Strategy: An Introduction to Game Theory
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Lecture - 53

Hello, welcome to mooc lectures on Strategy, An Introduction to Game Theory. In this module I am going to talk about bargaining.

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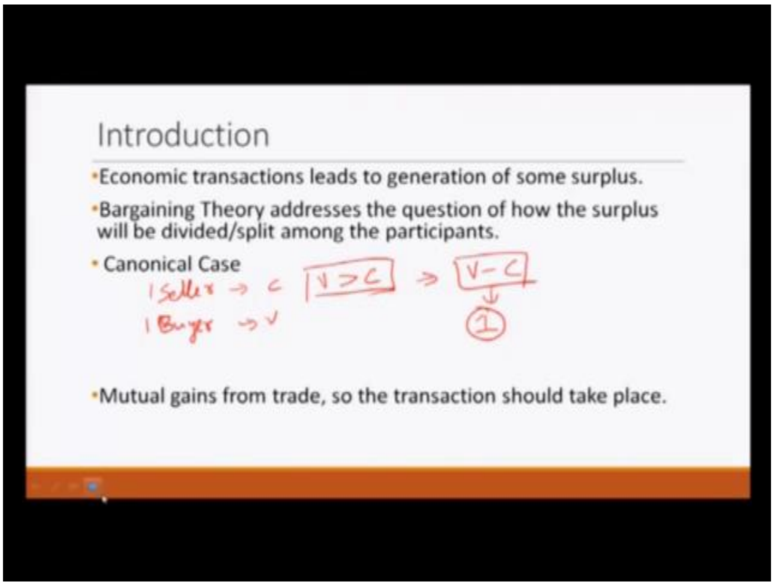

What is bargaining? Let us think about economic transaction, whenever a transaction takes place, a surplus gets generated. How that surplus should be split; that is the main topic of bargaining theory. Let us take a canonical case, in which we have one seller and there is one buyer. Let us say that seller has a, seller would like to sell an object at least at price C and buyer is willing to pay at max V, where V is greater than C.

So, definitely if this transaction takes place, then it would benefit seller and it would also benefit buyer. So, this transaction would generate the surplus of V minus C. So, basically bargaining theory addresses the question of how the surplus will be divided between these two participants. And as this transaction is mutually beneficial, so it should take place, but rather than talking about V minus C, we will normalize it to 1 and this can be any other number, but just for simplicity, we will take it as 1.

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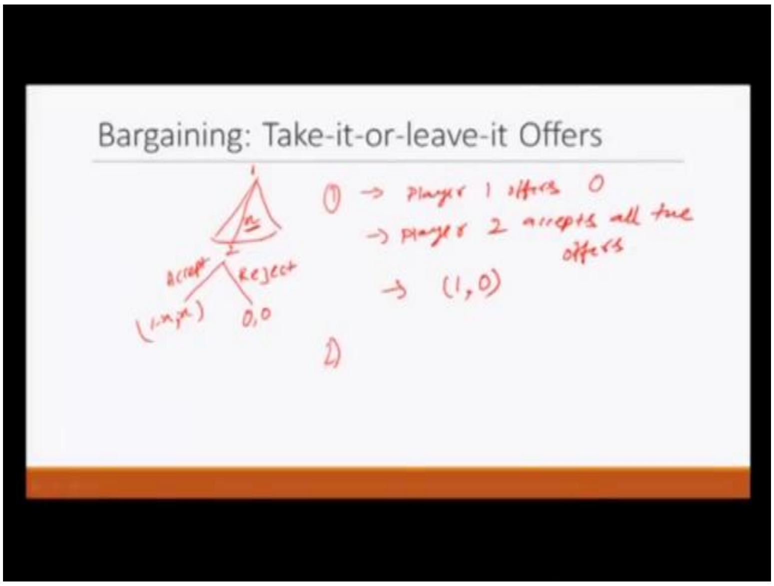

So, let as talk about a very simple form of bargaining; that is, take it or leave it offers. It may sound familiar, but let me repeat again, what happens, there is a player 1 and there is a player 2. So, two player game, similarly player 1 and player 2 can be buyers or seller, but we will get out of this buyer and seller setting. So, we will just talk about two players.

So, player 1 makes an offer to player 2 to divide this surplus of 1 and this is continuous offer. So, he can say that I will give you x and I will keep 1 minus x to myself and this value of x can be anything between 0 to 1. Then, once getting this 1s player 2 gets this offer, then player 2 decides either to accept or reject. If player 2 accepts, then player 1 gets 1 minus x and player 2 gets x, remember this x is the offer need to player 2 and if player 2 rejects, then both of them get equal to 0.

