Strategy: An Introduction to Game Theory
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Lecture- 03

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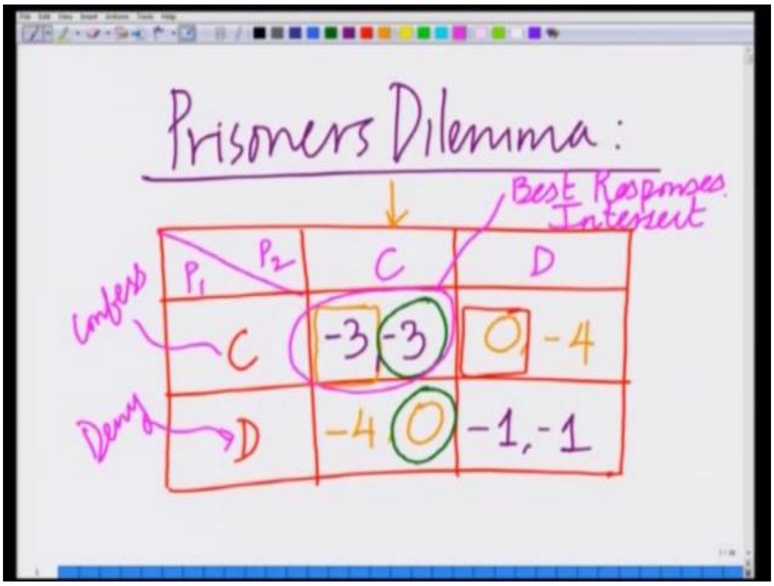

Hello everyone, welcome to another module in this online course strategy an introduction to game theory. And, in the previous module we had seen a simple example of a game, a game that we called or the game that is known as the prisoners dilemma. And, we said this is a simple game. Let me just try to describe this game again for the sake of continuity. Of course, we represented this game as a game table in which we represented along the rows the different possible actions of prisoner 1, along the columns the possible actions of prisoner 2.

And, of course, we said prisoner 1 and prisoner 2 can either confess which is represented. Of course, C stands for confess, D stands for deny. Similarly, C stands for confess for prisoner 2 and deny for prisoner 2. When prisoner 1 and prisoner 2 both confess, of course, each gets minus or 3 year prison sentence which means each gets a pay of minus 3. When prisoner 1 and prisoner 2 both deny each gets minus 1, a prison sentence of minus 1 each.

Now, of course, when prisoner 1 confesses and prisoner 2 denies, prisoner 1 get 0; the prisoner 2 denies gets a 4 year prison sentence which is minus 4. On the other hand, again when prisoner 1 denies and prisoner 2 confesses, prisoner 1 gets a 4 year prison sentence, minus 4; and prisoner 2 walks free he gets 0 years, alright. This is just a brief recap of the game table of the prisoners example of prisoners dilemma that we looked at in the last module.

What we want to do now is we want to try to analyze this game and start to understand how are the prisoners going to behave in this situation. What are the strategies that the prisoners are going to use? What is the thinking that is going to go on in the minds of these prisoners? What are they going to look at this game, look at their options or look at the possible actions that are available to them, and think about how to play this game?

Well, let us start by looking at the thought process of prisoner 1. Prisoner 1 is thinking. Well, let us say prisoner 2 is choosing to confess. If prisoner 2 is choosing to confess which means his action is basically he is confining the game to the first column. If prisoner 2 is confessing we are talking about the first column which corresponds to prisoner 2 confessing.

Well, if prisoner 2 indeed confesses which prisoner 1 by the way does not know about, this is only hypothetical, this is a thought process, this is only hypothetical is hypothesis that prisoner 1 is thinking about. If prisoner 2 confesses then it is better for prisoner 1 to confess. And, that you can clearly see that because if prisoner 1 confesses he gets a 3 year prison sentence. On the other hand, if he denies because prisoner 2 is confessing if prisoner 1 is going to deny then he is going to get a 4 a prison sentence because prisoner 2 is now going to act as a witness against prisoner 1, alright.

So, if prisoner 2 confesses then you can clearly see minus 3, the way to look at it from the game table is minus 3 is better than minus 4. So, the way we express it we say is that prisoner 1 is better of confessing than deny, if prison; and, I am going to mark this by an orange box which is if prisoner 2 confesses prisoner 1 is better of confessing than deny. And, this is known as the best response or BR.

