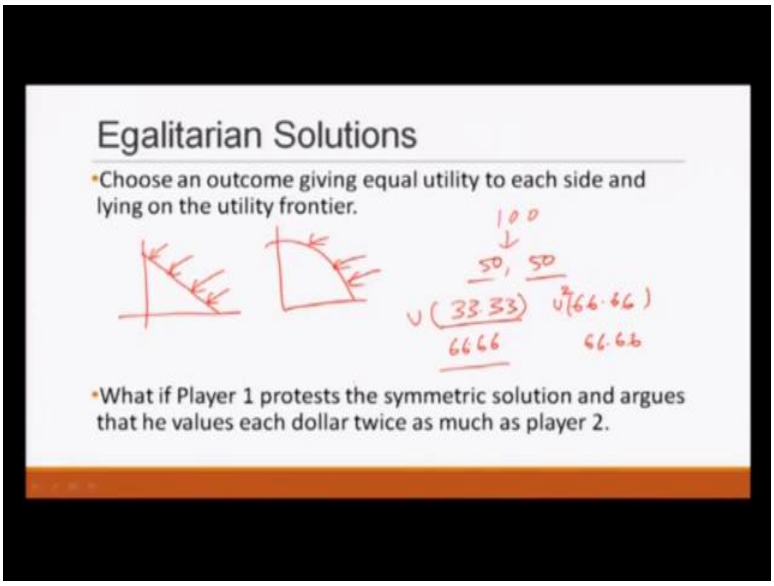

So, let us take about egalitarian solution first what is an egalitarian solution that it chooses an outcome giving equal utility to each side and lying on the utility frontier. So, let us talk about all four properties, what was the first property scale free, let us sold on for scale free. The second property was that no wastage of course, when we are talking about that the outcomes should lie one the utility front here what do we mean by utility front here that if this is the problem, then this would be a straight line is the utility frontier.

Similarly, in this case the utility frontier this even by the boundary. So, no wastage is satisfied. Similarly, that IIA is also satisfied, because if equal utility if an outcome giving equal utility is present even after applying removing some of the option that would be selected. So, IIA is satisfied if a problem is symmetric then everyone will get the same utility as it is given in the definition. So, it is symmetric how about the scale free.

So, let us take an example that would clear whether it is scale free or not, let us say we have to define 100 and it gets divided into 50, 50 to both of the players. Now, let us say player 1 protests and says that he values dollar or rupee twice as much as player 2. What would happen in that case? Because, the values dollar twice as much as the other player he would get 33.33 and other would gets 66.66.

Why? Because, this utility from 33.33 would be 66.66 and while the other players utility from 66.66 we can assume it is 66.66. So, it is of course, stupid for player 1 to argue that he values a many twice as much as player 2, but it is not scale free, because if is coming up with new utilities scheme then it the solution is no longer the same. So, egalitarian solution does not satisfy the scale free.

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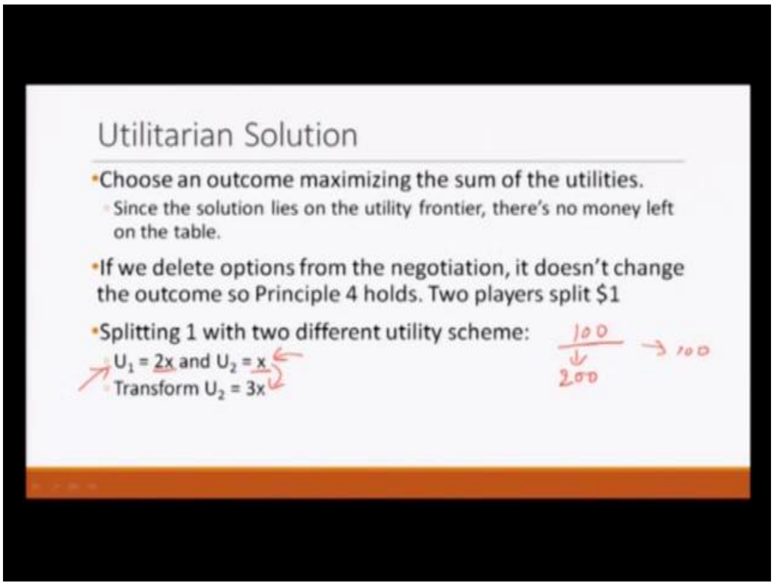

The next one is utilitarian solution, as suppose to egalitarian solution what happens in the utilitarian solution, it choose an outcome maximizing the sum of utilities. Since, the solution lies on the utility frontier, there is no money left on the table and it satisfies no wastage. Again here we have a also if it delete some of the option from the negotiation it does not change the outcome, so principle force that is IIA still holds.