We have already studied this game, this game is called ultimatum game, I recommend that you go back to the module on ultimatum game and revise it. What we studied that in this case, there is only one equilibrium and what is that equilibrium; that player 1 offers 0 and player 2 accepts all the offers. So, what is the outcome? The outcome is that player 1 gets 1 and player 2 gets 0, when we talked about ultimatum game, we also studied discrete version.

What happen in that, that there way two possibilities, one similar to this possibility and second one was that player 1 offers bear minimum, the discreet the bear minimum possible. And player 2 accepts all, but 0 offers, but today we are going to strict to this continuous virgin.

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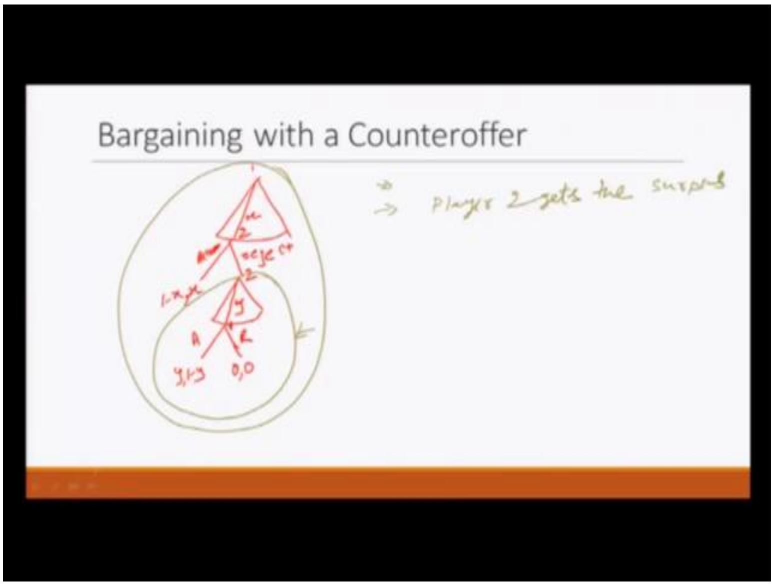

Now, let us extend this, take it or leave it offer, let us say that after player 1 makes an offer to player 2 and player 2 can of course decide whether to accept it or reject it. But, if player 2 decides to reject it, then player 2 gets to make another offer and then, player 1 gets to accept it or reject it. In both the cases game would end. So, let us describe it pictorially, player 1 makes an offer like earlier case, player 1's offer let us denote it by x, then player 2 gets to either accept or reject.

If player 2 accepts, then what happens player 1 gets 1 minus x and player 2 gets x and if player 2 rejects, player 2 gets to make a counteroffer. Let us say player 2 offer to player 1 is y, then player 1 gets to accept or reject, let me write it accept by A and reject by R. If player 1 accepts, then player 1 gets y and player 2 gets remaining 1 minus y and if player 1 rejects, then both of them get 0.

What happens in this game? This we did not study earlier, but let us focus on subgame of this whole game, let us focus on this subgame. The subgame that I just indicated is same as the ultimatum game with one difference. And what is that difference? That in this game player 2 is the one who makes the first offer and player 1 gets to decide whether to accept it or reject it. So, outcome is going to be very simple.

What would be the outcome? Here 2 would offer nothing and player 1 will accepts all the offer. So, now, let us focus on the whole game. What happens in the whole game? Player 1 makes that offer of x, if it is less than 1, then what happens, player 2 knows that by rejecting, he will get to keep the whole phi for himself, so player 2 would reject. So, two possibilities either player 1 offers the whole thing, the whole 1 unit to player 2 and player 2 accepts or player 1 offers something different.

And then, player 2 rejects and then, player 2 makes an offer of 0 to player 1 and then, player 2 will get to keep everything for himself. So, no matter which one we take, the outcome is always going to be that player 2 gets the whole thing, player 2 gets the surplus. And let me say that, what if we allowed player 1 to make counter of here at this point, then what would happened, again the last part would be same as the ultimatum game. And whoever gets to make the last offer, will get to keep everything. So, nothing changes, but this is not what we observe in typical bargaining situation.

