

Strategy: An Introduction to Game Theory
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Lecture - 34

Hello, welcome to another module in this Massive Open Online Course, Strategy An Introduction to Game Theory. Let us now consider looking at our, the Bayesian version of the battle of sexes game. We have derived the average payoffs of the boy in the last module for the different strategies of the different action combinations of the girl player of each type.

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Average Payoff Table for Boy:

	(C,C)	(C,H)	(H,C)	(H,H)
C	10	5	5	0
H	0	$\frac{5}{2}$	$\frac{5}{2}$	5

Possible Actions of Boy Possible strategy combinations for girl.

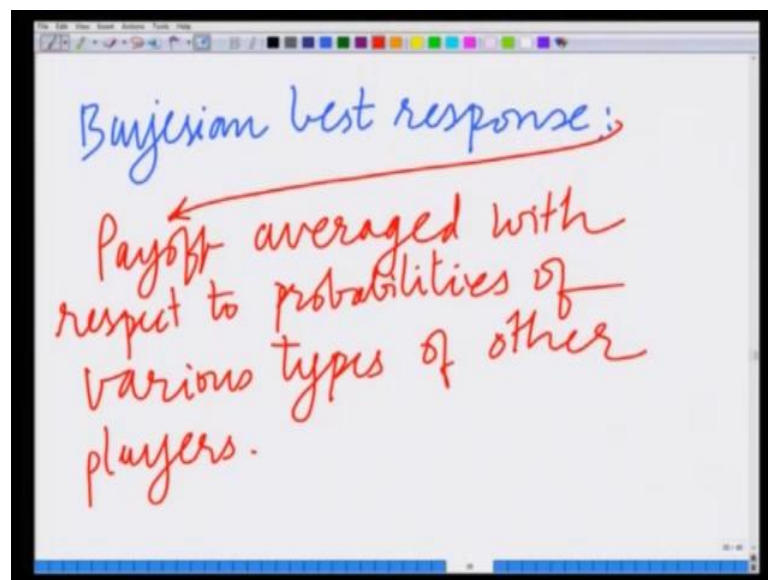
And now, let us try to see or let us try to invert, what are the best responses of the boy and that is for looking at this table; that is fairly simple. Because, if the girl player is choosing C comma C, then the best response of the boy is to choose C, because C gives him a payoff of 10, while H gives him a payoff of 0. If the girl is choosing C comma H; that is girl of type I is choosing C, girl of type U is choosing H, then the best response of the boy is again to use C, because C gives him a payoff of 5, while H gives him a payoff of 5 by 2.

Again, if the girl is choosing H comma C, then the best response of the boy is to again choose C, because C gives him a payoff of 5. While, if the girl is choosing H comma H; that is girl of type I is choosing H, girl of type U is choosing H, then the best response of

the boy is to choose H, which gives him a payoff of 5. So, what we have done now is we have illustrated the best responses of the boy.

So, if the girl is choosing C comma C, best response of the boy which gives him higher average payoff is to choose C. If the girl is choosing C H, best response of the boy is to choose C, girl is choosing H comma C, again best response of the boy is to choose C. However, if the girl is choosing H comma H, best response of the boy which gives him the higher average payoff is to choose H. So, we have figured out the best responses of the boy with respect to his average payoffs, average with respect to the types of the other player or the player 2.

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So, what we have done so far is, we have figured out the Bayesian best response. What is this Bayesian best response? That is, payoff average with respect to probabilities of various types of other players; that is what we are trying to figure out is, we are trying to figure out the payoff of the player boy; that is the average payoff of the player boy. Average using the probabilities corresponding to the different types of the other player and his payoff corresponding to the actions of the different types of the other player, therefore we have computed average payoff for the boy.

And we have seen, what is this Bayesian best response in this average sets. Now, let us go back, let us go and try to figure out, the payoff of the girl player. Now, remember girl,

the player of each type, the player knows his or her own type. So, the girl of type I knows her type, girl of type U knows her type.