But, how about scale free again we will see that there is a problem with the scale free. Let us say that player 1 utilities given by 2 x and player 2 is utilities given by x. What happens in this case? Because, the aim is to maximize the sum of the utilities everything will be given to player 1 and nothing will be given and to player 2 why, let us say again you have to defined 100 rupees, if you give it to player 1, it will translate into 200, because 2 multiplied by x if you give it to be a player 2 it will translate into 100.

But, you are attempt is to maximize the sum of utilities, so you will give everything to player 1, but let us rescale the utility again. Now, let us say that player 2 utility is represented by 3 x, now in this case everything will be given to player 2 and nothing will be given to player 1, so utilitarian solution is also not scale free.

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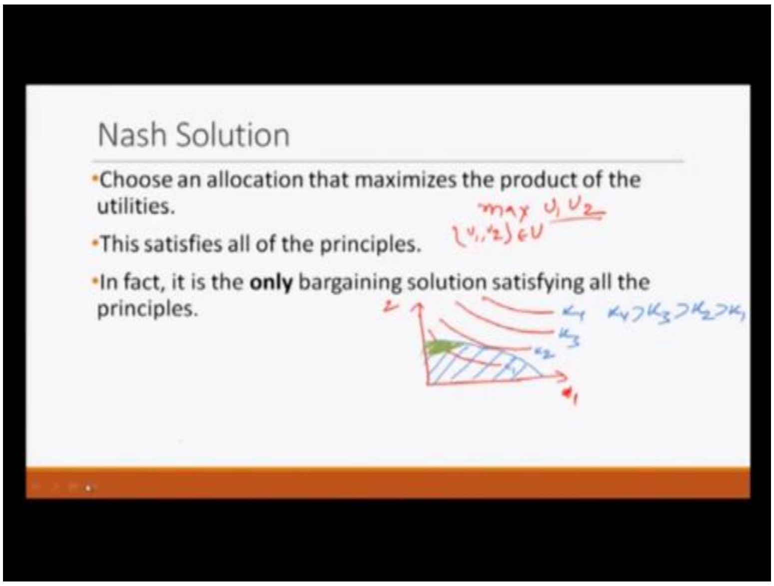

Now, we are going to talk about Nash solution, what did the Nash propose he said the choose an allocation that maximizes the product of the utilities. So, maximize u 1 multiplied by u 2, such that u 1 comma u 2 belongs to the utility possibility set, it satisfies all the principle, let us say how because if we have to, if the situation is exactly the same for both the players then what happens, if you it will fall little bit of mathematics.

But, let us pay attention u 1 if we have on x axis u 1 or y axis u 2 are the utility of the player 2 an here utility of player 1 u 1 multiplied by u 2 this is an equation of hyperbola, so it will be like this. And what is the aim of the Nash solution to achieve the highest hyperbola possible. So, let us take a bargaining problem, what is the aim given the these are the possibilities try to attempt maximum highest possible hyperbola of course, we can number also k 1 k 2 k 3 k 4 clearly k 4 is greater than k 3 is greater than k 2 is greater than k 1.

So, it is very, very clear if we take out some of the option, let us say if we remove these options here still the outcome would not change as long as this outcome is present we will have this particular outcome selected by the Nash solution. So, IIA is satisfy no wastage is satisfied the scales free is also satisfied.

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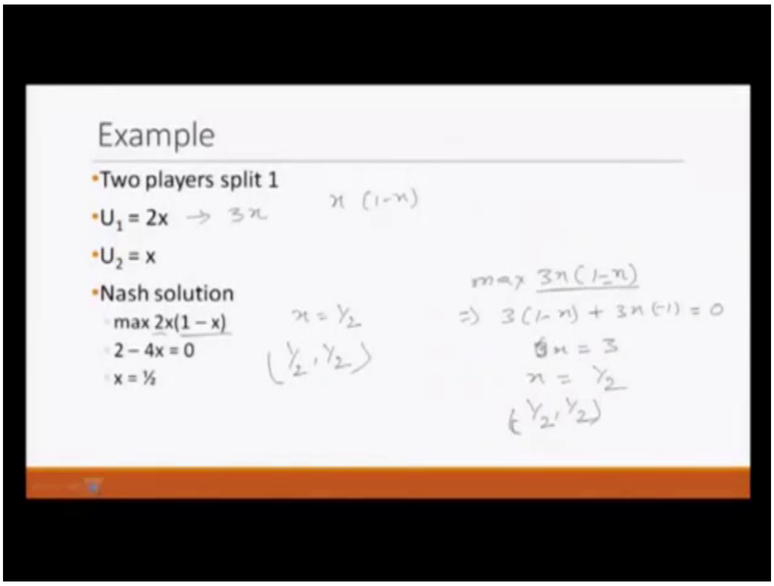

