Strategy: An introduction to Game Theory
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Lecture – 47

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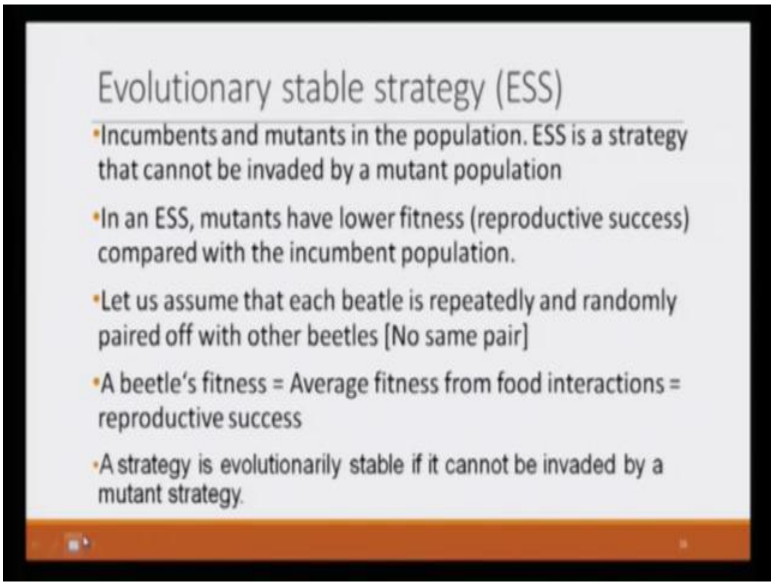

So, let us talk about Evolutionary Stable Strategy in little detail. What we have? Now, let us say that there are two types, one is incumbent type, which was present earlier and now there is also a new type mutant type in the population. ESS or Evolutionary Stable Strategy is a strategy that cannot be invaded by a mutant population, what do we mean by engage and that would become clear.

If an animal, if a particular characteristics has it is evolutionary stable, what it means that animals having this particular characteristics with, will have higher fitness in comparison to of the same kind of animal with different characteristics. So, if the animal will have higher fitness, they will have higher reproductive success and they would sustain in the population. What can we do here, in this size game we can say that each beetle is repeatedly and randomly paired off with other beetles, because we want to know average fitness.

So, average fitness would be determined over longer period of time through which, because the beetle would interact with different kind of beetles, invasion would become clear soon.

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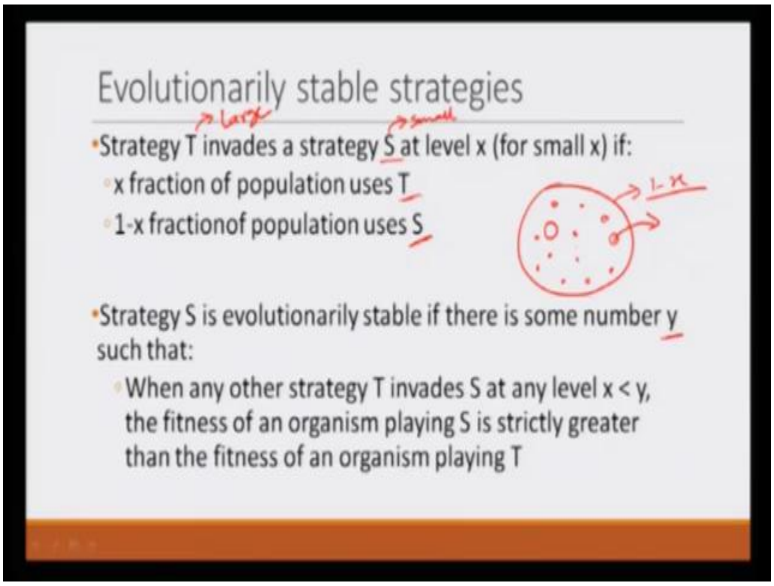

Now, what we say strategy T invades a strategy S at level x, if x fraction of population uses T and 1 minus x fraction of population uses S. What we mean? Let us say, that beetle world is entirely made of a small beetles of course, my poor picture, but and this is the entire population. And, now let us say x fraction of large beetles are introduced that is what we are saying, that here strategy S is we can says small and here we say strategy T is large.