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Bayesian BOS:

	C	H
C	10, 5	0, 0
H	0, 0	5, 10

$P(I) = \frac{1}{2}$

	C	H
C	10, 0	0, 10
H	0, 5	5, 0

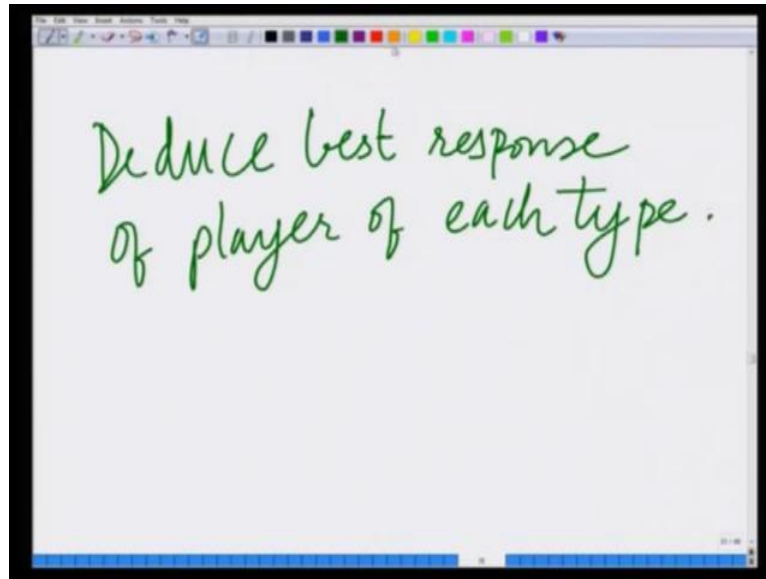
$P(U) = \frac{1}{2}$

So, to figure out the payoffs of the girl of each type, let us go back to the game table, from the game table we can see that, if the girl is of type I, again we use the standard technique to figure out the best response. Because, there is only one type of boy, if the boy is choosing C, then the best response of the girl is to choose C. If the boy is choosing H, then the best response of the girl is to choose H, because H gives her a payoff of 10, while C gives her a payoff of 0.

On the other hand, if girl is of type U, then, we are in the second table and in this scenario, if the boy is choosing C, then the best response of the girl is to choose H, because H gives a payoff of 10. While, if the boy chooses H, best response of the girl is to choose C, because C gives her a payoff of 5. So, what we have done now is, we have illustrated the best responses of the girl of each type.

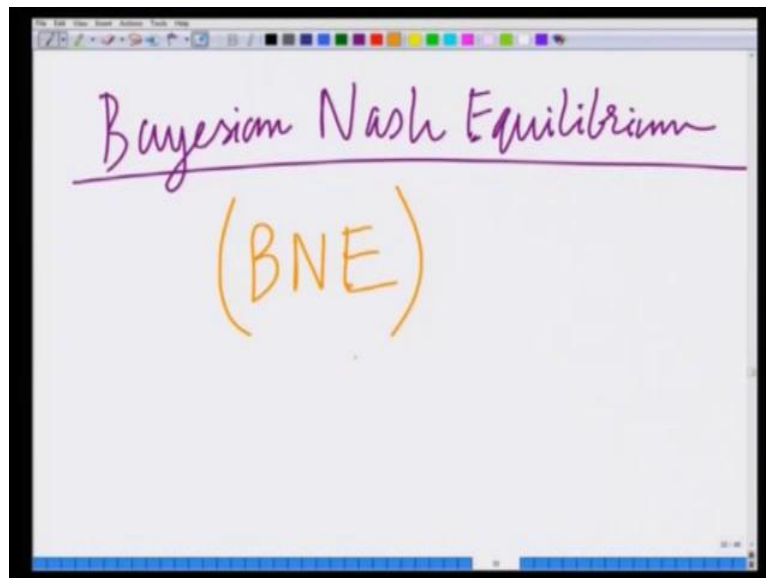
The best response we have done is we first derive the best responses of the girl of type I, followed by looking at the best response of the girl of type U that is, what is the best response of the girl when she is interested or of type I. What is the best response of the girl or player 2 of type U, which is uninterested and we have figured out the best responses of the player of both types. So, we have figured out, so illustrated, so we have to derive best response.

