Strategy: An Introduction to Game Theory
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Lecture - 48

Welcome to mooc lectures on a Strategy, An Introduction to Game Theory. In this module, we are going to compare evolutionary stable strategy and Nash equilibrium.

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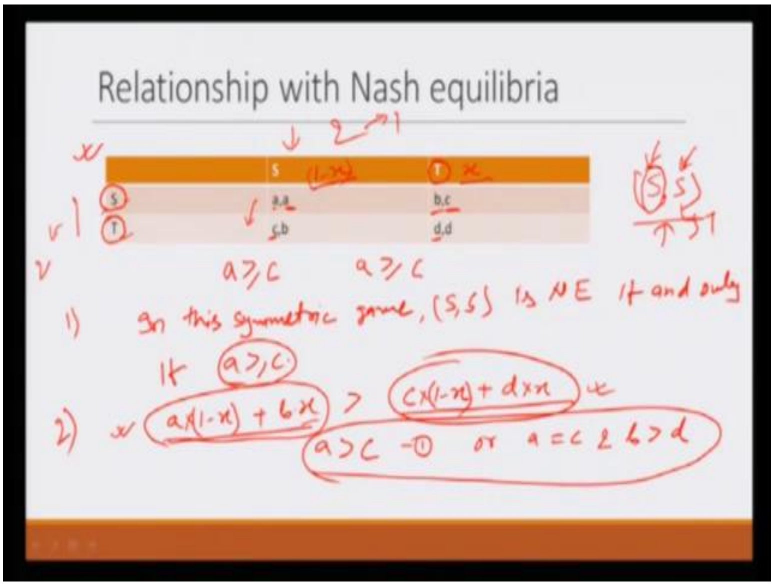

You must be thinking, how can we compare strategy with an equilibrium, after all equilibrium in game theory means a strategy profile. Meaning, that in equilibrium we have one is strategy for all the players. But, here whenever we are talking about evolutionary game, we are talking about symmetric game. What do we mean by a symmetric game? Let us say, that in games in which players identity is not important.

Let us say, here we have player 1 and here we have player 2. Here, even if we call this player, player 2 and this player 1, nothing would change; payoff is corresponding to a particular strategy. So, in symmetric game the identity of the player is not important and payoff to the strategy is always the same. So, here in evolutionary game, what we are saying, what we are getting that evolutionary stable strategy is one that survives the invasion from any mutation.

So, here, if we are saying S is the evolutionary stable strategy, then basically we can think of a corresponding strategy profile as S S, to which we will compare the Nash equilibrium. Now, let us say, if S S is a Nash equilibrium, then what is the condition required, it means that player 1 best response is S, given player 2 is playing S. And what does this best response mean; that player 1 would not like to deviate, would not like to change his strategy given the other player is playing S.

And when would a player not like to change his strategy? When by changing his strategy, he does not expect to gain anything. So, this is possible given that other player is player S, what can player 1 do, either play S or play T. So, from S, he can deviate to T, but when this deviation is not a good idea, when a is greater than or equal to c. So, player 1 would not deviate, when a is greater than or equal to c.

Remember, we have already hypothesis that we want S comma S to be the Nash equilibrium of this game. So, player 1 does not have any incentive to deviate to T, when a is greater than or equal to c. Similarly, what can player 2 to do? Player 2 is player S given that player 1 is playing S. When player 2 would like to deviate? When by changing from S to T, he gets strictly greater payoff.

So, what is the condition again, so that he does not want to change from S to T, when this a is greater than this c or greater than or equal to. So, again we get a is greater than or equal to c. So, in this symmetric game S comma S, let me write in this symmetric game S comma S is any, if and only if, a is greater than or equal to c. This is the requirement.

Now, if we want to see, what is the condition, so that S is evolutionary stable strategy. What does it mean? That S is able to survive and attack an invasion from a mutation strategy called T. So, let us say that population is earlier entirely made of animal, which who have hardwired the strategy S. Now, let us say a mutation T appears and they invade this population of S at label x and x is typically, we take it as a very small.