So, strategy large invades a strategy S at level x, if among the whole population 1 minus x fraction of population uses S that is small and x fraction of population uses T. So, formal definition, strategy S is evolutionary stable if there is some number y, this number y is not very important it just says that there exist some x, which is greater than 0. And when any other strategy T invades S at any level x which is less than y, the fitness of x is strictly greater than the fitness of an organism playing T.

Let us, understand it little bit, what is happening here, we have already talked about the invasion. Now, what happens that if average fitness of a small beetle is higher than the average fitness of large beetle, then an average small beetle would have higher reproductive success. Their proportion in the population would increase, it also means that the proportion of population of large beetle would decrease, x is anyways small and eventually it will become equal to 0.

So, over the period small beetle would be able to withstand an invasion of large beetle. If a strategy is able to withstand such invasion, then that strategy is called evolutionary stable strategy.

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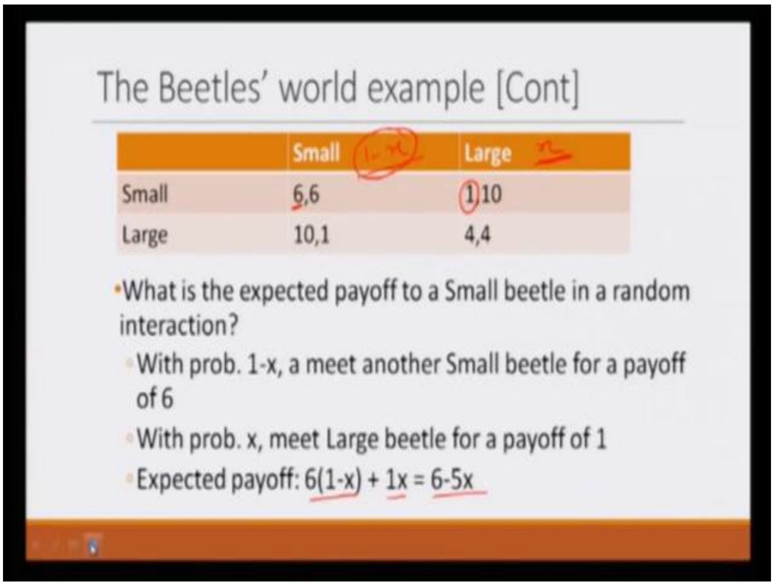

Now, let us do it mathematically, in this particular size game can we call small an evolutionary stable strategy, let us check. What we say, that small is already present and large is invading, so after the invasion large is with proportion x and small is with proportion 1 minus x. When can be call that small is evolutionary stable, when on average in this new environment new, what is the new environment that a small beetle are of proportion 1 minus x and large beetles are of proportion x.

In this new environment, if a small beetle does better than large beetle then of course, small would be evolutionary stable. But, if in this changed environment if large does better than small, then we cannot say small is an evolutionary stable strategy, so this is what we have to check. So, again repeating the same thing 1 minus x and x, what would be the expected payoff of a small beetle.

Let us say in any interaction, if small beetle with interacting with the any other beetle, what is the probability that other beetle is also small 1 minus x, because 1 minus x proportion of the beetles are small. So, that beetle would obtain 6 with probability 1 minus 6 and 1 with probability x. So, what would be the expected value on average?

What we can say that small beetle would obtain 6 multiplied by 1 minus x plus 1 x, so it would be 6 minus 5 x.