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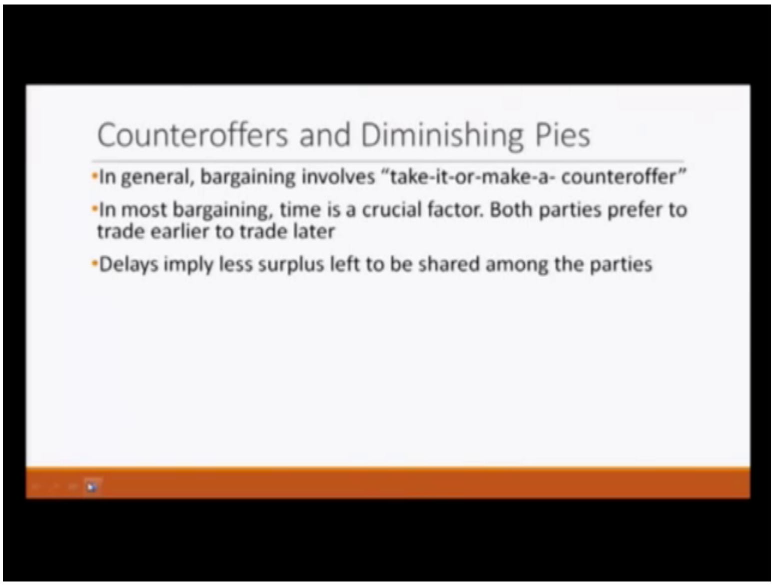

So, we will add some more assumption here. So, what happens that in most of the bargaining, time is a crucial factor, both parties would prefer to trade earlier than to trade later. So, basically delay implies less surplus, left for the both parties to be shared among themselves. So, now, we are going to modeled this.

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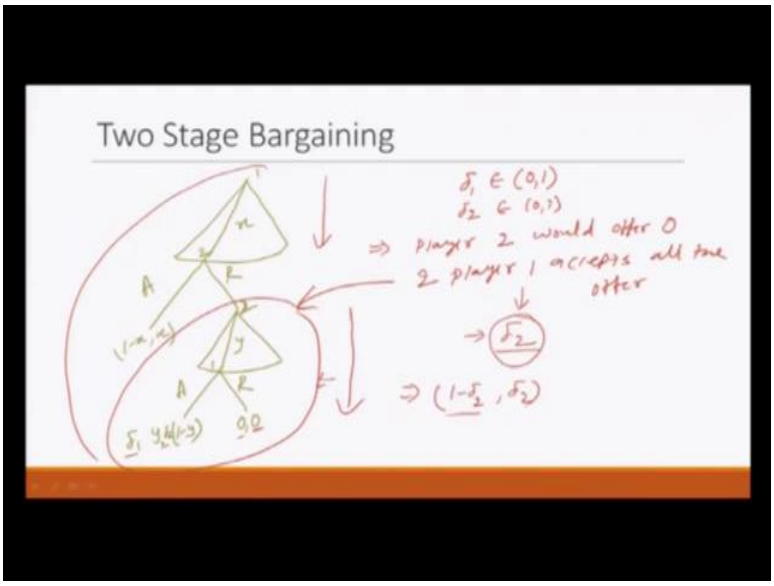

So, what would happened, again same as the earlier game, but here we will take a discount factor that will model the value for time. Remember, this count factor when we were discussing the infinitely repeated game, we discuss the discount factor. So, the similar known thing we will be using here. So, here what happens, player 1 makes an offer to player 2, offer of x and then player 2 either gets to accept it or reject it.

If player 1, player 2 accepts then player 2 gets x and player 1 gets 1 minus x, if player 2 rejects, then player 2 gets to make a counter offer and let us say this time player 2 offers y to player 1. Now, player 1 can either accept or reject, in both the cases the game would end. If player 1 accepts, then player 1 gets y and player 2 gets 1 minus y, but we also want to model the value of chain. So, for that we are using the discount factor.

So, let us say for player 1 discount factor is delta 1 and for the player 2 discount factor is delta 2. So, here we use delta 1 and here we have delta 2 and if player 1 rejects at this stage, both of them will get 0. Now, what should be the equilibrium here? It should be player 2 and then, player 1 accepts or rejects. So, what happens here, again let us focus on smaller game; that is the subgame of the whole game, this game like earlier this game is very similar to the ultimatum game with two differences.

One that player 2 makes an offer first and then, player 1 gets to accept or reject and second difference is that there is a discount factor involved. But, if you pay attention discount factor appears multiplicatively and it is safe to assume that discount factor is between 0 and 1, but never equal to 0 or 1. And all the time in all the subgames, we will be comparing the payoffs to, the reject payoff that is always going to be 0.