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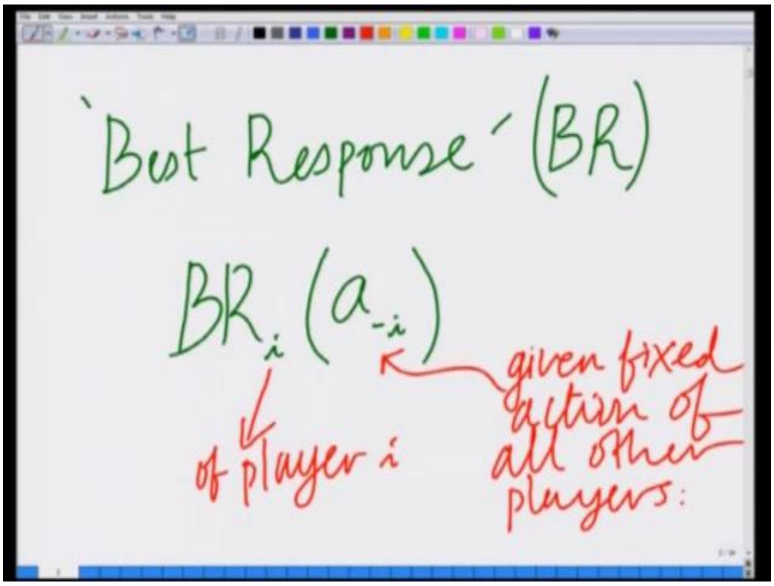

What is the best response? The best response of player i for a fixed action of all, fixed action combination of all, the player, other players. So, we are talking about best response of player i, given the fixed action of all the other players. We are talking about the best response of player i, given the fixed action of all the other players.

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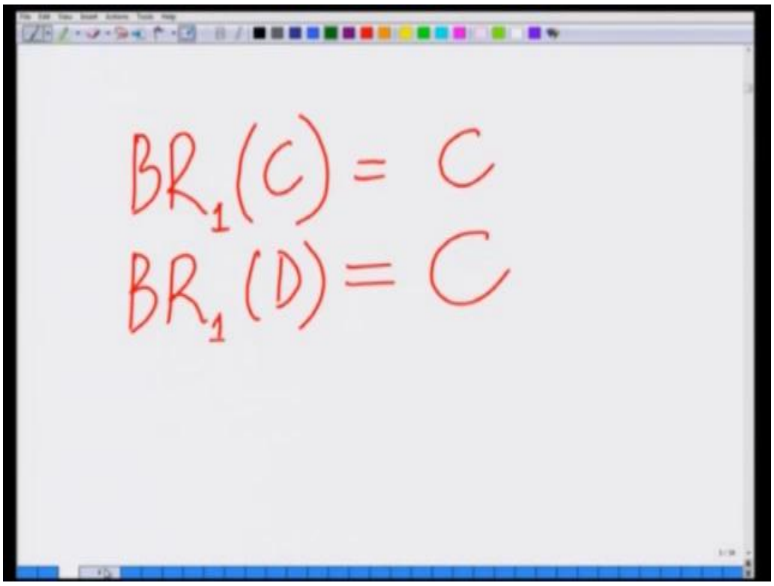

For instance, best response of player 1, what we just illustrated is best response of prisoner 1, given the action of the other player which is to confess is to obviously what we saw is to confess; that is given that prisoner 2 that is other player has confessed best response of player 1 is to confess. Now let us look at if player 2 denies, if player 2 denies if player 1 confesses he gets 0 years in prison. On the other hand, if he denies he gets 1 year in prison.

So, again the best response of player 1 if player 2 is denying under the fixed action that player 2 denying is to confess, right. So, if player 2 is denying confession makes him walk free, denying gives him 1 year in prison. So, basically, the best response of player 1 to deny is also to confess; that is a best response of the player 1 to deny of player 2 is also to confess.

On the other hand, now let us look at the thought process of player 2. We have looked at the thought process of player 1; now let us look at the thought process of player 2. Of course, player 2 is also thinking, if or prisoner 2 is also thinking if prisoner 1 is confessing. Prisoner 1 is confessing in the sense that he is choosing row 1 then if prisoner 2 confesses he gets 3 years in prison. But if prisoner 2 denies because now there is evidence against him he gets 4 years in prison. So, is better of confessing is best response is confessing indeed because this game is symmetric between prisoner 1 and prisoner 2.

And similarly, if prisoner 1 is denying that is he is choosing the second row then again prisoner 2 is better of confessing because confessing makes him walk free; well, denying gives him 1 year in prison. So, again he is better of confessing which is I am marking by the circles.