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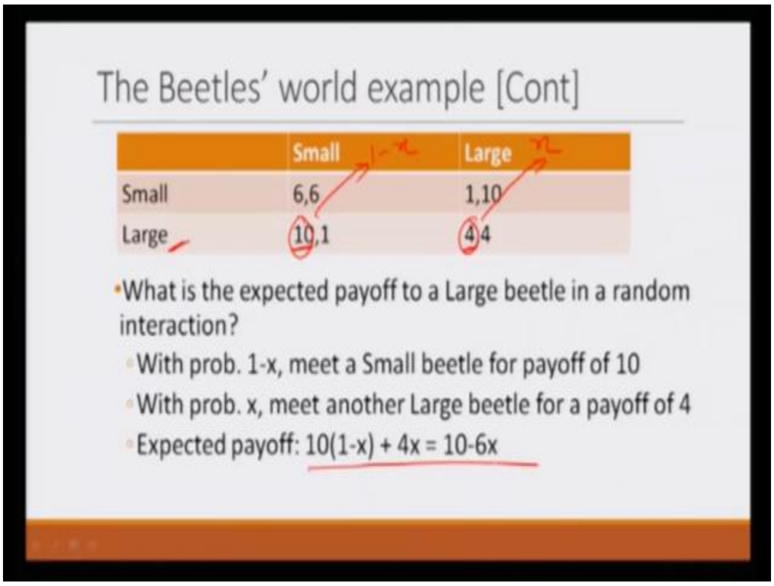

What would be the expected payoff to a large beetle in this new changed environment? With probability 1 minus x, a large beetle would meet another small beetle; in that case payoff is 10 for the large beetle and with probability x this large beetle would again meet a large beetle. So, in that case payoff is 4, so how much is the expected payoff, 10 multiplied by 1 minus x plus 4 multiplied by x, so we get 10 minus 6 x.

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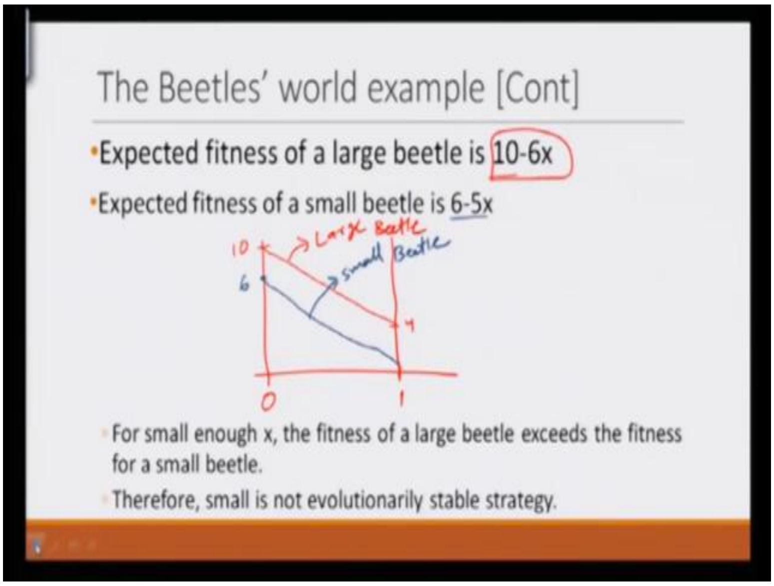

So, expected fitness of a large beetle in this environment is 10 minus x unexpected fitness of small beetle is 6 minus 5 x. We can draw the graph and see, we can compare than the average fitness, what we have extend change from 0 to 1. So, let us draw for the large beetle first, when x is equal to 0, it means we do not have any other large beetle in the population, this is just the one.

Then, the expected payoff is 10, why, because each time it interact with the other beetle, other beetle is definitely going to be a small beetle and it would capture all the resources, so in that case 10. And, if x is equal to 1, that means the whole population is made of large beetle. So, this beetle will always be interacting with the large beetle, in that case we have already seen the payoff is 4, from here also we will get 6 multiplied by 1, so 10 minus 6 is 4, so let us say here is 4.

And, since it is a straight line we just need two points to draw this line and of course, this is the, this is for the large beetle. Now, let us do for the small beetle, when x is equal to zero means the whole population is made of small beetle, so small beetle would always interact with the small beetle and the payoff is going to be equal to 6, which is also clear, if you put x is equal to 0, this will be equal to 6, so this is 6.