So, we multiply a positive number with a positive number, we always get a positive number which is greater than 0 and here payoff is 0. So, we do not have to worry that much about the discount factor. So, what happens here, what should player 1 do? There are several, there are infinite number of subgames, what should player 1 do, player 1 in all the subgame player 1 is better off by accepting.

But, in one subgame, he is indifferent between accepting and rejecting; that subgame is in which player 2 offers 0. So, ultimately what would happen that if we repeat the same logic that will learned during the ultimatum game, what do we get, player 2 would offer 0 and player 1 accepts all the offer. I am writing this, assuming that this sub game is the whole game. So, what happens ultimately, if player 2 offers 0 to player 1, he gets to keep the whole phi, the whole surplus for himself; that is equivalent to 1.

But, because of the delay, time delay, this is the stage 1 and this is the stage 2. So, there is a discount factor that would come into the play. So, one we translate into delta 2. So, player 2 would get delta 2. Now, let us come back to the whole game, what happens in the whole game, if player 1 offers anything less than delta 2, then player 2 can reject and then, he will get to keep the whole surplus for himself that will give him delta 2.

So, player 1 would not offer anything less than delta 2, because if player 1 offers less than delta 2, if player 2 rejects and makes another offer for player 1, the ultimate result what we are getting, player 1 will keep everything for himself and gives nothing to player 1. So, player 1 is better off by offering delta 2 or greater than delta 2, because in that case player 2 would be better or by accepting.

And of course in this case, player 1 should not offer more than delta 2, because he can always increase his payoff by decreasing his offer to up to delta 2. So, what happens ultimate result, in this case, player 2 end up getting delta 2 and player 1 end up offering 1 minus delta 2. So, of course, there would not be any delay, write in the beginning, player 1 would offer delta 2 to player 2 and player 2 will accept this offer. And ultimate result, the result would be that player 1 ins a getting 1 minus delta 2 and player 2 will end up getting delta 2.

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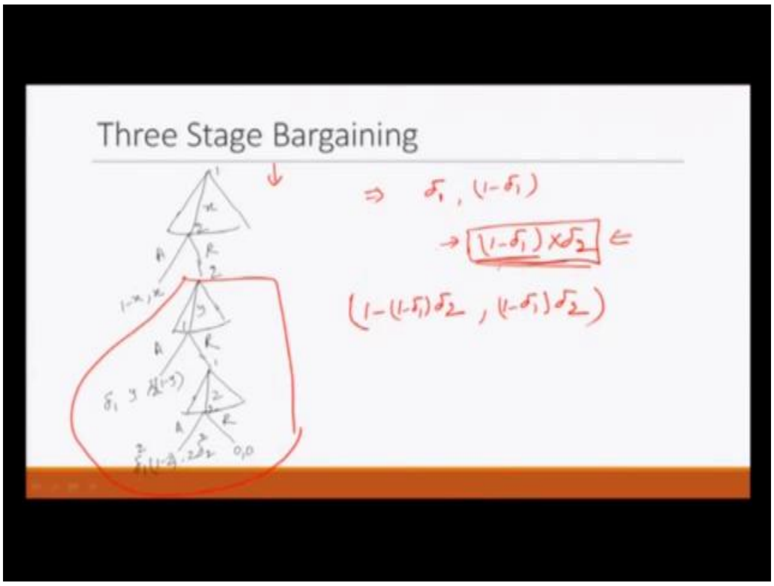

We have already discuss the equilibrium, let as look at the three stage bargaining, just one more stage. So, in three stages, what we are going to do the same thing, but we will add one more stage. What happens player 1 makes the first offer and player 1 again like previous cases, player 1 offers x to player 2, player 2 has can accept or reject. If player 2 accepts then player 2 gets x and player 1 gets to keep the remaining 1 minus x. If player 2 rejects, then player 2 gets to make a counteroffer of y for player 1. Player 1 can again accept or reject, if player 1 accepts, then player 1 gets y and player 2 gets 1 minus y.

But, after delay of one stage, so we will use discount factor once, delta 1 for player 1 and delta 2 for player 2 and if player 1 rejects, then player 1 gets to make another offer and that is z, let us say to player 2. And in this case, player 2 can either accept or reject, but in both the cases game would end, if player rejects now in this case, then both will get 0 and if player 2 accepts, then player 2 gets z and player 1 gets 1 minus z.