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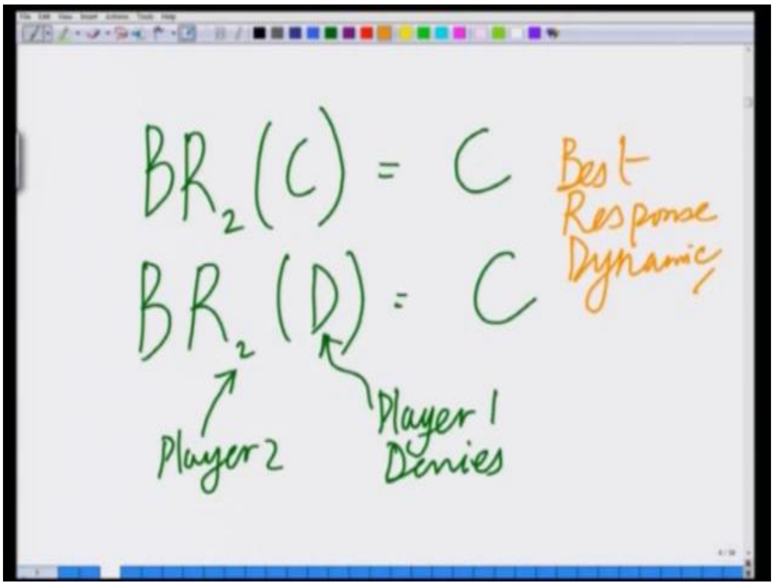

So, his best responses we have already looked at the best responses of player 1, best response of player 2, given player 1 confesses is to confess. Also, here you can see best response of prisoner 2 given player D 2, player 1 denies is also to confess. Best response of player 2 given player 1 denies is also to confess. This is the best response of player 2 given player 1 denies is also to confess, right. So, we have the 4 best responses. This is known as the best response dynamic of the game. This is known as the best response dynamic, right.

Let me just write this down here. This is known as the best response strategy of the best response dynamic of this game. And, what is interesting is if you go back to this game now and you can see now that there is a special box in this game and that special box everyone should be able to see that is where there are there is a both square and a circle in the same box; that is where the best responses are intersecting; this is where the best responses intersect.

That is, in outcome c comma c, prisoner 1 is playing the best response to prisoner 2 because the prisoner 2 is choosing C, best response of prisoner 1 is C. If prisoner 1 is choosing C, best response of prisoner 2 is to choose C. For instance, that is not true in the box D comma C. In the box D comma C, prisoner 2 is playing the best response but prisoner 1 is not playing the best response because if prisoner 2 chooses C best response of prisoner 1 is choose to C not D, right.

That is also not happening in box C comma D. It is also not happening in box D comma C because here none of them are playing their best response. It is only in box C comma C that both the players are playing their best response. This is known as the Nash equilibrium of the game. So, this is one of the fundamental concepts of a game.

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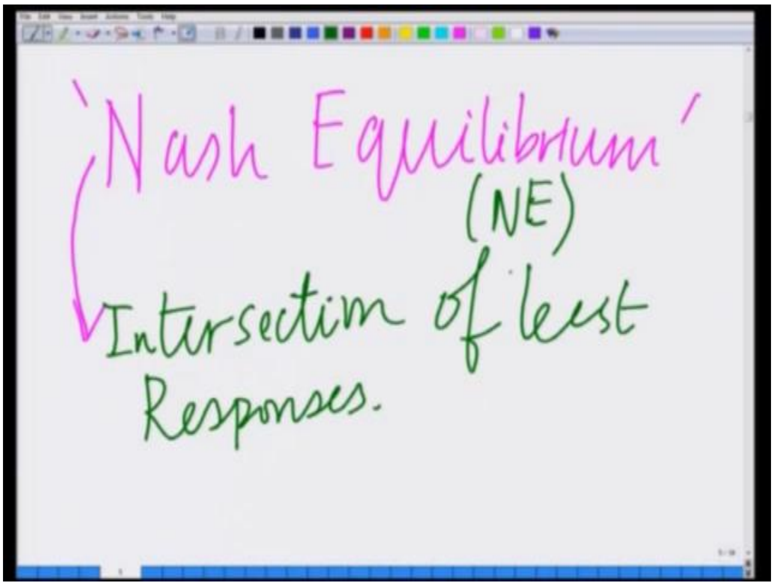

This is known as a Nash which we are going to keep coming back to again and again. And, this is a very important concept; a concept on which the entire theory of games is based is the concept of a Nash equilibrium. What is a Nash equilibrium? Simply put a Nash equilibrium is the intersection of, intersection; that is to say that each player is playing his best response to the actions of all the other players.