Of course, it is not to the scale, so do not worry the only thing is that it is lower and when x is equal to 1, in that case payoff is 1, so this and this is for small beetle. So, what do we see, that a large beetle always does better than small beetle, so no matter how small proportion of large beetles are introduced in the population or they get introduced on their own, they would have better fitness on average. And, so over the time their population would increase and they would eventually, why pound the population of small beetle.

So, starting with small beetle population you will get the large beetle population. So, clearly small is not evolutionary stable, that is what we write that not only for any x, for all x the fitness of large beetle exceed the fitness of a large beetle, therefore small is not a evolutionary stable strategy.

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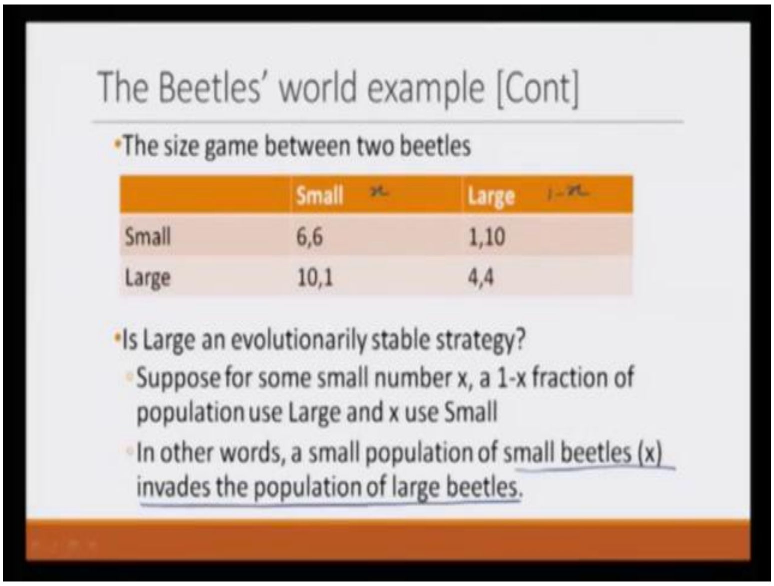

Now, the next question is a very similar, can we say that large is large an evolutionary stable strategy. What do we need to do? We cannot use this distribution that we are having here, 1 minus x and x, what do we have to do, we have to start again and say that population is entirely made of, let us say large beetles and a small population of small beetles are introduced or they get introduced.

Let us say in the new set up, their proportion is x and for the large proportion is 1 minus x. Now, we have to do again in this new environment, how a large beetle would do or ((Refer Time: 10:39)) a small beetle, so we have to calculate. Basically, what is happening here, that a small population of small beetles invades the population of large beetles.

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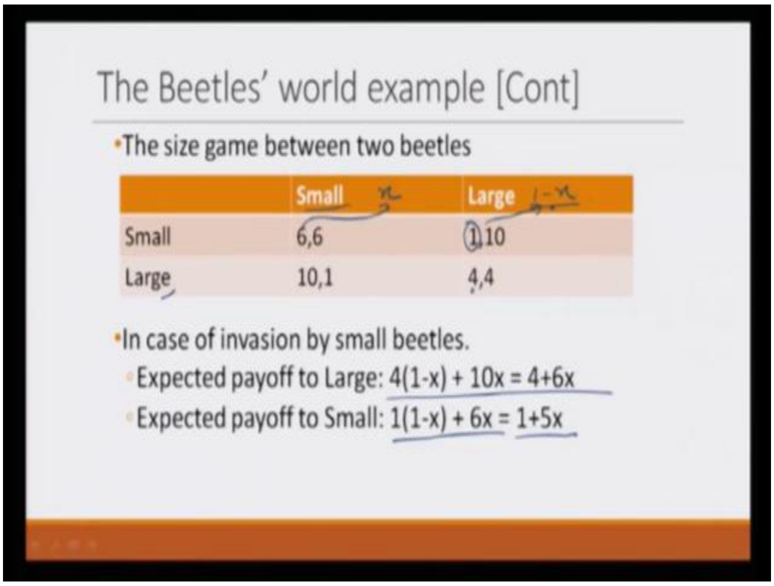