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So, we have to deduce of player of each best response of player of each type. So, we have figured out the best response of girl or player 2 of type I, also derived the best responses of player 2 of type U. So, now, we have to deduce the Bayesian Nash equilibrium of the game.

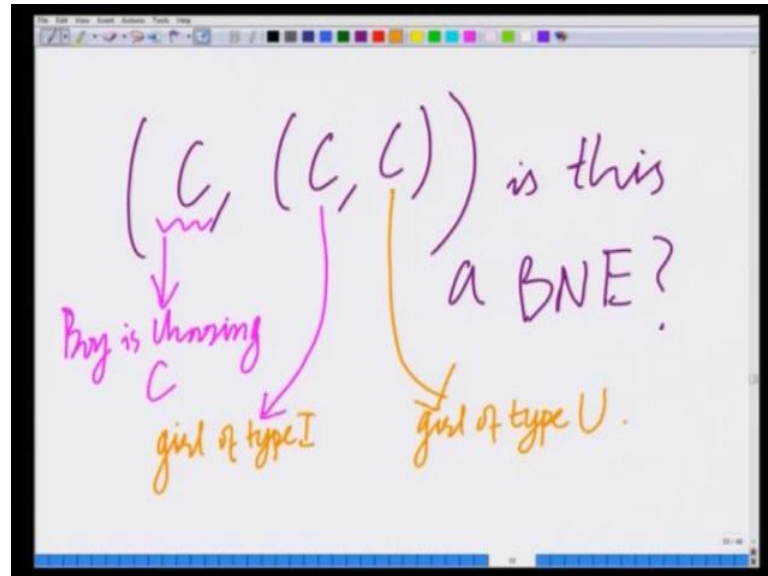
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So, now, the next thing that we have to do is to deduce the Bayesian Nash equilibrium or the BNE of this game. So, what we have to do now is to basically deduce the Bayesian Nash equilibrium or the BNE of this game. Let us go back to the best responses of the

boy, now if you look at the best response of the boy, best response of the boy to C, C of the girl is to choose C.

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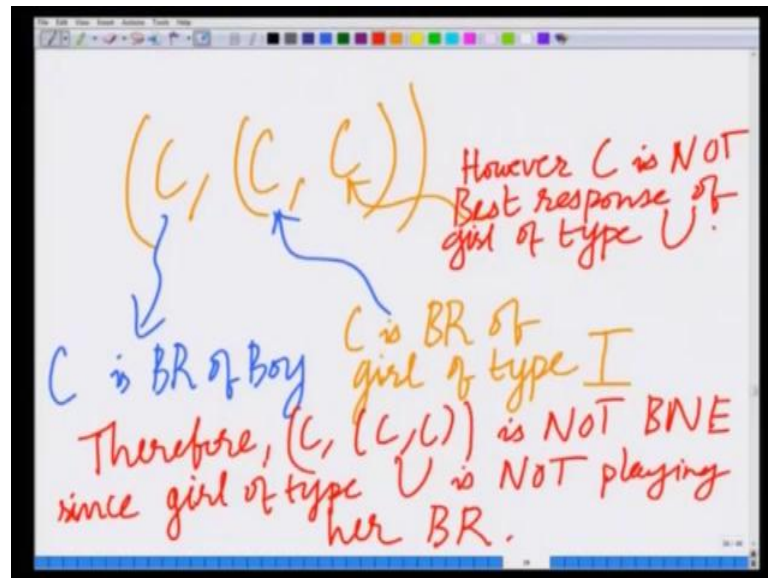


So, C, C, so what we want to do is, let us check, if C comma C, C is this, a Bayesian Nash equilibrium. Remember, Bayesian Nash equilibrium, where each player is playing his or her best response. Now, in C comma C, C; that is where boy is choosing C, girl of type I is choosing C and girl of type U is choosing C. We have seen that C of the boy is the best response to C comma C of the girl; that is where girl of type I is choosing C and girl of type U is choosing C.

And now we have to figure out, if each girl of each type is playing her best response, because we have seen that C of the boy is the best response to C, C of the girl. For this to be a Nash equilibrium, C C of the girl has to also be a best response to C of the boy, which means girl of type I should be playing her best response, girl of type U should also be playing her best response.