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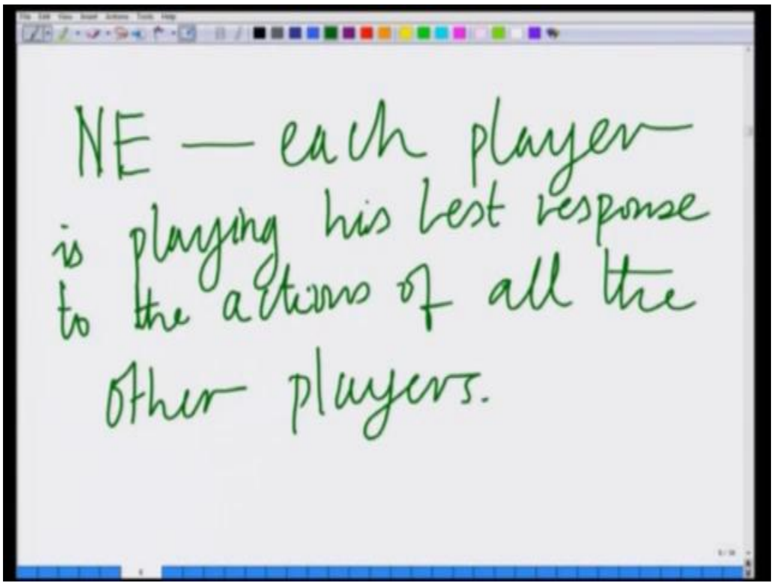

That is each player in a Nash equilibrium which we are going to denote by NE. This Nash equilibrium is frequently abbreviated by NE. In NE each player is playing his best response to the actions of all the other players. Indeed, because if you go back, again look at it, c comma c is a nash equilibrium because for player 1 c is a best response to c of player 2; and for player 2 for a player 2's perspective c is his best response to c of player 1. So, each player is playing the best response to the other, right.

So, this is the place where the best responses are intersecting and therefore, this is known as the nash equilibrium. Because the best response of prisoner 1 to c of prisoner 2 is c, and the best response of prisoner 2 to c of prisoner 1 is c. So, this is known as the nash equilibrium where the best responses are intersecting; that is each player is playing his best response to the actions of all the other players.

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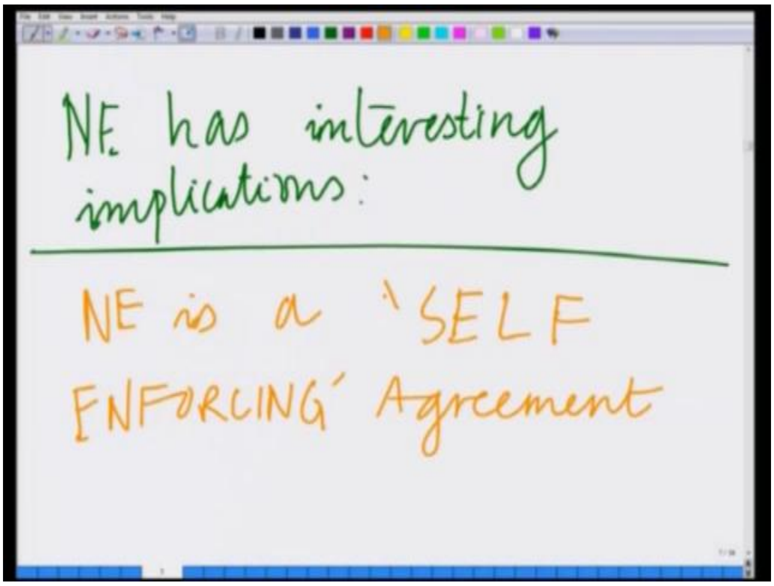

And this nash equilibrium has interesting implications. NE, the nash equilibrium has interesting implications. This nash equilibrium can also be understood intuitively as a self sustaining outcome. NE is a self sustaining is; the NE is basically a self enforcing; this can also be thought of as a self enforcing. What do we mean by a self enforcing agreement? A self enforcing agreement is basically a sort of agreement which does not need any other extraneous agency to enforce it, try to understand it better.

If you go back and take a look at this game, let us for instance assume that before going into their interrogation rooms, prisoner 1 and prisoner 2, agree to choose the outcome deny, deny, which is better for both of them because each one receives only a 1 year prison sentence. Is this agreement self enforcing? Well, not really.