So, when this invasion is not possible? When S is evolutionary stable strategy, when the fitness of S in presents of this is small t is greater than the fitness of on average an animal having this T strategy. So, let us say how much is the payoff of S, payoff of S would be a multiplied by 1 minus x. So, a multiplied by 1 minus x plus b multiplied by x, this is the payoff to strategy S.

How much would be the payoff to a strategy T? Again an animal having T strategy would encounter an animal having S strategy with probability 1 minus x. So, in that case, the payoff is c. So, c multiplied by 1 minus x and with probability x, an animal having strategy T or characteristics T would encounter the other animal having the same characteristics T is x and in that case payoff is d, so d multiplied by x. So, S is evolutionary stably strategy, if we have this entity is greater than this entity.

Notice, that this has to be true for sum x, so even let us say b and d are very, very high number. What happens we can take x as a very small number, let us say 0.00001 or so. So, ultimately this identity will be greater than this, in case when a is greater than c. So, this is the one condition we obtained, but there also, there is another condition in which the left hand side is greater than right hand side.

And what is that condition, let us say if a is equal to c, in that case, if b is greater than d, then we will have this side left hand side greater than right hand side for all values of positive x. So, these are, a is equal to c and but then, we need b has to be greater than d. So, what do we obtain? That in symmetric game S comma S is a Nash equilibrium, if and only if a is greater than or equal to c, of course, a, b and c are given in the biometrics stable described above. And similarly in the same game, S is evolutionary stable, if a is greater than c or a is equal to c and b is greater than d.

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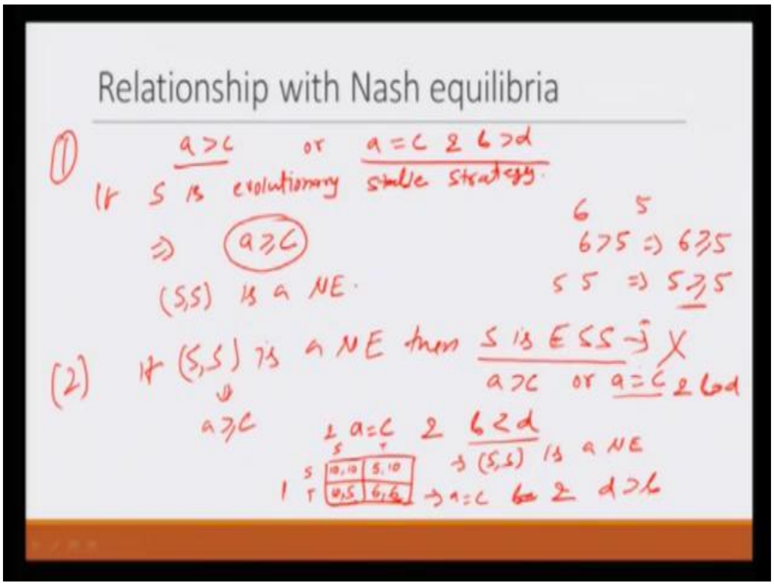

If we pay attention to these two criteria, they are very similar, because they both contain this relationship between a and c. Now, what we have to see that, if S is evolutionary stable strategy, then can we say that S comma S is definitely a Nash equilibrium. So, if S is evolutionary stably strategy, then the condition is that a is greater than c or a is equal to c and b is greater than d if S is evolutionary stable.

Both, either this is true or this is true, in both the cases, it is clear that a is greater than or equal to c. When a is greater than c, it implies that a is greater than or equal to c and when a is equal to c, then it also again implies that a is greater than or equal to c. For example, we can say if 6 and 5 we are comparing, if 6 is greater than 5, then it also implies 6 is greater than or equal to 5.