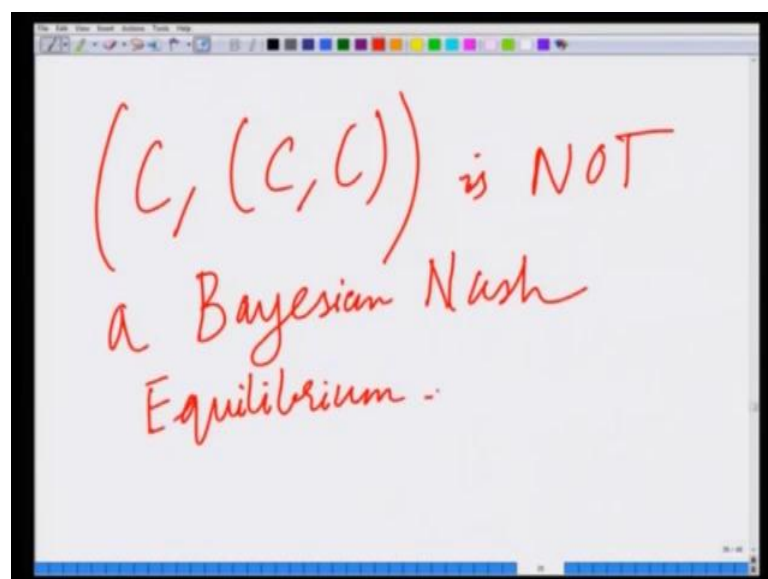
Let us check, if this C is indeed, the scenario ((Refer Time: 08:33)) and we go here and we see from the Bayesian game table is that, if the boy is choosing C, then C is the best response of girl of type I. However, if boy is choosing C, the best response of girl of type U is H and not C, so if boy is choosing C, while C of girl of type I is the best response, C of girl of type U is not the best response.

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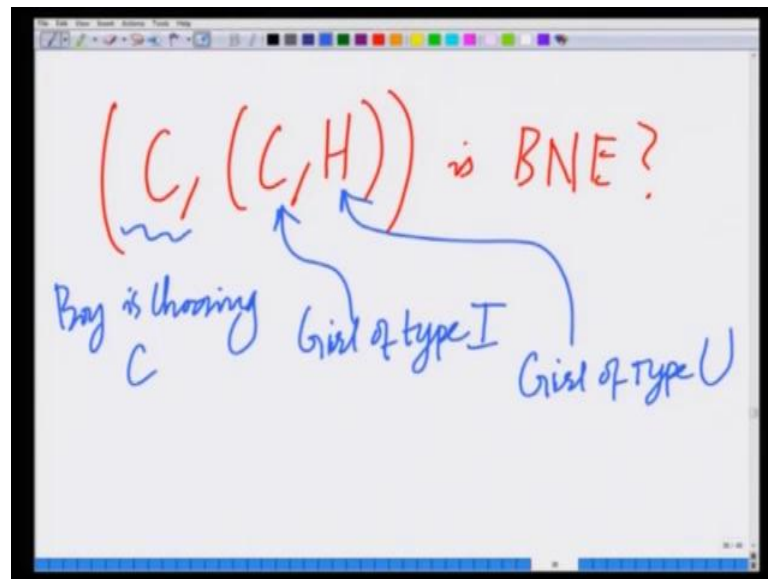
So, in this scenario that is, if you look at C comma C , we have analyzed it that C is best response of boy, C is best response of girl of type I. But, C however, C is not best response of girl of type U. Therefore, C comma C comma C is not a Bayesian Nash equilibrium, because girl of type U is not playing her best response. Therefore, C comma C comma C is not Bayesian Nash equilibriums, since she is not playing her best response.

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So, C, C is C, C, C ; that is, so therefore, this C, C, C is not a Bayesian Nash equilibrium. That is the outcome, where boy is choosing C , girl of type I is choosing C , girl of type H is choosing also choosing C is not a Bayesian Nash equilibrium. Let us now go back to again the table of the average payoff of the boy ((Refer Time: 11:20)). Let us consider C, H of the girl; that is girl of type I is choosing C , girl of type U is choosing H , best response of the boy is to choose C . Let us ask, whether C, C, H is the Bayesian Nash equilibrium.