So, payoff would be very similar, here we have 1 minus x x, a large beetle would get, a large beetle would make large would interact with other large 1 minus x proportion of times. So, it is going to be how much 4 multiplied by 1 minus x and x proportion of time it would be interacting with the small beetle, so the payoff would be 10 x the sum is going to be 4 plus 6 x.

Similarly, expected payoff to small or expected fitness of a small is going to be 1 multiplied by 1 minus x coming from here, 1 multiplied by 1 minus x and 6 multiplied by x, if we sum these to up we will get 1 plus 5 x. So, what do we get in case of a invasion by a small beetle the expected fitness of a large beetle is 4 plus 6 x and expected fitness of a small beetle is 1 plus 5 x.

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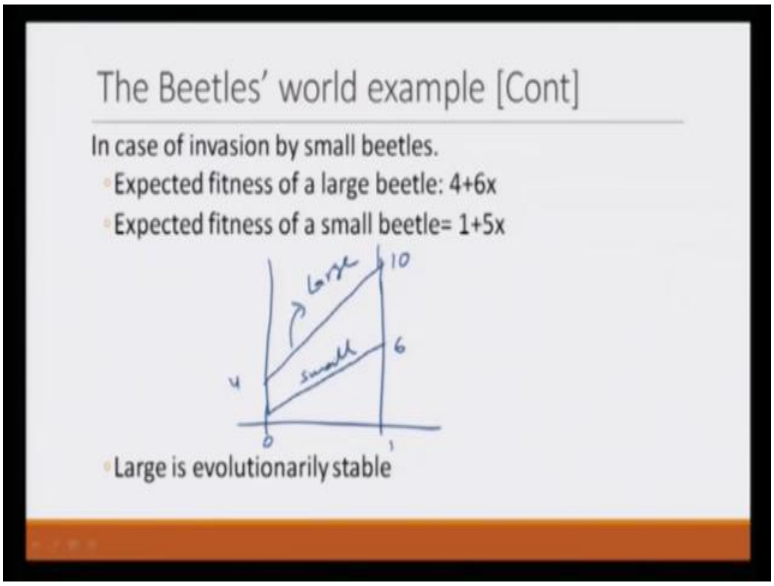

So, if we draw, which are be x x, where is from 0 to 1 for the large beetle is starting from 4 it goes up to10 4 to 10 and for the small beetles starting from one it goes to 6 this is for large beetle and this is for small beetle. Large beetle always has better fitness than the small beetle, so small beetle would have not be able to invade the population of large beetle, so large is evolutionary stable.

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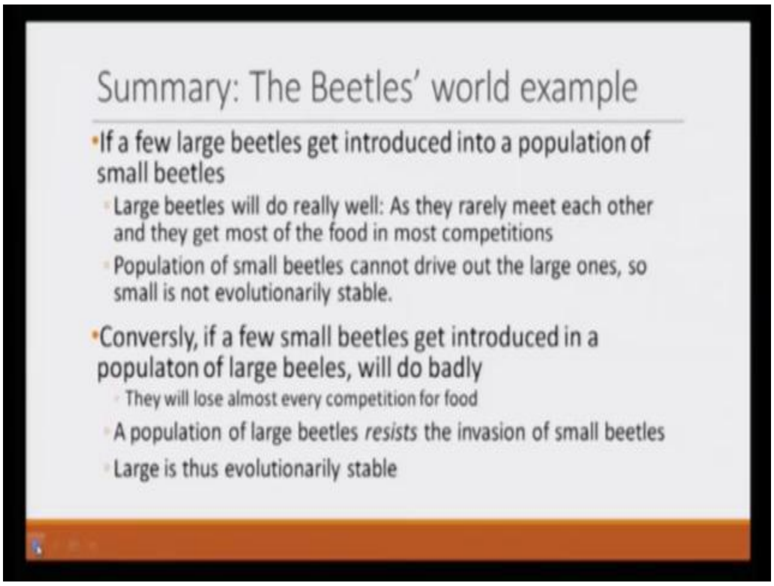