How? We will see let us take an example, we will see that is ((Refer Time: 23:52)). So, let us take this example in which two players would like to split 1, utility of player 1 is given by 2 x utility of player 2 x given by x. What is the Nash solution? Nash solution would, because the player 1 is getting x then player 2 will get 1 minus x. So, x and 1 minus x since utility of player 1 is 2 x. So, what we need to do here is, maximize 2 x multiplied by 1 minus x and if you maximize x is equal to 1 by half both the players will get one half and one half.

Let us change the utility of player let us stretch it and let us say the utility of player 1 is 3 x, but would be the now new solution, now here we will have to maximize 3 x multiplied by 1 minus x and if you do the maximization again what do we get, first order condition that we differentiate it with respect to 2 x what do we get 3 multiplied by 1 minus x plus 3 x and here minus 1 equal to 0 and what do we get, 6 x is equal to 3. So, x is equal to half again both the players get one half and one half, so it is also scale free.

So in fact, it is only barraging solution that satisfies all the principle, although I discuss these four principles, but you do not have to strict to only these four particular principles. If you think about the symmetry argument it is kind of hard wired in our mind that if everyone is in the same situation then the bargaining solution. So, divide the pie equally, similarly no wastage also make sense that if benefit has to be giant it they player should giant should that particular benefit. So, there should it to be any dispute about these two principles, one regarding symmetry, secondary adding no wastage how about to other one.

The third one that we have talked about is, scale free that is also widely excepted that it is good idea to have an outcome that is have a solution concept that is scale free. The most problematic one is independence of irrelevant alternative the forth one is. So, several solutions have been proposed one notable is ((Refer Time: 26:32)) he solution I am not getting in to it which takes out IIA and gives another criteria and comes up with another solution concepts.

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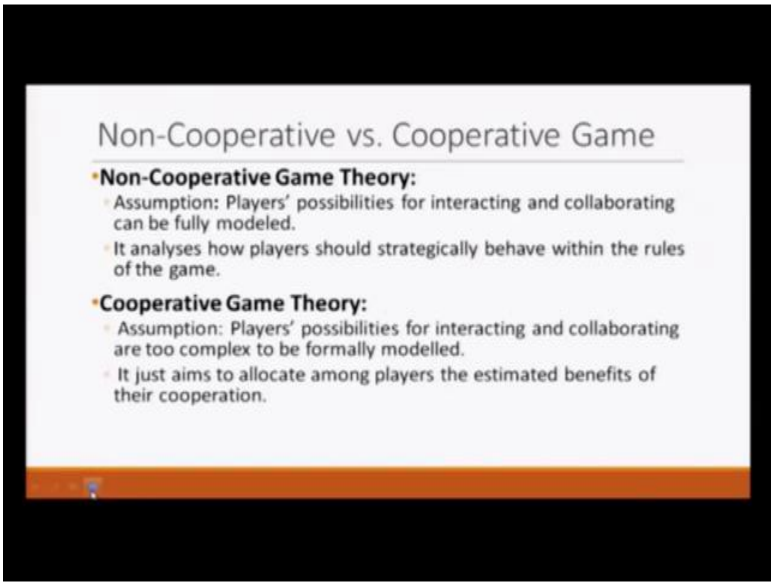

So, that is it for the bargaining axiomatic bargaining I want to close this module just by distributing, two different branches of game theory that is non corporative and corporative game theory. Most of the things that we have discussed, so for in this course except today that we did in bargaining everything we discussed was non corporative game theory. What is non corporative game theory? We assume that players possibilities of interacting and collaborating can be fully model.

We know how players move, what are their actions available, what would be the pay off these particular combination of action would be taken. So, it analyzes how player should strategically behave within the rules of the game, as suppose to non corporative game theory, we have corporative game theory in which the basic assumption is players possibilities for interacting or collaborating are too complex to be formally modeled.

Exhausted aims to allocates, allocate among player the estimated benefit of that operation. So, these are the two major branch is to two branches of game theory, this course is primarily devoted to non corporative game theory, but we though it is a nice idea to just introduce the notion of corporative games.

Thank you.