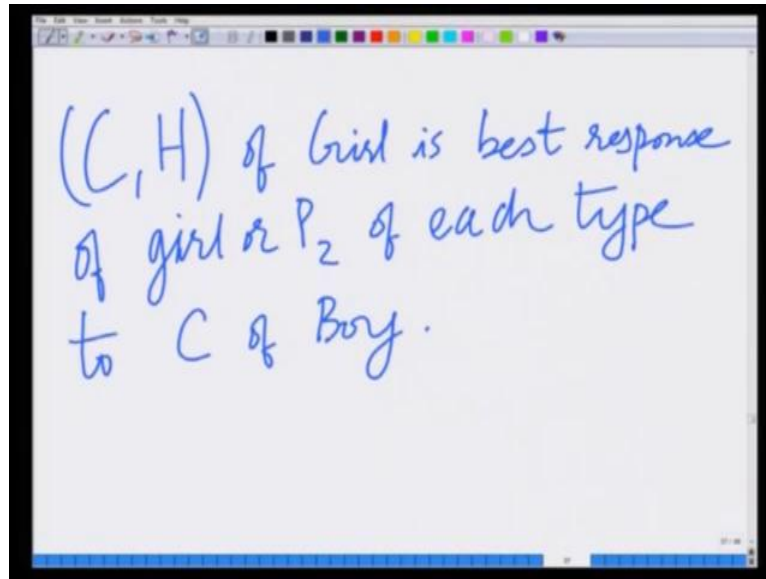
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Let us ask the question, if C, C, H is the Bayesian Nash equilibrium. Well, where boy is choosing C , girl of type I is choosing C , girl of type U is choosing H , well C of the boy is the best response to C, H of the girl; that we have already seen from the average payoff table. Now, we have to see if C, H of the girl; that is where girl of type I is choosing C , a girl of type U is choosing H is the best response to C of the boy.

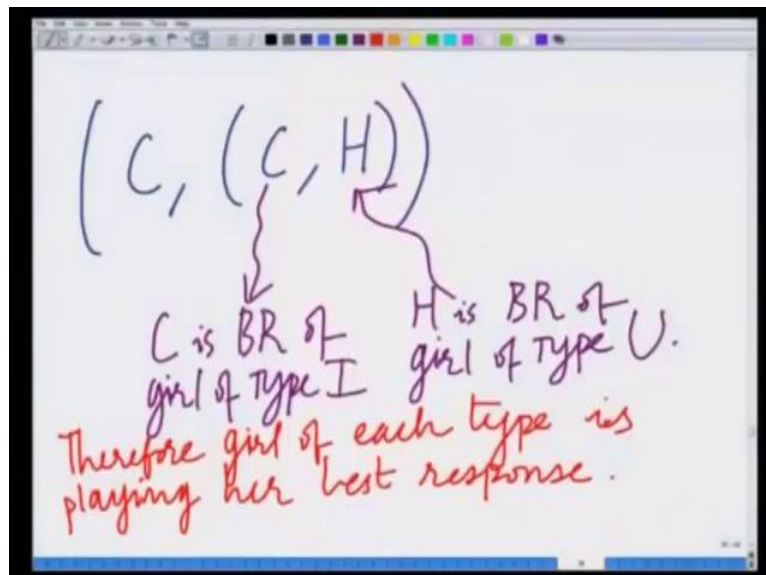
And to figure that out let us go back to our Bayesian game table, ((Refer Time: 12:38)) if the boy is choosing C , C of girl of type I is the best response and if boy is choosing C , H of girl of type U is also the best response. So, therefore, C, H for the girl is the best response for girl of each type to C of the boy.

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So, C comma H, so what we are saying here and you can verify that by looking at the Bayesian game tables is that C comma H of girl is best response of girl or p_2 of each type to C of boy. That is when boy is choosing C, C is the best response of girl of type I and H is best response of girl of type U.