And similarly, although 5 and 5 are equal, but when we write 5 is greater than or equal to 5, it is not wrong, it is true, because 5 is equal to 5 and what this is saying, 5 is either greater than 5 or 5 is equal to 5. So, both are two, so we obtain this result. So, this implies if S is evolutionary stable strategy S comma S is a Nash equilibrium. This is what we obtained.

Now, can we say if S comma S is a any, then S is evolutionary stable strategy, how about this is a statement, let us check, if S comma S is Nash equilibrium, then we obtain a is greater than or equal to c. If we want S to be evolutionary stable strategy, then what is the requirement either a is greater than c or a is equal to c and b is equal to d. Now, we can always construct an example in which a is equal to c and b is less than d.

Let me write here, a is let us say 10, b is let us say 5, c is as we have to say a is equal to c, so c is 10 and what we need b should be less than d, so d should be 6. I have written only the payoff of player 1 and here we have player 2. If we transpose this matrix, then we will obtain the payoff of player 2 and that would be 10, here we would have how much 10, here we will have 5 and here we will have 6.

In this let us say this is S T, S T, S comma S is any, but S is not evolutionary stable y, because 10 is equal to 10, but 6 is greater than 5 a is equal to c, but d is greater than b. What is the requirement for S to b evolutionary stable strategy; that if a is equal to c, then b has to be greater than d, but this is not what we obtain. We obtain here that a is equal to c, but and d is greater than b.

So, we cannot say that if S comma S is the Nash equilibrium, then S is evolutionary stable strategy, this statement is false. So, they are not equivalent, it only means that one implies evolutionary stable strategy implies the Nash equilibrium, but other way around is not true.

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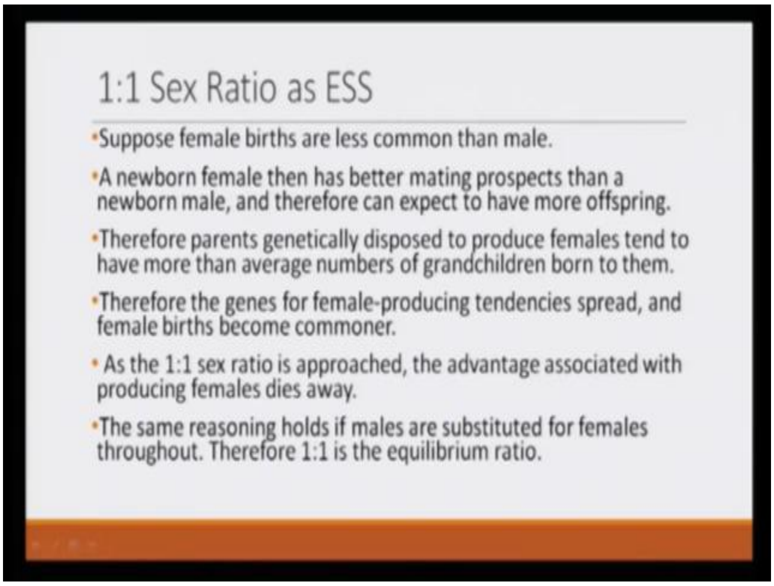

Now, let us do two examples, just to wrap up this chapter and we will do one example using the sex ratio. Typically, what we observe is that, if all the animal world, most of the animals have or present in sex ratio of 1 is to 1. So, let us say there are in any specious, there are two types of animals, one which would on average give more birth to females or other which on average would give more worth to males. It may sound wrong, but let us, but approximate is that, there are two types, one only gives birth to male and other gives worth to only female.

I know this is not true, but just for the simplicity of argument, let us assume. The argument would not change, if we replace gives only birth to female to an average move birth to more number of females, it would remain the same. So, let us assume that female births are less common than male. It means the type which gives birth to male are in proposition higher than half in the population.

What would happen that an new born female, then would have better mating prospect, because for one female, there would be more than one male and therefore, we can expect that female would have more off spring, more children, more kid. Therefore, the parents genetically dispose to produce female tend to have more than average numbers of grandchildren whole born to them.