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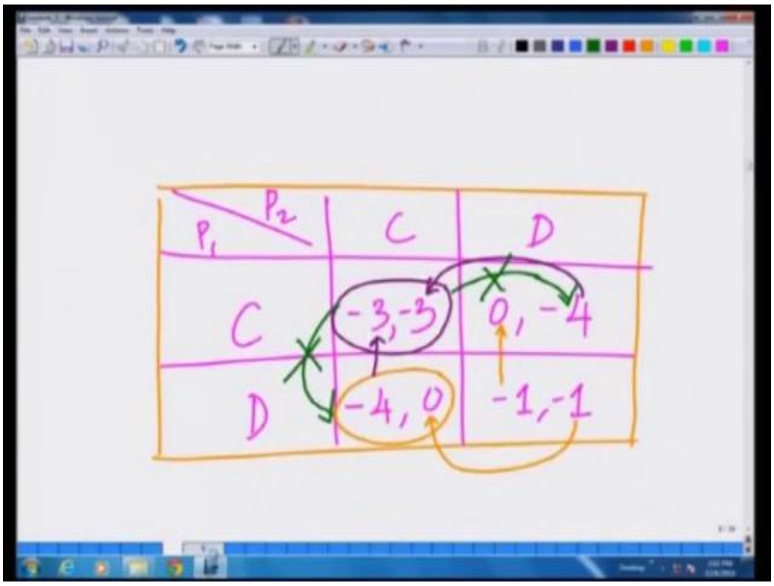

Let us write this game again clearly so that I can illustrate my point which is basically C D, C D, p 1, p 2, minus 3, minus 3, minus 1, minus 1, 0, minus 4, minus 4, 0, is this agreement self enforcing? If both the player choose on D, is this agreement self enforcing? You can clearly see, no; because the moment they go into their interrogation rooms prisoner 1 has an incentive to deviate; he was an incentive to deviate to confess which will give him 0 years in prison.

In fact, prisoner 2 also has an incentive to deviate which will change or reduces prison sentence from 1 year to 0. So, even though they agree that they are going to choose D, this agreement is not self sustaining because the moment they go into their interrogation rooms they have an, each one has an incentive to deviate.

How about D C if prisoner 1 chooses and prisoner 2 agree on D C? Well, clearly you can see that in this scenario prisoner 1 is not happy because if he chooses D he gets 4 years in prison. So, he has an incentive to deviate to confess; that is if he is choosing the initially agree that prisoner 1 is going to deny and prisoner 2 is going to confess, prisoner 1 is going to have an incentive to confess. So, it is not a self sustaining agreement.

Similarly, you can clearly see again C comma D is also not a self sustaining agreement. Because if prisoner 1 chooses C, prisoner 2 chooses D prisoner 2 has an incentive to deviate to C. So, the only self sustaining agreement is in fact, c comma c, why? Because if both prisoner 1 and prisoner 2 agree; prior to the interrogation if they agree that they are going to confess then they are going to indeed confess because neither has an incentive to deviate because if prisoner 2 deviates from C to D he ends up increasing his prison sentence to minus 4. Therefore, he is not; there is no incentive to deviate for prisoner 2.

Similarly, for prisoner 1 if he chooses to deviate from C to D, he is going to end up increasing his prison sentence from minus from 3 years to 4 years. So, he has no incentive. So, you can clearly see, C C is the only outcome; the action outcome or the strategy outcome in this game from which no prisoner has an incentive to unilaterally deviate that is deviate from this outcome. Therefore, C C is a self-enforcing agreement. That is if both prisoners agree on C C they are indeed going to choose C C in their interrogation room.

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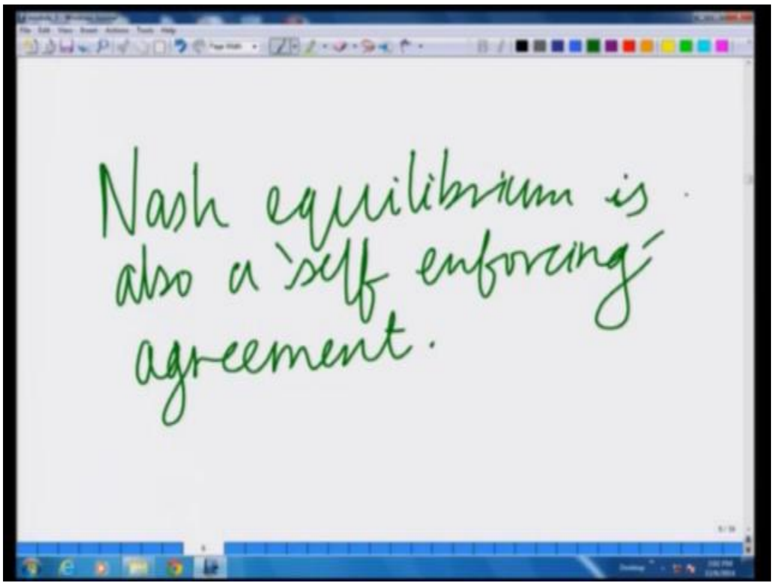

And therefore, the Nash equilibrium is also a self enforcing agreement.