So, let us some rise, what we have learn, that if a few large beetle get introduced in to a population of small beetles large beetles will do really well. Why? As they rarely meet each other and they get most of the food in most competition, because most of the competitions are happening with the small beetle, so they do really well. And, population of small, therefore population of small beetles cannot drive out the large once, so that small is not evolutionary stable.

What happens? If the converse is true, if a few small beetles get introduce in a population of large beetles, they do very badly. Why? Because, most of the competition they will have large with large beetles and they will do worst the large beetles. So, a population of large beetles will be able to resist the pop invasion of small beetles, therefore large is evolutionary stable.

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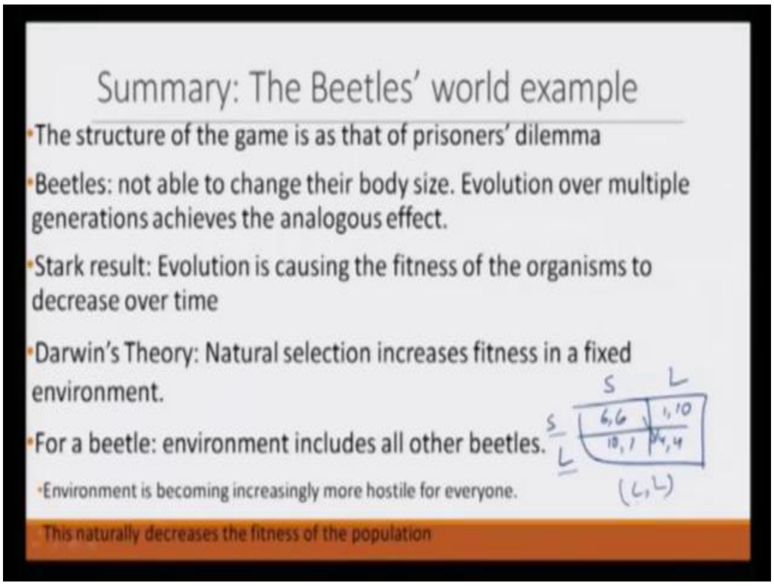

If, you pay attention to this particular game, let me draw it for you 6 comma 6 4 comma 4 1 comma 10 and 10 comma 1 this is small this is large this is small this is large. If you look at it that is same as prisoners', but dilemma notice, we did not use that logic, we cannot use this logic. Because, beetles cannot rationally decide to becomes small or large they cannot strategically decide, but if you just look at the payoffs, then it is same as prisoners' dilemma, here S l is strictly dominates S.

So, L comma L is the only Nash equilibrium in this case, because both the players up, here I have by mistake written m this is L, so for player 2 also L is strictly dominates S. So, we have, in fact dominant strategy equilibrium and which is of course, the Nash equilibrium, but we cannot use the same logic. Now, if we pay attention to other fact one more think, that we learn, that over the period fitness average fitness is decreasing from 6 to 4.

We begin with that average fitness is 6 and now the average fitness is 4. So, that is also problematic, because, what we have learn from evolutionary biologic that is not very important for game theory by just to complete the chapter, we should say that natural fitness is not increasing over the time. So, where did Darwin go wrong or, where did we go wrong the thing is Darwin did not go wrong, because on Darwin talked about that natural selection increases fitness in fixed environment here.

The environment is not fixed environment itself is changing environment itself is becoming those style for everyone. So, which is naturally decreasing the fitness of the population, so it does not contradicts that Darwin's theory. That is it will again in the next module, we are going to compare evolutionary stable strategy with Nash equilibrium. And, we would also check what happens in the case of hawk and dove game, which one is evolutionary stable.

Thank you.