But, this happens after two stage, so we will use we will discount it twice, first delta 1 and again with delta 1. So, it will appear multiplicatively delta square 1 here and delta square 2 here. So, now, what happens if we pay attention to this subgame, this is same as the two stage bargaining and what did will learn that the player, who offers first offers delta that the amount equal to the discount factor of other player to that player.

So, here player 2 is making that offer. So, what player 2 will do, player 2 will offer delta 1 to player 1 and then, player 2 will get to keep 1 minus delta 1, but this, if we see from here, then we have to apply the discount factor. And so what is happening, player 2 is basically getting 1 minus delta 1 multiplied by delta 2, because now we are shifting see whenever we shift one stage backward, what do we do, we use discount factor 1's.

So, seen from here, player 2 is getting 1 minus delta 1 multiplied by delta 2, if we repeat the logic that we gave earlier, what would happen player 1 if makes and offer at least as largest this, then player 2 will accept. Otherwise, player 2 will reject and if player 2 rejects, then player 1 will end up getting 0. So, player 1 it is a better idea it is player 1 has incentive to of make an offer at least has largest this. So, what would happen, player 1 will make an offer equivalent to 1 minus delta 1 multiplied by delta 2 to player 2 and will get to keep 1 minus delta 1 multiplied by delta 2 and this will be the outcome.

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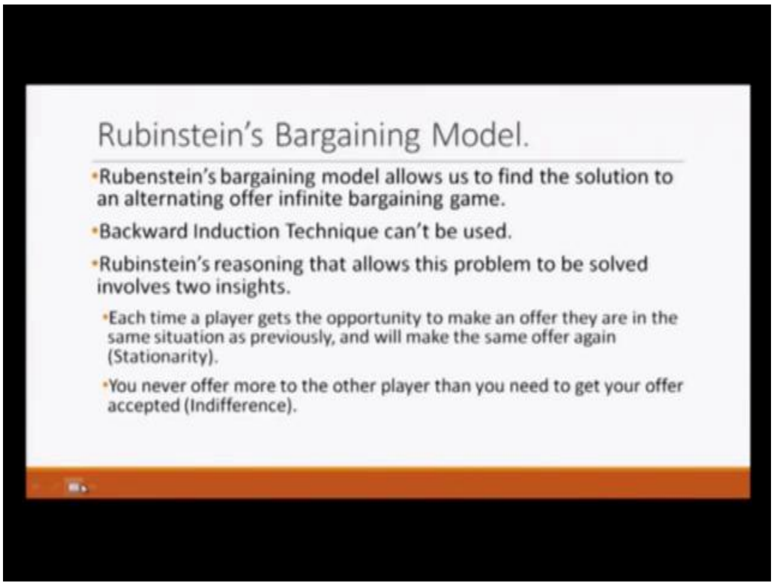

Now, before we get into a particular bargaining model, I would like to add that three stage, four stage, you can get up to a finite number of T stages, I recommend that you do it, you will get a very nice result. But, if thing about a scenario, when we have this alternative bargaining, alternative offer bargaining for infinite period; that it can go on forever. In that case, we cannot use the backward induction technique, why because to backward induction technique, we have to start with the subgame, which directly ends with that, directly reads to the terminal node.

Because, if we have infinitely long game, then we do not nowhere the terminal node is we cannot use the backward induction technique. So, Rubinstein a very famous economist, he are good that, we can use two inside to solve this problem. So, he suggested that each time a player gets the opportunity to make an offer, they are in the same situation has previously and will make the same offer again.

Why, because if it is infinitely repeated game, then having one more a stage will not take you nearer to the last, there is no last stage, but it will not take nearer to the end of the game. So, you will again make the same offer, second inside that if he had, you never offer more to the other player, then you need to get your offer accept it and this logic, we have been using it earlier also.

((Refer Time: 20:12)) Like here, we said that player 1 needs to make an offer at least has large 1 minus delta 1 multiplied by delta 2 and therefore, he will offer only this much, nothing more, because offering more when getting less for himself. So, he will offer nothing more. So, he reports the idea of his stationarity and in difference to solve this problem.