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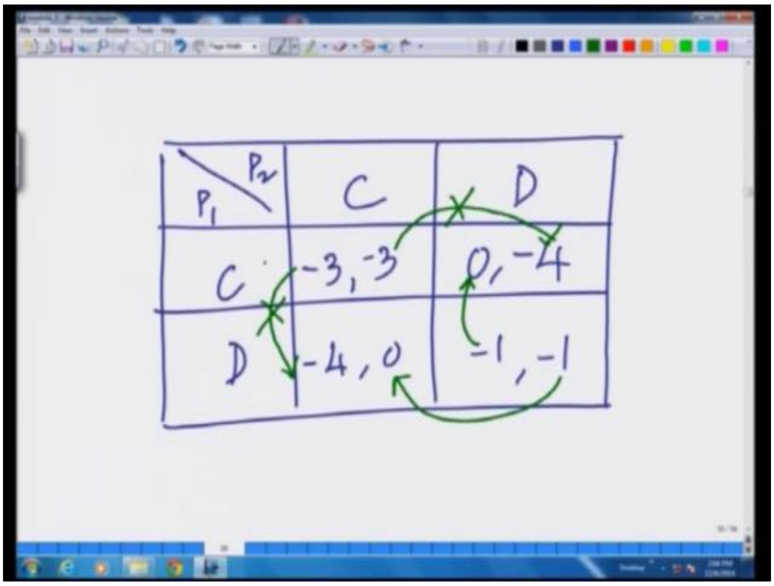

What is another way to talk about a Nash equilibrium? A Nash equilibrium also as we said is that is where no player has an incentive to deviate unilaterally, right. Let us talk about that again. I mean, although we have talked about it let us try to go through that again. Let me just draw the game table, right; C D, C D, p 1, p 2, minus 3, minus 3, minus 1, minus 1, minus 4, 0, 0, minus 4.

If you look at outcome C C then prisoner 2 does not have an incentive to deviate because if he deviates from C to D he can get minus 4; he is going to increase his prison sentence to 4 years. Similarly, if prisoner 1 deviates from C to D he is going to increase his prison sentence to minus 4. So, he does not have an incentive to deviate.

How about D D? From D D both have an incentive to deviate because from D D prisoner 2 can deviate to 0, prisoner 1 can also deviate to confess. Similarly, how about C D that is confess for prisoner 1, deny for prisoner 2? From this prisoner 2 has an incentive to deviate to confess. Same thing with deny for prisoner 1 and confess for prisoner 2. Prisoner 1 has an incentive to deviate from deny to confess to reduce his prison sentence from 4 years to 3 years, right.

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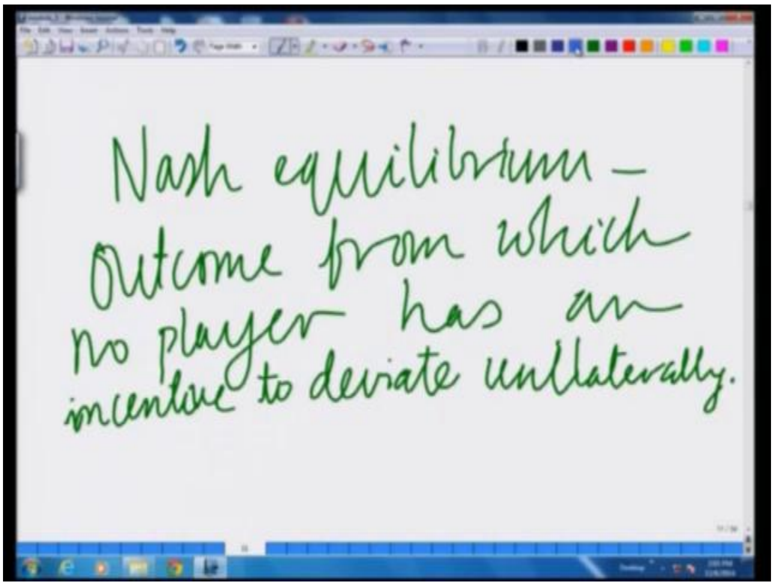

So, the only possible outcome from which no one has an incentive to deviate unilaterally. Unilaterally means by himself, is the outcome confess, confess. So, Nash equilibrium is also frequently stated as outcome from which no player has an incentive to deviate unilaterally. So, a Nash equilibrium is an outcome from which no player has an incentive to deviate unilaterally. That is the keyword here, unilaterally, no player has an incentive to deviate unilaterally.