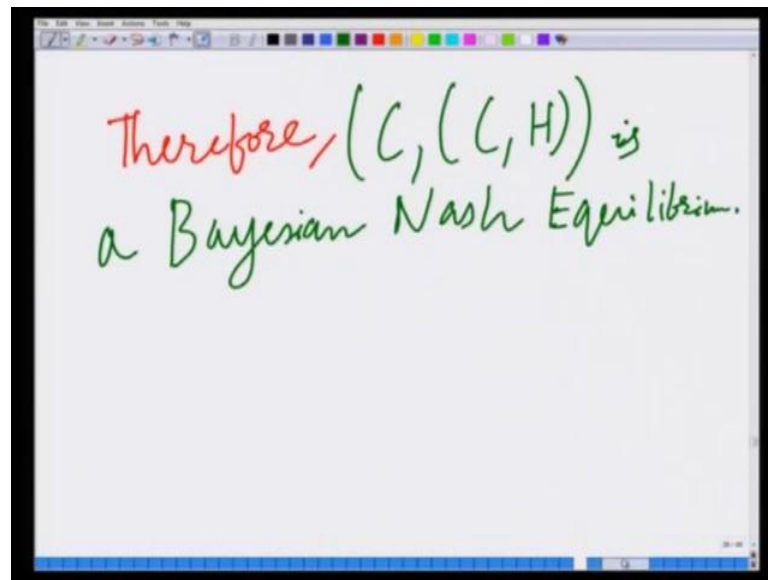
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That is what we are saying this, when boy is choosing C, C is best response of girl of type I and H is best response of girl of type U. Therefore, player 2 of each type; that is girl of each type is playing her best response; therefore, player 2 girl of each type is

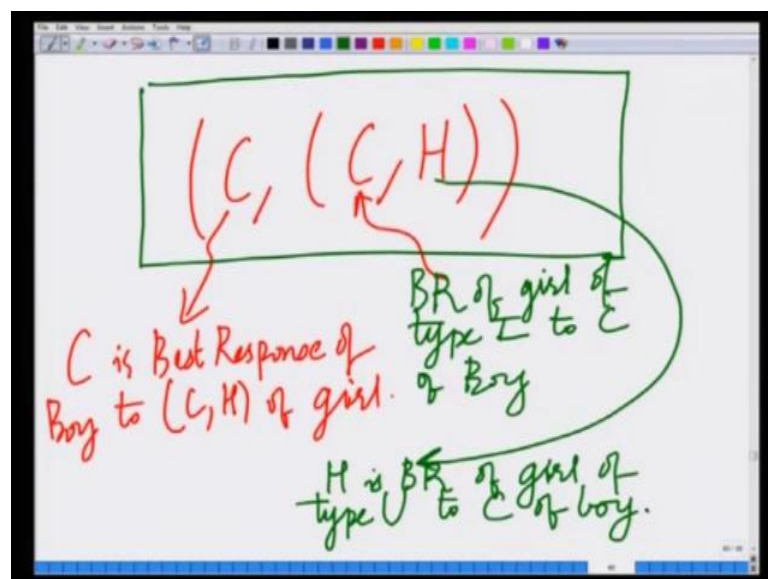
playing her best response. Therefore, if you at look at C comma C comma H , C of the boy is best response to C comma H of the girl, C comma H of the girl is the best response of the girl of each type to C of the boy. Therefore, C comma C comma H is the Bayesian Nash equilibrium.

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Therefore, we can say C comma C comma H is a Bayesian Nash equilibrium.