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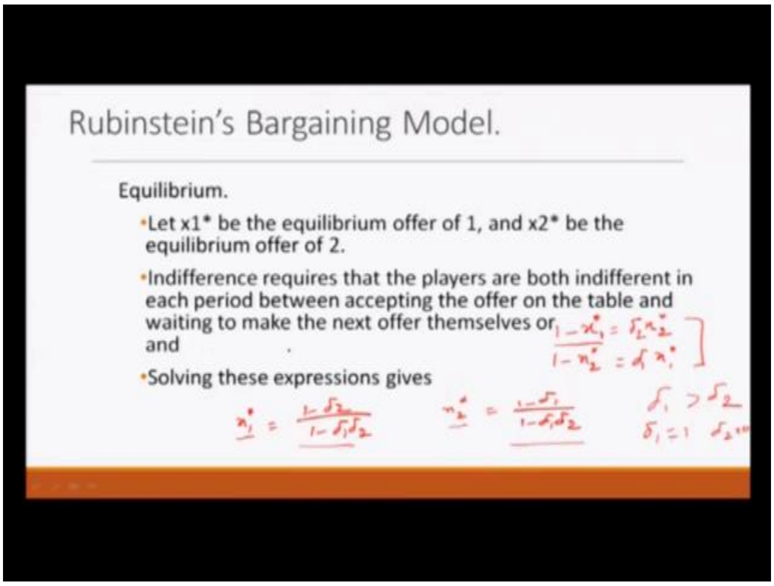

So, let us just layout the problem and solve it, let us say that x 1 star be the equilibrium offer of player 1 and x 2 star be the equilibrium offer of player 2. In difference requires that the players are both in different in each period between accepting the offer on the table and waiting to make the next offer themselves. So, what we can say that 1 minus x 1 let us say that will be the offer for himself, this will be equivalent to, what he will get from the second player delta x 2.

Similarly, 1 minus x 2 star will be equal to delta 1 x 1 star, if we solve this, what do we get, x 1 star would be equal to 1 minus delta 2 divided by 1 minus delta 1, delta 2 and x 2 star will be 1 minus delta 1 divided by 1 minus delta 1, delta 2. I would like to in reemphasize, how did I get these two things, remember the two notions that we just discussed stationarity and indifference.

And indifference means that player the offer that is being made, the player should be indifferent between what he would get if his offer gets accept it or in the next stage the other players offer get x accept it. And in the equilibrium, because at we are talking about infinite stages, it should be stationary; it should not change from one stage to another stage. So, that is how, I am getting these two expression.

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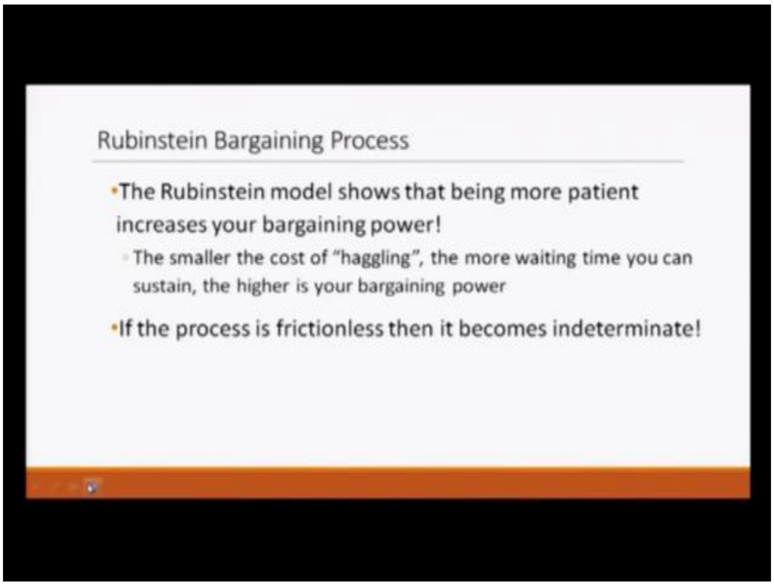

So, what does the Rubinstein bargaining process tell us; that Rubinstein model shows that being more patients increases your bargaining power. Let us compare here ((Refer Time: 22:47)) that more patients means higher delta, let us say, when you have delta 1 is greater than delta 2, what happens in this case, we can take an extreme example, where delta 1 is equal to 1 and delta 2 is equal to 0. In this case, what happens everything player 1, everything goes to all the surplus goes to player 1 and player 2 gets nothing.

So, clearly having more patients meaning getting more surplus and if the process is friction less, then it becomes in determinate. What does it mean that if delta 1 and delta 2 both are equal to 1. Then, what happens, then we are not able to decide in this system. So, that is all for in non-cooperative bargaining, in the next module, we are going to discuss cooperative bargaining.

Thank you.