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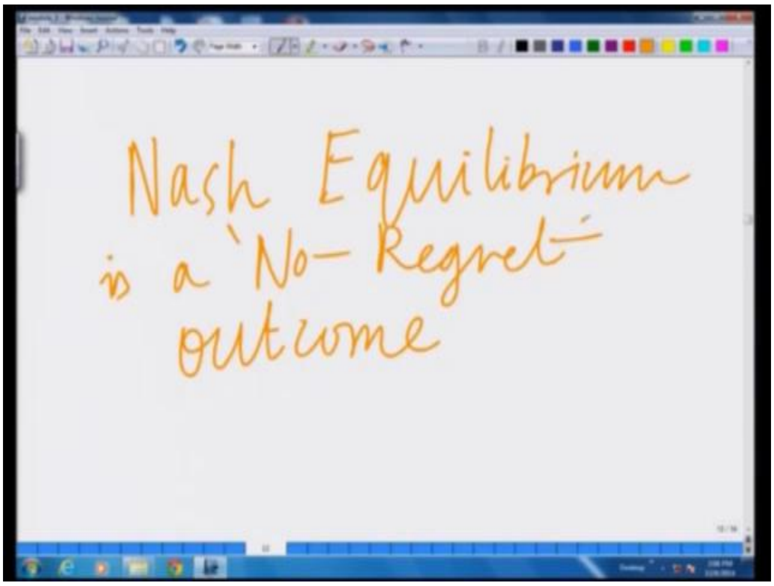

Another way to look at the Nash equilibrium is that the Nash equilibrium is a no regret. What do you mean by no regret outcome? Let us go and look at it again.

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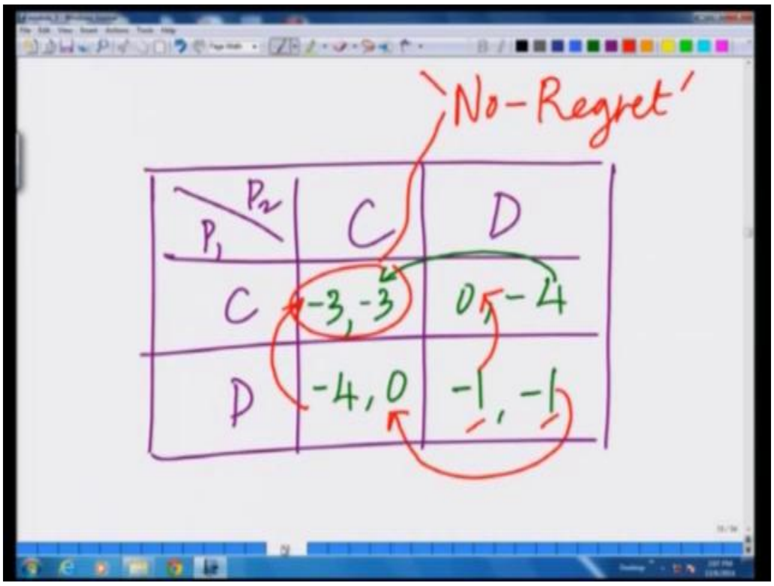

If you look at the game of prisoners dilemma you can again see that if the outcome is confess, confess then none of prisoner 1 or prisoner 2 have any regrets. Because you cannot further improve your pay off. But if the outcome is confessed, denied then prisoner 2 has a regret because he can always he feels he should have chosen confess instead. Similarly, if the outcome is denied, confess then prisoner 1 has a regret because he feels he should have chosen confess rather than deny.

And, of course, if the outcome is deny, deny then both of them have a regret; both of them feel that prisoner 1 feels he should have chosen confess, prisoner 2 feels he should have also chosen confess. Therefore, in this case in any other outcome other than confess, confess both of them have regret. So, C comma C is the only no regret outcome. This is the only no regret outcome which is the Nash equilibrium of which is the only no regret outcome which is the Nash equilibrium of this game, right.

So, there are multiple ways to understand a Nash equilibrium. Nash equilibrium is fundamentally where the best responses intersect each player; each individual player is playing his best response to the collective actions of all the other players. And this is true of each every individual player; not just 1 player, but every individual player is playing his best response. As a result, this is the self enforcing or a self fulfilling or a self enforcing agreement.

This is also an outcome from which none of the players have an incentive to deviate unilaterally, right. And also, this is also a no regret outcome. In this outcome none of the players have any regrets that they could have improved their payoff or utility by choosing a different action, right. So, a Nash equilibrium has all this interesting interpretations and is one of the fundamental most important concepts of a game; or, one of the most important tools used to analyze or one of the most important paradigms used to analyze a game.