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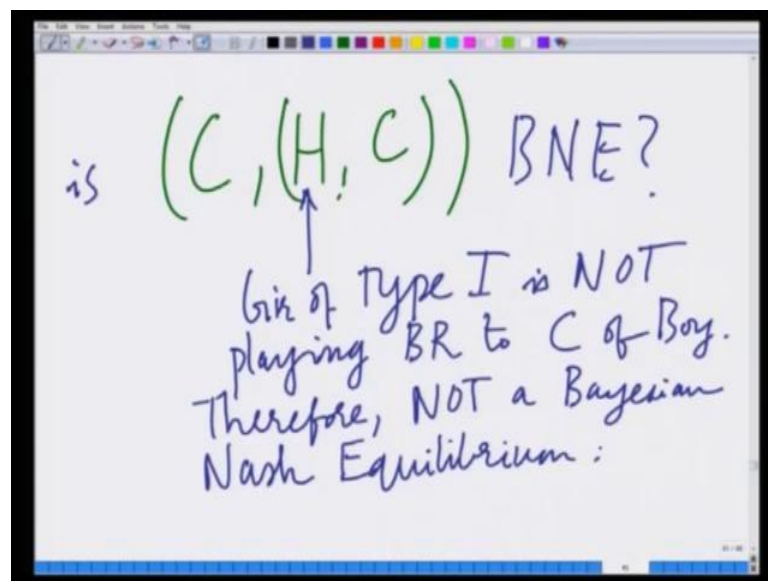
And how do dissect this, we dissect this, we looked at does by saying that basically C , if girl is choosing C comma H , C is best response of boy to C comma H of girl and C best

response of girl of type I to C of boy and H is best response of girl of type U to C of boy. So, this is C comma C comma H is indeed a Bayesian Nash equilibrium, because C of the boy is best response to C comma H strategy C comma H of the girl.

And C comma H is the best response of girl of each type to C of the boy; that is C of the girl of type I is best response to C of the boy and H of girl of type U is best response to C of the boy. So, this is the Bayesian Nash equilibrium in which each player of each type is playing is or her best responses. So, C comma C comma H, this is indeed the Bayesian Nash equilibrium and this is interesting, because now we have considered a game with uncertainty in payoff.

So, where at the boy does not know the exact nature of his operant. Similarly, we can consider other scenarios. Let us consider the rest of the scenarios. For instance, ((Refer Time: 17:36)) if we can look at H comma C of the girl, the best response of the boy is C.

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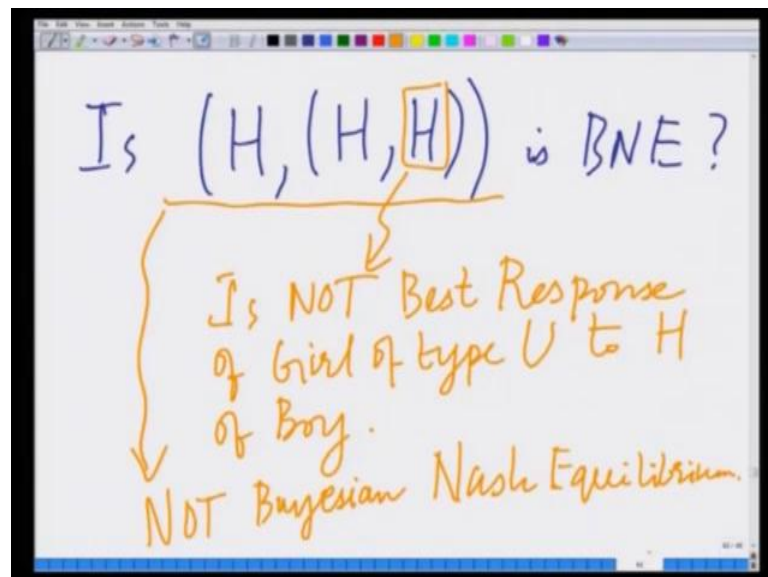
So, we can ask, so we can consider, H comma C of the girl best response of the boy is C. So, if the girl is using H comma C, C is the best response of boy. So, we can ask is C comma H comma C is this the Bayesian Nash equilibrium, well boy is choosing C, girl of type I is choosing H, girl of type U is choosing C.

We can quickly see, try to see from the, ((Refer Time: 18:14)) if this is the best response of the girl, if boy is choosing C best response of girl type I is to choose C not H. If boy is

choosing C best response of girl of type I is to choose C. Therefore, if you go back here, we can see that girl of type I is not playing best response to C of boy. Therefore, this is not a Bayesian Nash equilibrium.

Remember, each girl, why each girl each player of each type has to play her is other best response. If the girl of type I is choosing H; that is not her best response, because her best response to C of the boy is to choose C. So, therefore, this is not a Bayesian Nash equilibrium, how about the other one ((Refer Time: 19:33)) H best response of the boy to H comma H of the girl is to choose H.