So, this tendency that this attribute this characteristics giving worth to female will spread in the population and there proportion in the population would rise and eventually, what would happen that 1 is to 1 sex ratio would approach. So, the advantage associated with the producing female would die away. The same reasoning would hold true, it males are substituted for female throughout; that is by an equilibrium, we see 1 is to 1.

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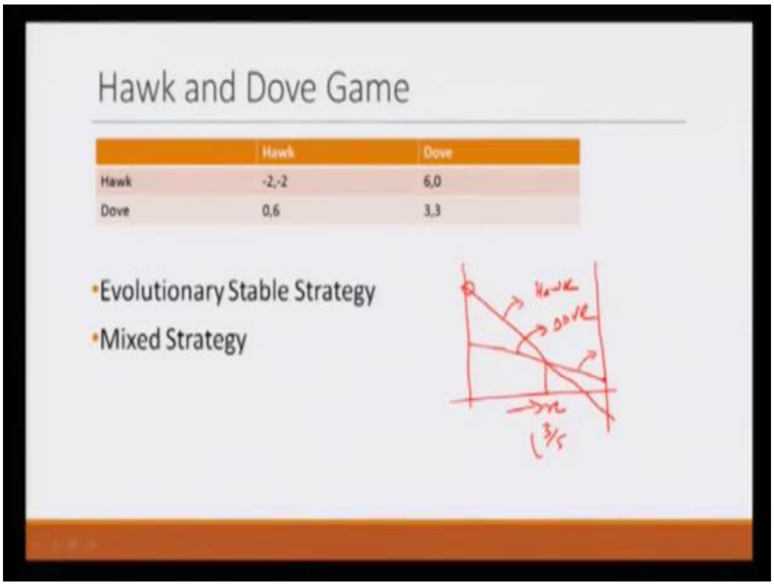

Other example I just want to touch briefly, we started the evolutionary game theory by taking this example of hawk and dove game. Can we say that hawk is an evolutionary stable, remember we have some, I do not remember the value, but we had this kind of payoff table we had. This was for hawk and this was for dove and this is the x increase in x.

Notice that, none of hawk or dove are evolutionary stable. So, if you have only a population of dove and if you introduce some hawk, what would happen, the hawk would do on average better then dove, hawk would have more fitness. So, they would spread in the population and eventually, the dove it would not be stable the population would not be only with dove type.

Similarly, if we start only with hawk type and we introduce the dove type, dove type, this is this one for dove. So, dove type would do better than a hawk type. So, their proportion in the population would increase. So, we can say in hawk and dove game, there is no purest strategy; that is evolutionary stable, but we have already seen that, there exist. If I remember correctly, the ratio was 3 by 5 in the particular example that we had given, that 3 by 5 of one type and two by 5 of the other type; that is stable; that we have seen.

So, to bring that in to this discursion, we have to define something like mixed evolutionary stable strategy. So, what do we mean by mixed evolutionary stable is strategy, it can be explain in two different manner, one that population is entirely made of two different types of animal, one are hawk type, another are dove type. And this particular mixed is a stable in the sense that if population of one of the fraction of one of the type goes up, the population would ultimately come back at the original place.

So, that is one interpretation, second interpretation would be that, there are different animals and the mixed there is strategy like sometime, they play hawk type and sometime play, they play dove type. So, here what it means that only this particular mixing is a stable. In the sense that if a player starts, a player, there is a new type of the animal, which mixes in some other proportion that would not be able to invade, the population which was mixing with radio 3 by 5 and 2 by 5.

It would be a nice exercise for you to do and check using this particular definition of mixed evolutionary stable strategy. So, in hawk and dove game, what do we see that none of this strategies, purest strategies are evolutionary stable strategies, but there exists one mixed evolutionary stable strategies; that is it for the evolutionary game theory.

Thank you very much.