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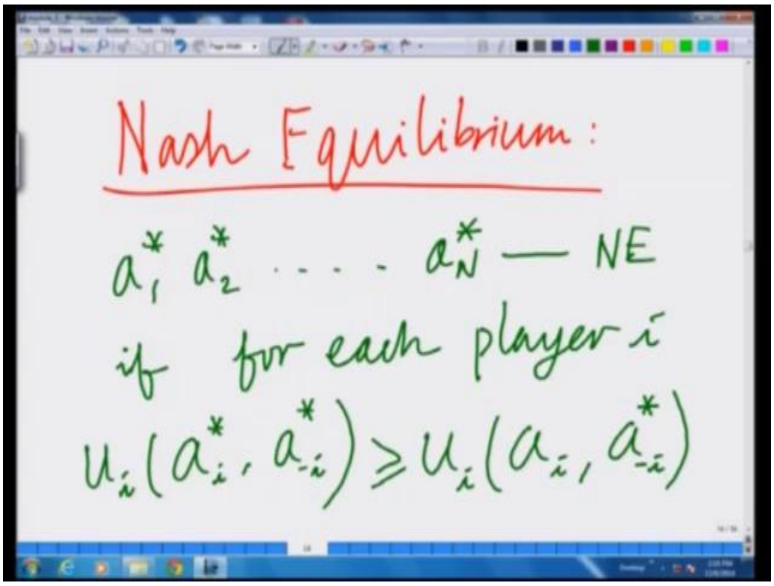

Let us come, let us formally define the Nash equilibrium before closing this module. We call an action profile, a 1 star, a 2 star, so on, a n star, as a Nash equilibrium. If this is a Nash equilibrium, for each player i his payoff u i of a i star comma a minus i star, remember this is the notation that we use, a i star is action of player i an equilibrium action, a minus i star is the equilibrium action for all the other players, this is greater than or equal to the payoff of player i from any other action i give for a fixed action profile a minus i star of all the other players.

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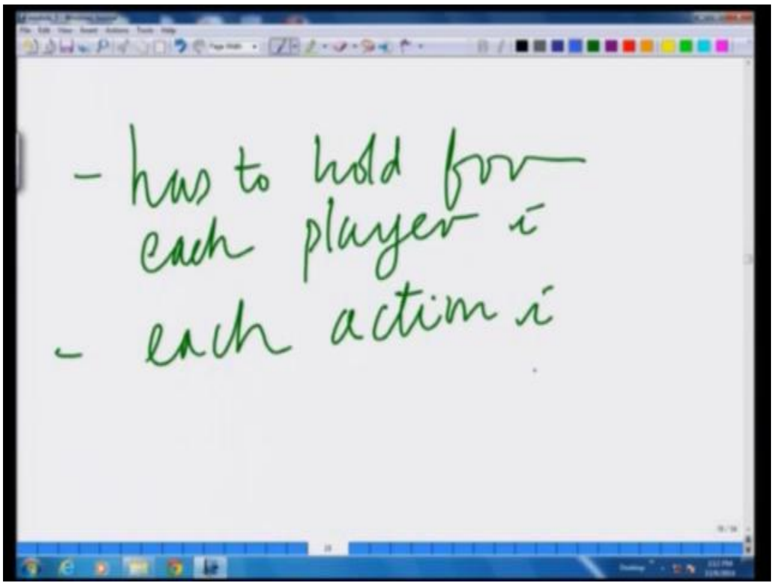

And, this has to hold for each player i and each action a i. Let us think about this. What does it mean? It means that a i star that is given a minus i star, fixed a minus i star of all the other players, a i star yields a higher payoff compared to any action, any other action a i for player i. So, therefore, means a i star is the best response. This means, a i star is the best response to, a minus i star.

So, player i is playing his best response a i star. And, this has to hold for all the players i which means each player has to play his best response. So, player i is playing the best response, and this is true for all the players i, that is all players are playing their best response. So, this is indeed the definition of the nash equilibrium.

So, the nash equilibrium can be represented mathematically using u i a i star that is we call up strategy profile or natural profile a 1 star, a 2 star, upto a n star, in an n player game; remember, I forgot to mention this, in a general n player game; we call this as a nash equilibrium if u i of a i star, a minus i star is greater than or equal to u i. The payoff from any action a i comma the fixed action profile a minus i star of all the other players. Which means a i star is the best response to a minus i star.

And, this is true for all the players i. That is each player i is playing his best response. So, this is the definition of the nash equilibrium, alright. So, this completes this module. This is slightly important module because we developed several, start up with the most important concept of game theory that is the nash equilibrium. So, try to understand it thoroughly before you proceed on to the next modules because this is an idea, key idea that we are going to keep coming back again and again to analyze the properties of games, the behavior of these different agents or these different players in these various games.

Thank you, thank you very much, and we will conclude this module here. Thank you.