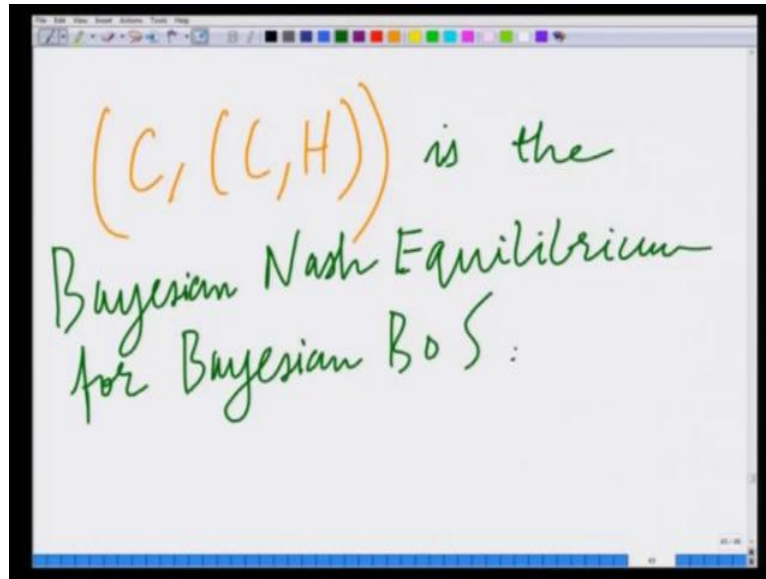
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So, let us check if H comma H comma H is, let us ask the question is H comma H comma H is this a Bayesian Nash equilibrium, well again if you go back to the table, ((Refer Time: 20:02)) you can see that, if boy is choosing H then H is the best response of girl type of I. But, C is the best response of girl of type U, therefore, H is not in the best response of girl of type U to H of the boy.

So, if you go back here, we can see that this H this is not best response of girl of type U to H of best response of girl of type U to H of the boy. So, therefore, this is also not Bayesian, this is not a Bayesian Nash equilibrium. So, we eliminated all possible choices and therefore, we have deduced that the only Bayesian Nash equilibrium of this game is ((Refer Time: 21:23)) C comma C comma H. Therefore, C comma C comma H is only Bayesian Nash equilibrium.

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Therefore, C comma C comma H is a Bayesian Nash equilibrium of this game, so fine when we conclude this, what we are saying is that basically C comma C comma H is the Bayesian Nash equilibrium for the Bayesian battle of sexes game. So, for the Bayesian version of the battle of sexes game in which there is uncertainty for the boy, there is uncertainty regarding the type and hence, the payoffs of the operant player or the other player; that is the girl.

We have to look at a Bayesian this game is Bayesian nature, we have look at a Bayesian Nash equilibrium. And we are saying that the region Nash equilibrium of this game is basically C comma C comma H , were the boy is always choosing C girl of type I is choosing C girl of type U is choosing H , C is the best response of the boy in terms of his average payoff, averaged with respect to the probabilities of girl player of type I and type U .

And C comma H is the best response of the girl in the sense that C is the best response of the girl of type I to C of the boy at H is the best response of the girl of type U to C of the boy. Therefore, C comma C comma H is a Bayesian Nash equilibrium, where each player of each type is playing his or her best response. So, let us stop this module with here with this, I am assuming I given a fairly clearly idea of what the Bayesian Nash equilibrium is about.

First motivated the idea of a Bayesian game, we are looked at the concept of average payoffs and then, we have looked at the Bayesian Nash equilibrium in terms of the best response. But, in terms of the best response of each player of each type; that is a Bayesian Nash equilibrium, it is an enhancement or a refinement or an advanced version of the simple idea of Nash equilibrium; that we have look for none Bayesian games.

That is where each player knows exactly; that is where each player has only a single time and the game table is therefore, precise there is no uncertainty. The Bayesian games are used to model scenarios, where there is uncertainty. So, let us stop this module with here and we will take another example in the next module.

Thank you very much.